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A Trident Fuzzy Number and its Arithmetic Operations

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ABSTRACT: This paper introduces the new concept called the Trident Fuzzy Number. Here the Trident Fuzzy Number along with different forms of Trident Fuzzy Numbers are discussed, the α -cuts and its arithmetic operations for all forms of Trident Fuzzy Number is also discussed with a suitable numerical example.

KEYWORDS: Trident Fuzzy Number, alpha-cuts, membership function, Arithmetic Operation.

I. INTRODUCTION

This paper introduces the new concept called the Trident Fuzzy Number; the $\alpha - cuts$ and the different forms of Trident Fuzzy Numbers along with its arithmetical operations are discussed with a suitable numerical example. Here in this paper, there are four sections: first section deals with Introduction, the second section deals with the Representation of Trident Fuzzy Number, third section deal with different forms of Trident Fuzzy Number such as Triangular Trident Fuzzy Number, Trapezoidal Trident Fuzzy Number and Pentagonal Fuzzy Number etc., its alpha-cuts and Arithmetic Operations for these Trident Fuzzy Numbers with a suitable numerical example and finally the conclusion based on our study.

II. RELATED WORKS

Fuzzy Sets are introduced by Lotfi.A.Zadeh [1] (1965). The representation and application of fuzzy numbers are introduced by S.Hilpern [2] (1997). Later Shan-Huo Chen and Chin Hsun Hseih [3](2000) gives an idea about the Representation, Ranking, Distance and Similarity of L-R type fuzzy number and applications. Christer Carlsson and Rober Fuller[4] (2001) gives an idea on possibilistic mean value and variance of fuzzy numbers. Later Chen and Wang [5] (2008) introduced Fuzzy distance of trapezoidal fuzzy numbers. A.Nagoorgani [6] (2012) deals with a new operation on triangular fuzzy number for solving fuzzy linear programming problem and in the same year arithmetic operations on generalized trapezoidal fuzzy number and its applications is given by Sanhita Banerjee and Tapan Kumar Roy [7] and Salim Rezvani [8] introduced the ideaabout a new method for rank, mode, divergence and spread of generalized exponential trapezoidal fuzzy numbers. Then Rajarajeswari and Sahaya Sudha.A [9] (2013) introduced the ranking of hexagonal fuzzy numbers for solving objective fuzzy linear programming problem. T.Pathinathan and K.Ponnivalavan [10] (2015) introduced the reverse order triangular and pentagonal fuzzy numbers and in the same year A. Felix and A.Victor Devadoss [11] introduced decagonal fuzzy number under uncertain linguistic environment and also A.Thamaraiselvi and R.Shanthi [12] introduced to solve fuzzy transportation problem with generalized hexagonal fuzzy numbers.A.PraveenPrakash and M.GeethaLakshmi [13] (2016) introduced the sub-trident ranking using fuzzy numbers.

III. REPRESENTATION OF GENERALIZED TRIDENT FUZZY NUMBER

A Trident Fuzzy Number $\tilde{A}$ of any fuzzy subset of the real line $\Re$, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions: [11]

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i. $\mu_{\tilde{A}}$ is a continuous mapping from $\Re$ to [0,1].

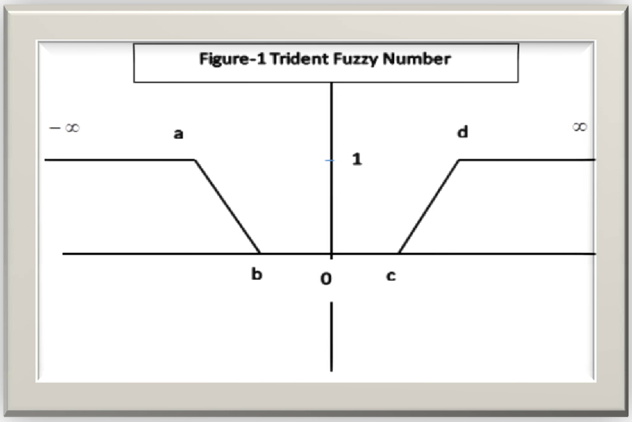
ii. $\mu_{\tilde{A}}(x) = 1^{1/3}, -\infty < x \le a_T$.

iii. $\mu_{\tilde{A}}(x) = [L(x)]^{1/3}$ is strictly decreasing on $[a_T, b_T]$.

iv. $\mu_{\tilde{A}}(x) = w^{1/3}, b_T \le x \le c_T$.

v. $\mu_{\tilde{A}}(x) = [R(x)]^{1/3}$ is strictly increasing on $[c_T, d_T]$.

vi. $\mu_{\tilde{A}}(x) = 1^{1/3}, d_T \le x \le \infty$, where $0 \le w < 1$ and a_T, b_T, c_T, d_T are real numbers.



We denote this type of generalized Trident Fuzzy Number as $\tilde{A} = (a_T, b_T, c_T, d_T; w)_{LR}$. When w=0 , we denote this type of generalized Trident Fuzzy Number as $\tilde{A} = (a_T, b_T, c_T, d_T)_{LR}$. When L(x) and R(x) are straight line then $\tilde{A}$ is Trapezoidal Trident Fuzzy Number and we denote it as (a_T, b_T, c_T, d_T) [3].

IV. DIFFERENT FORMS OF FUZZY NUMBERS, ITS ALPHA CUT, AND ITS ARITHMETIC OPERATIONS

The Different Forms of Trident Fuzzy Numbers are as follows:

A. TRIANGULAR TRIDENT FUZZY NUMBER(T_RTFN)

A Triangular Trident Fuzzy Number is given by $\tilde{A} = (a_{1_T}, a_{2_T}, a_{3_T})$ where all $a_{1_T}, a_{2_T}, a_{3_T}$ are real numbers and its membership function is represented as follows:

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$$\mu_{\tilde{A}}(x) = \begin{cases} 1^{1/3} & for \quad x < a_{1_T} \\ \left[\dfrac{a_{2_T} - x}{a_{2_T} - a_{1_T}}\right]^{1/3} & for \quad a_{1_T} \le x \le a_{2_T} \\ 0 & for \quad x = a_{2_T} \\ \left[\dfrac{x - a_{2_T}}{x - a_{2_T}}\right]^{1/3} & for \quad a_{2_T} \le x \le a_{3_T} \\ 1^{1/3} & for \quad x > a_{3_T} \end{cases}$$

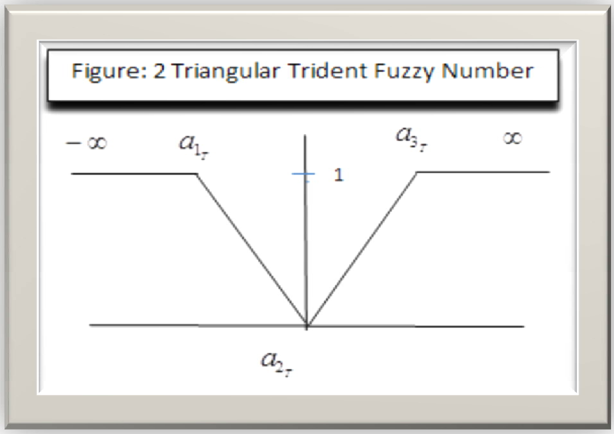

Figure: 2 Triangular Trident Fuzzy Number

Example: 1

A Triangular Trident Fuzzy Number (TT$_{ri}$FN) $\mu_{\tilde{A}}(x) = (-1,0,1)$ and its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & for \quad x < -1 \\ [-x]^{1/3} & for \quad -1 \le x \le 0 \\ 0 & for \quad x = 0 \\ [x]^{1/3} & for \quad 0 \le x \le 1 \\ 1 & for \quad x > 1 \end{cases}$$

i. $\alpha - cut$ for Triangular Trident Fuzzy Number

The $\alpha - cut$ of Trapezoidal Trident Fuzzy Number is given by $[-x]^{1/3} = \alpha$ and $[x]^{1/3} = \alpha$. for example, $\tilde{A}_\alpha = (-\alpha^3, \alpha^3)$. If $\alpha = 0.5$ then $\tilde{A}_{0.5} = (-0.125, 0.125)$ and if $\alpha = 1$ then $\tilde{A}_1 = (-1,1)$.

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ii. Arithmetic Operations for Triangular Trident Fuzzy Number

- **Addition for Two Triangular Trident Fuzzy Number**

  If $\tilde{A}_T = (a_{1_T}, a_{2_T}, a_{3_T}), \tilde{B}_T = (b_{1_T}, b_{2_T}, b_{3_T})$ are two Triangular Trident Fuzzy Number then the addition is given $\tilde{A}_T + \tilde{B}_T = (a_{1_T} + b_{1_T}, a_{2_T} + b_{2_T}, a_{3_T} + b_{3_T})$.

  Example: 1.1

  If $\tilde{A}_T = (-0.8, 0.0.6), \tilde{B}_T = (-0.2, 0.0.4)$ are two Triangular Trident Fuzzy Number then $\tilde{A}_T + \tilde{B}_T = (-1, 0, 1)$.

- **Subtraction for Two Triangular Trident Fuzzy Number**

  If $\tilde{A}_T = (a_{1_T}, a_{2_T}, a_{3_T}), \tilde{B}_T = (b_{1_T}, b_{2_T}, b_{3_T})$ are two Triangular Trident Fuzzy Number then the subtraction is given by $\tilde{A}_T - \tilde{B}_T = (a_{1_T} - b_{1_T}, a_{2_T} - b_{2_T}, a_{3_T} - b_{3_T})$.

  Example: 1.2

  If $\tilde{A}_T = (-0.4, 0.0.6), \tilde{B}_T = (-0.2, 0.0.4)$ are two Triangular Trident Fuzzy Number then $\tilde{A}_T - \tilde{B}_T = (-0.2, 0, 0.2)$.

A. TRAPEZOIDAL TRIDENT FUZZY NUMBER($T_RT_{RI}FN$)

A Trapezoidal Trident Fuzzy Number is given by $\tilde{A} = (a_{1_T}, a_{2_T}, a_{3_T}, a_{4_T})$ where all $a_{1_T}, a_{2_T}, a_{3_T} a_{4_T}$ are real numbers and its membership function is represented as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1^{1/3} & for \quad x < a_{1_T} \\ \left[\dfrac{a_{2_T} - x}{a_{2_T} - a_{1_T}}\right]^{1/3} & for \quad a_{1_T} \le x \le a_{2_T} \\ 0 & for \quad a_{2_T} \le x \le a_{3_T} \\ \left[\dfrac{x - a_{3_T}}{a_{4_T} - a_{3_T}}\right]^{1/3} & for \quad a_{3_T} \le x \le a_{4_T} \\ 1^{1/3} & for \quad x > a_{4_T} \end{cases}$$

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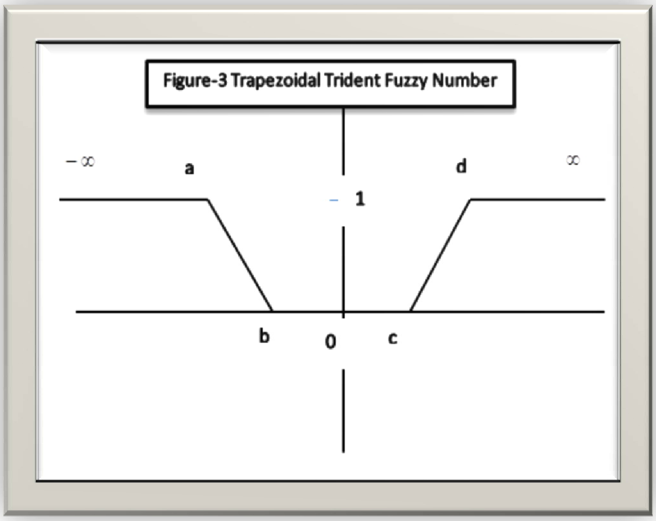

Example: 2

A Trapezoidal Trident Fuzzy Number ($T_rT_{ri}FN$) $\mu_{\tilde{A}}(x) = (-1,-0.8,0.6,1)$ and its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & for \quad x < -1 \\ \left[\dfrac{-0.8-x}{0.2}\right]^{1/3} & for \quad -1 \le x \le -0.8 \\ 0 & for \quad -0.8 \le x \le -0.6 \\ \left[\dfrac{x-0.6}{0.4}\right]^{1/3} & for \quad 0.6 \le x \le 1 \\ 1 & for \quad x > 1 \end{cases}$$

i. $\alpha - cut$ for Trapezoidal Trident Fuzzy Number

The $\alpha - cut$ of Trapezoidal Trident Fuzzy Number is given by $\left[\dfrac{-0.8-x}{0.2}\right]^{1/3} = \alpha$ and $\left[\dfrac{x-0.6}{0.4}\right]^{1/3} = \alpha.$

For example, $\tilde{A}_\alpha = \left[-(0.2\alpha^3 + 0.8), 0.4\alpha^3 + 0.6\right].$

If $\alpha = 0.4$ then $\tilde{A}_{0.4} = (-0.8128, 0.6256)$ and if $\alpha = 0.9$ then $\tilde{A}_{0.9} = (-0.9458, 0.8916).$

ii. Arithmetic Operations for Trapezoidal Trident Fuzzy Number

- **Addition for Two Trapezoidal Trident Fuzzy Number**

  If $\tilde{A}_T = (a_{1_T}, a_{2_T}, a_{3_T}, a_{4_T}), \tilde{B}_T = (b_{1_T}, b_{2_T}, b_{3_T}, b_{4_T})$ are two Trapezoidal Trident Fuzzy Number then the addition is given $\tilde{A}_T + \tilde{B}_T = (a_{1_T} + b_{1_T}, a_{2_T} + b_{2_T}, a_{3_T} + b_{3_T}, a_{4_T} + b_{4_T}).$

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Example: 2.1

If $\tilde{A}_T = (-0.8,-0.6,.0.4,0.6), \tilde{B}_T = (-0.2,-0.3,0.2,0.2)$ are two Trapezoidal Trident Fuzzy Number then $\tilde{A}_T + \tilde{B}_T = (-1,-0.9,0.6,0.8).$

- **Subtraction for Two Trapezoidal Trident Fuzzy Number**

If $\tilde{A}_T = (a_{1_T}, a_{2_T}, a_{3_T}, a_{4_T}), \tilde{B}_T = (b_{1_T}, b_{2_T}, b_{3_T}, b_{4_T})$ are two Trapezoidal Trident Fuzzy Number then the subtraction is given by $\tilde{A}_T - \tilde{B}_T = (a_{1_T} - b_{1_T}, a_{2_T} - b_{2_T}, a_{3_T} - b_{3_T}, a_{4_T} - b_{4_T}).$

Example: 2.2

If $\tilde{A}_T = (-0.6,-0.4,0.8,0.9), \tilde{B}_T = (-0.3,-0.4,0.6.0.8)$ are two Triangular Trident Fuzzy Number then $\tilde{A}_T - \tilde{B}_T = (-0.3,0,0.2,0.1).$

C. PENTAGONAL TRIDENT FUZZY NUMBER ($PT_{RI}FN$)

A Pentagonal Trident Fuzzy Number is given by $\tilde{A} = (a_{1_T}, a_{2_T}, a_{3_T}, a_{4_T}, a_{5_T})$ where all $a_{1_T}, a_{2_T}, a_{3_T} a_{4_T}, a_{5_T}$ are real numbers and its membership function is represented as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1^{1/3} & for \quad x < a_{1_T} \\ \left[\dfrac{a_{2_T} - x}{a_{2_T} - a_{1_T}}\right]^{1/3} & for \quad a_{1_T} \le x \le a_{2_T} \\ \left[\dfrac{a_{3_T} - x}{a_{3_T} - a_{2_T}}\right]^{1/3} & for \quad a_{2_T} \le x \le a_{3_T} \\ 0 & for \quad x = a_{3_T} \\ \left[\dfrac{x - a_{3_T}}{a_{4_T} - a_{3_T}}\right]^{1/3} & for \quad a_{3_T} \le x \le a_{4_T} \\ \left[\dfrac{x - a_{4_T}}{a_{5_T} - a_{4_T}}\right]^{1/3} & for \quad a_{4_T} \le x \le a_{5_T} \\ 1^{1/3} & for \quad x > a_{5_T} \end{cases}$$

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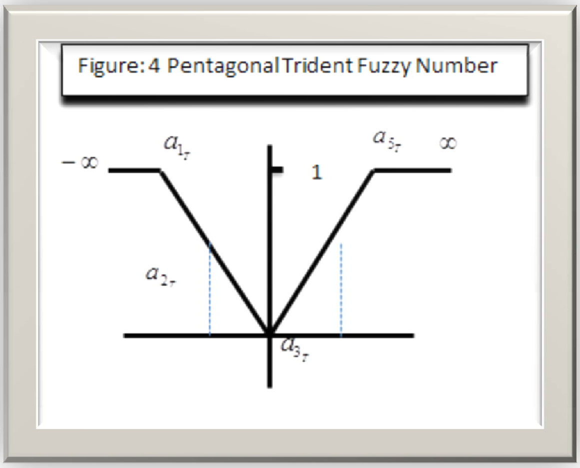

Figure: 4 Pentagonal Trident Fuzzy Number

Example: 3

A Pentagonal Trident Fuzzy Number (PT$_{ri}$FN) $\mu_{\tilde{A}}(x) = (-1,-0.6,0.7,0.8)$ and its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & for \quad x < -1 \\ \left[\dfrac{-0.6-x}{0.4}\right]^{1/3} & for \quad -1 \le x \le -0.6 \\ \left[\dfrac{-x}{0.6}\right]^{1/3} & for \quad -0.6 \le x \le 0 \\ 0 & for \quad x = 0 \\ \left[\dfrac{x}{0.7}\right]^{1/3} & for \quad 0 \le x \le 0.7 \\ \left[\dfrac{x-0.7}{0.1}\right]^{1/3} & for \quad 0.7 \le x \le 0.8 \\ 1 & for \quad x > 0.8 \end{cases}$$

A. $\alpha - cut$ **for Pentagonal Trident Fuzzy Number**

The $\alpha - cut$ of Pentagonal Trident Fuzzy Number is given by $\left[\dfrac{-0.6-x}{0.4}\right]^{1/3} = \alpha$, $\left[\dfrac{-x}{0.6}\right]^{1/3} = \alpha, \left[\dfrac{x}{0.7}\right]^{1/3} = \alpha, \left[\dfrac{x-0.7}{0.1}\right]^{1/3} = \alpha$

for example, $\tilde{A}_\alpha = \begin{cases} \left[-(0.4\alpha^3 + 0.6), 0.6\alpha^3\right], -1 \le x \le 0 \\ \left[0.7\alpha^3, 0.1\alpha^3 + 0.7\right] \quad ,0 \le x \le 0.8 \end{cases}$

If $\alpha = 0.2$ then $\tilde{A}_{0.2} = \begin{cases} [-0.6032, -0.0048], -1 \le x \le 0 \\ [0.0056, 0.7008] \quad, 0 \le x \le 0.8 \end{cases}$

and if $\alpha = 0.1$ then $\tilde{A}_{0.1} = \begin{cases} [-0.6004, -.0006], -1 \le x \le 0 \\ [0.0007, 0.7001] \quad, 0 \le x \le 0.8 \end{cases}$

ii. Arithmetic Operations for Pentagonal Trident Fuzzy Number

- **Addition for Two Pentagonal Trident Fuzzy Number**

If $\tilde{A}_T = (a_{1_T}, a_{2_T}, a_{3_T}, a_{4_T}, a_{5_T}), \tilde{B}_T = (b_{1_T}, b_{2_T}, b_{3_T}, b_{4_T}, b_{5_T})$ are two Pentagonal Trident Fuzzy Number then the addition is given $\tilde{A}_T + \tilde{B}_T = (a_{1_T} + b_{1_T}, a_{2_T} + b_{2_T}, a_{3_T} + b_{3_T}, a_{4_T} + b_{4_T}, a_{5_T} + b_{5_T})$.

Example: 3.1

If $\tilde{A}_T = (-0.6, -0.4, 0, .0.2, 0.4), \tilde{B}_T = (-0.4, -0.2, 0, 0.4, 0.6)$ are two Pentagonal Trident Fuzzy Number then $\tilde{A}_T + \tilde{B}_T = (-1, -0.6, 0, 0.6, 1)$.

- **Subtraction for Two Pentagonal Trident Fuzzy Number**

If $\tilde{A}_T = (a_{1_T}, a_{2_T}, a_{3_T}, a_{4_T}, a_{5_T}), \tilde{B}_T = (b_{1_T}, b_{2_T}, b_{3_T}, b_{4_T}, b_{5_T})$ are two Trapezoidal Trident Fuzzy Number then the subtraction is given by $\tilde{A}_T - \tilde{B}_T = (a_{1_T} - b_{1_T}, a_{2_T} - b_{2_T}, a_{3_T} - b_{3_T}, a_{4_T} - b_{4_T}, a_{5_T} - b_{5_T})$.

Example: 3.2

If $\tilde{A}_T = (-0.6, -0.4, 0.8, 0.9), \tilde{B}_T = (-0.3, -0.4, 0.6.0.8)$ are two Triangular Trident Fuzzy Number then $\tilde{A}_T - \tilde{B}_T = (-0.3, 0, 0.2, 0.1)$.

V. CONCLUSION

This paper aims to introduce the new concept called the Trident Fuzzy Number. The advantage of this proposed method is to get a more accurate result, so that the vagueness will be reduced. Also the Trident Fuzzy Number is very simple and easy to apply in problems.

REFERENCES

1. Lotfi.A.Zadeh, "Fuzzy Sets", Information and Control, vol.8, pp.338-353, 1965.
2. S.Hilpern, "Representation and application of fuzzy numbers", Fuzzy Sets and Systems, Vol.1, No.2, pp.259-268, 1997.
3. Shan-Huo Chen and Chin Hsun Hseih, "Representation, ranking, distance and similarity of L-R type fuzzy number and applications", Australian Journal of Intelligent Information Processing Systems: Australia, vol.6, n0.4, pp.217-229,2000.
4. ChristerCarlsson and Rober Fuller, "On possibilistic mean value and variance of fuzzy numbers",Fuzzy Sets and Systems,Vol.122,pp.315-326,2001.
5. Chen and Wang, "Fuzzy distance of trapezoidal fuzzy numbers and applications", International Journal of Innovative Computing, Information and Control Vol.4, No.6, pp.1445-1454, 2008.
6. A.Nagoorgani, "A new operation on triangular fuzzy number for solving fuzzy linear programming problem", Applied Mathematical Sciences, Vol.6, No.11, pp.525-532, 2012.
7. Sanhita Banerjee, Tapan Kumar Roy, "Arithmetic operations on generalized trapezoidal fuzzy number and its applications", Turkish Journal of Fuzzy Systems, Vol.3, No.1, pp.16-44, 2012.

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8. Salim Rezvani, "A new method for rank, mode, divergence and spread on generalized exponential trapezoidal fuzzy numbers", Turkish Journal of Fuzzy Systems, vol.3, No.2, pp.98-107, 2012.
9. Rajarajeswari and Sahaya Sudha.A, "Ranking of hexagonal fuzzy numbers for solving objective fuzzy linear programming problem", International Journal of Computer Applications, Vol.84, No.8, 2013.
10. T.Pathinathan and K.Ponnivalavan, "Reverse order triangular and pentagonal fuzzy numbers", Annals of Pure and Applied Mathematics, Vol.9, No.1, pp.107-117, 2015.
11. A. Felix and A.Victor Devadoss, "A new decagonal fuzzy number under uncertain linguistic environment", International Journal of Mathematics and its application: Vol.3, pp.89-97, 2015.
12. A.Thamaraiselvi and R.Shanthi, "Solving fuzzy transportation problem with generalized hexagonal fuzzy numbers", ISOR Journal of Mathematics Vol.11, No.5, pp.8-13, 2015.
13. A.PraveenPrakash and M.GeethaLakshmi, "sub-trident ranking using fuzzy numbers", International Journal of Mathematics And its Applications, Vol.4, No.2, pp.143-150, 2016.