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Algorithms for Finding an Efficient Dominating Set of an Interval Graphand a Split Dominating Set of a Circular-Arc Graph

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ABSTRACT: Interval graphs are rich in combinatorial structures and have found applications in several disciplines such as traffic control, Ecology, Biology, Computer Science etc. .Circular-arc graphs are introduced as generalization of Interval graphs. If we bend the real line into a circle, then any family of intervals of the real line is transformed into a family of arcs of the circle. In this paper we are presenting an algorithm for finding an efficient dominating set of an interval family if it exists and Also an algorithm for finding a non- split dominating set of circular-arc graph.

KEYWORDS :Interval Graph, Circular-arc graph, Dominating set, Dominating number, Efficient Dominating set, Split dominating set.

I.INTRODUCTION

Let $I = \{I_1, I_2, ..., I_n\}$ be an interval family. Each interval I_i in I has represented by $I_i = [a_i, b_i], for\ i = 1, 2, ..., n$. Here a_i is called the left endpoint and b_i is called the right end point of I_i . Without loss of generality , we may assume that all endpoints of the intervals in I are distinct numbers between 1 and 2n. The intervals are labeled in the increasing order of their right endpoints. Two intervals I_i and I_j are said to be intersect each other if they have non-empty intersection. An interval family I is said to be proper if no interval in I is contained in another interval.

A graph $G = (V, E)$ is called an interval graph if there is a one-to-one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I intersect. Let G be a graph with vertex set V and edge set E .

The open neighbourhood set of a vertex $v \in V$

$N(v)$ or $nbd(v) = \{u \in V / uv \in E\}$

The closed neighbourhood set of a vertex $v \in V$

$N[v]$ or $nbd[v] = nbd(v) \cup \{v\}$

The neighbourhoodnumber[1] of G is defined as the minimum cardinality of a neighbourhood set of G . A vertex in a graph G dominates itself and its neighbours. A subset D of V is said to be a dominating set of G if every vertex in $< V - D >$ is adjacent to some vertex in D . The theory of domination in graphs was introduced by Ore and Berge. The domination number γ is the minimum cardinality of a dominating set. A dominating set S of a graph G is called an efficient dominating set[6-7] if $|N[v] \cap S| = 1$, for every vertex $v \in V(G)$. That is, a dominating set S is efficient if and only if every vertex is dominated exactly once.

For each interval i , $nbd[i]$ denotes the set of all intervals which intersects i (including i). Let $\max(i)$ denotes the largest interval in $nbd[i]$. Let us now define $NI(i)$ of the interval i as below.

$NI(i) = j$ if $b_i < a_j$ and there does not exist an interval k such that $b_i < a_k < a_j$. If there is no such j , then $NI(i) = null$.

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Let $A=\{A_1, A_2, ..., A_n\}$ be a family of arcs on a circle C. Each endpoint of the arc A_i is assigned a positive integer called a co-ordinate. The endpoints are located at the circumference of C in ascending order of the values of the co-ordinates in the clockwise direction.

Suppose that an arc begins at c and ends at point d in the clockwise direction. Then we denote such an arc by [c, d] and the points c and d arc called respectively the head point and tail point of the arc. The arcs are given labels in the increasing order of their head points. If the head point of an arc is less than the tail point of the arc then the arc is called a forward arc. Otherwise it is called a backward arc. A is called a proper arc family if no arc in A contains another arc.A graph $G=(V,E)$ is called a circular-arc graph if there is a one - to - one correspondence between V and A such that two vertices in V are adjacent if and only if their corresponding arcs in A intersect.

Let us now denote the arc family by $A=\{1,2,......,n\}$, where arc $i=A_i$ and G is its corresponding Circular-arc graph. We assume that G is a connected graph. Circular. Let $G=(V,E)$ be a graph. A dominating set[2-5] D of a graph $G=(V,E)$ is a split dominating set if the induced sub graph $<V-D>$ is disconnected. The split domination number [9] Υ_s(G) of G is the minimum cardinality of a split dominating set. A dominating set D of a graph $G=(V,E)$ is a non-split dominating set, if the induced sub graph $<V-D>$ is connected . The non-split domination number Υ_{ns}(G) of G is the minimum cardinality of a non-split dominating set.Kulli .V.R et.all [6] introduced the concept of split and non split domination in graphs and also in Maheswari, B et all. Define $nbd^-[i]$ is the set of all backward intersecting arcs to i and $nbd^+[i]$ is the set of all forward intersecting arcs to i and contained in i. Define $d^-(i)$ is the number of backward intersecting arcs to i and $d^+[i]$ is the number of all forward intersecting arcs to i and contained in i .Define $fAdj(i)$ is the arc which **is** first contained in i. If there is no contained arc in i then $fAdj(i)$ is the arc which **is** first forward intersecting arc to i. Define $f\max(i)$ is the maximum arc in $nbd^+[i]$

An algorithm is a step by step specification on how to perform a certain task. The steps in the algorithm must be simple, unambiguous and be followed in a prescribed order. Further , we will insist that algorithm to be effective. That is, it must always sole the problem in a finite number of steps.

II. PROPOSED ALGORITHM

A.Algorithm for finding an efficient dominating set of an interval graph

Input: Interval family $I=\{1,2,3,......,n\}$

Step 1: *Find* max(1).
Step 2: *If* $d(\max(1))=2$
    Step 2.1: $S=\{1\}$
  Else
    Step 2.2: $S=\max(1)$
Step 3: LI = The largest interval in S.
Step 4: *If* $NI(LI)=null$ then
    goto step 7
  else
    goto step 5

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Step 5: If there exists an interval v in $nbd[\max(NI(LI))]$
which is also belongs to $nbd[LI]$ then
Step 5.1: *Efficient* dominating set does not exists
and goto step 7
Step 6: $S = S \cup \{\max(NI(LI))\}$ goto step 3
Step 7 : *End*

Out Put : An efficient dominating set of an interval graph of given interval family if it exists.

III.THEOREMS

Theorem 1 :

Let G be a graph corresponding to an interval family. Let $D = \{x_1, x_2\}$ be a minimum dominating set x_1 dominates $S_1 = \{1,2,.....,i\}$ x_2 dominates $S_2 = \{i+1,,.....,n\}$ then an efficient domination occurs in G where $S_1 \cap S_2 = \varnothing$ and cardinality $|N(x_2) \cap D| = 1, \forall x_2 \in I$.

Proof:

Let $I = \{I_1, I_2, ..., I_n\}$ be an interval family. Let G be an interval graph of corresponding to an interval family. Let us find the minimum dominating set $D = \{x_1, x_2\}$. Suppose x_1 dominates $S_1 = \{1,2,.....,i\}$ and x_2 dominates $S_2 = \{i+1,,.....,n\}$ when $S_1 \cap S_2 = \varnothing$ ....(A)

We have to show that $|N(x_2) \cap D| = 1, \forall x_2 \in I$

If possible suppose that $|N(x_2) \cap D| \neq 1, \forall x_2 \in I$

This implies that $|N(x_2) \cap D| = \varnothing$, or $|N(x_2) \cap D| > 1$

Take any vertex $r \in S_1$

If $|N(x_2) \cap D| = \varnothing$

$\Rightarrow |N(r) \cap D| = \varnothing$

But r is adjacent to any vertex in D.

So $|N(x_2) \cap D| = \varnothing$ is wrong.

If $|N(x_2) \cap D| > 1$

$\Rightarrow |N(r) \cap D| > 1$

$\Rightarrow$ 'r' is adjacent to both x_1 and x_2 .

This implies x_1 and x_2 dominates r which is contradicts to our hypothesis $S_1 \cap S_2 = \varnothing$

So $|N(x_2) \cap D| > 1$ is wrong.

Therefore $|N(x_2) \cap D| \neq 1, \forall x_2 \in I$ is wrong.

Hence $|N(x_2) \cap D| = 1, \forall x_2 \in I$.

$|N(x_2) \cap D| = 1, \forall x_2 \in I$ implies D is an Efficient dominating set.

Hence the theorem

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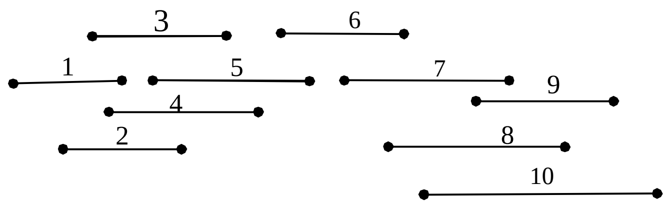
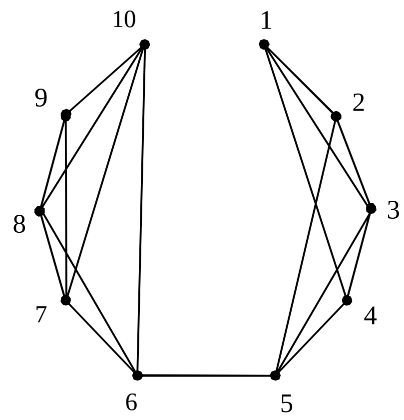
Example



Interval family 1



Interval Graph 1

Procedure

Input: Interval Family $I = \{1, 2, ..., 10\}$

$Step\ 1: \max(1) = 4$

$Step\ 2: S = \{4\}$

$Step\ 3: LI = 4$

$Step\ 4: NI(4) = 6$

$Step\ 5: \max(6) = 10$

$Step\ 6: S = \{4\} \cup \{10\} = \{4, 10\}$ and goto step3

$Step\ 7: LI = 10$

$Step\ 8: NI(10) = null$

$Step\ 9: End$

Output: $\{4, 10\}$ is an efficient dominating set.

Theorem 2:

Let $D = \{s_1, s_2, ..., s_k\}, k \leq n$ be a minimum dominating set of given interval family $I = \{I_1, I_2, ..., I_n\}$. If there exists two intervals $p, q \in D$ such that any interval s_1 intersects both $p, q \in D$ then minimum dominating set is not an efficient dominating set.

Proof:

Given that $I = \{I_1, I_2, ..., I_n\}$ is an interval family and $D = \{s_1, s_2, ..., s_k\}, k \leq n$ is a minimum dominating set.

Suppose there exists two intervals $p, q \in D$ such that any interval s_1 intersects both $p, q \in D$.

That is, $s_1 \in nbd[p]\ and\ s_1 \in nbd[q]$

Now we have to prove that D is not an efficient dominating set.

That is, we have to prove that $|N(s_1) \cap D| \neq 1$

Since $s_1 \in nbd[p]\ and\ s_1 \in nbd[q]$, p, q are neighbors of s_1.

This implies that $p, q \in N(s_1)$.

But $p, q \in D$.

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So $p,q \in N(s_1)$ and $p,q \in D$.

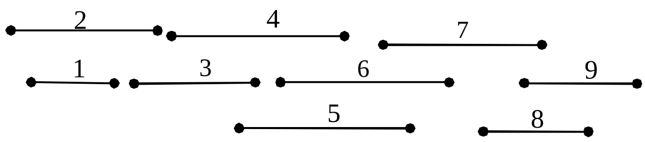
$\Rightarrow p,q \in N(s_1)$ and D

$\Rightarrow p,q \in N(s_1) \cap D$

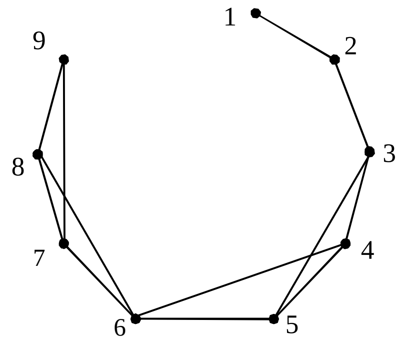
$\Rightarrow |N(s_1) \cap D| = 2 \neq 1$

Therefore D is not an efficient dominating set

Example



Interval Family 2



Interval Graph 2

In the above example D = {2,6,9} is a minimum dominating set and s_1 = 7, p = 6 and q = 9.

$|N(s_1) \cap D| = |\{6,8,9\} \cap \{2,6,9\}| = |\{6,9\}| = 2$.

Therefore {2,6,9} is not an efficient dominating set.

B.Algorithm for finding split dominating set of a circular-arc graph

Algorithm for minimum dominating set of a circular-arc graph.

Input : Circular-arc family

Output : Minimum dominating set of circular-arc graph of given circular-arc family.

$Step\ 1\ :\ T\ = \{\, j \,/\, j\ \in\ nbd^{+}[1]\ \}$.

$Step\ 2\ :\ P = \{\, j \,/\, j \in T\ and\ j \in nbd[k],\ \forall\ k\ in\ T\ \}$.

$Step\ 3\ :\ MDS = \max imum\ vetex\ in\ P$

$Step\ 4\ :\ LI\ =\ The$ largest interval $in\ MDS$.

$Step\ 5 : Find\ NI(LI)$.

$Step\ 6 : If\ NI(LI) < LI\ then\ goto\ Step\ 9$.

$Step\ 7\ :\ Find\ \max(NI(LI))$

$Step\ 8\ :\ MDS = MDS \cup \max(NI(LI))\ goto\ Step\ 4$.

$Step\ 9\ :\ End$.

Algorithm for finding split dominating set of a circular-arc graph

$Step\ 1$: Count = 0.

$Step\ 2\ :\ T\ = \{\, j \,/\, j \in nbd^{+}[1]\ \}$.

$Step\ 3\ :\ P = \{\, j \,/\, j \in T\ and\ j \in nbd[k],\ \forall\ k\ in\ T\ \}$.

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$Step\ 4\ :\ If\ \ d^{-}(1)=1\ and\ there\ is\ no\ (u,v)\ such\ that$

$u \in nbd^{-}(1)\ v \in nbd^{+}(1)$

$Step\ 4.1\ :\ SD=1.$

Else

$Step\ 4.2\ :\ SD\ =\ The$ largest $highest$ deg$ree\ arc\ in\ P.$

$Step\ 5\ :\ a=SD.$

$Step\ 6\ :\ c_1=nil.$

$Step\ 7\ :\ LI=The$ largest $arc\ in\ SD.$

$Step\ 8\ :\ If\ \ Count\ \geq\ 2\ goto\ step\ \ 15$

$Step\ 9\ :\ If\ \ there\ is\ no\ (u,v)\ such\ that\ u \in\ nbd^{-}(LI)\ and$

$v\ \in\ nbd^{+}(LI).$

$Step\ 9.1\ :\ Count\ \ =\ \ Count\ \ +1.$

$Step\ 9.2\ :\ If\ \ Count\ \geq\ 2\ goto\ step\ \ 15$

$Step\ 10\ :\ Find\ p\ \ =\ \ The\ Number\ of\ \ arcs\ in\ nbd[LI]$

$such\ that\ there\ is\ no\ any$ intersecting $arc\ to\ that\ arc\ except\ LI.$

Endif

$Step\ 11\ :\ If\ \ such\ p\ exists\ then$

$Step\ 11.1\ :\ Count\ \ =\ \ Count\ \ +\ p.$

$Step\ 11.2\ :\ If\ \ Count\ \geq\ 2\ goto\ step\ 15$

Endif.

$Step\ 12\ :\ Find\ q\ \ =\ \ The\ Number\ of\ \ vertices\ with\ d^{+}(x)=1\ or\ d^{-}(x)=1,\ d^{+}(x)=1\ and\ \ d^{-}(x)=1$

$such\ that\ x \in nbd^{+}[LI]\ \ and\ \ \ x \neq\ fAdj(LI)\ and\ \ x=\ f\ \max(LI).$

$Step\ 13\ :\ If\ \ such\ q\ exists\ then$

$Step\ 13.1\ :\ Count=Count\ \ +\ q.$

$Step\ 13.2\ :\ If\ \ Count \geq 2\ goto\ step\ 15$

Endif.

$Step\ 14\ :\ If\ \ there\ is\ \ (u,v)\ such\ that$

$u \in\ nbd^{-}(LI)\ and\ u<LI$

$and\ v\ \in\ nbd^{+}(LI).$

$Step\ 14.1\ :\ If\ \ c_1 \neq Nil\ and\ \ v=\ fAdj(c_1)$

$Step\ 14.1.1\ :\ Find\ NI(LI).$

$Step\ 14.1.2\ :\ If\ \ there\ is\ no\ \ (u,v)\ such\ that$

$u \in\ nbd^{-}(NI(LI))\ and$

$v\ \in\ nbd^{+}(NI(LI)).$

$Step\ 14.1.2.1\ :\ SD=SD \cup \{NI(LI))\}$

$goto\ step\ 7$

Endif

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Endif

Else

$Step\ 14.2\ :\ Count = Count + 1.$

$Step\ 14.3\ :\ Take$ largest $v.$

$Step\ 14.4\ :\ SD = SD \cup \{v\}\ then\ goto\ 7.$

$Step\ 14.5 :\ c_1 = v$

Endif.

$Step\ 15\ :\ Find\ NI(LI).$

$Step\ 16\ :\ If\ NI(LI) \in nbd[a]\ goto\ \ Step\ 21.$

$Step\ 17\ :\ If\ Count\ < 2$

$Step\ 17.1\ :\ If\ \ there\ is\ no\ (u,v)\ such\ that$

$u \in nbd^{-}(NI(LI))\ and\ v \in nbd^{+}(NI(LI)).$

$Step\ 17.1.1\ :\ If\ d^{+}(NI(LI)) = 0\ then$ $Step\ 17.1.1.1\ :\ SD = SD \cup j,\ where\ j\ is$ fmax and $adjacent\ to\ NI(LI)\ goto\ step\ \ LI$ else

$Step\ 17.1.2\ :\ SD = SD \cup \{NI(LI)\}\ goto\ step\ \ 7.$

Endif.

$Step\ 17.2\ :\ If\ \ there\ is\ \ (u,v)\ such\ that$

$u \in nbd^{-}(NI(LI))\ and\ v \in nbd^{+}(NI(LI)).$

$Step\ 17.2.1\ :\ c_1 = v.$

$Step\ 17.2.2\ :\ SD = SD \cup \{v\}\ goto\ 7.$

$Step\ 18\ : \max(NI(LI)).$

$Step\ 19\ :\ SD = SD \cup \{max\ (NI(LI))\}.$

$Step\ \ 20 :\ If\ \ max(NI(LI)) \notin nbd[a]\ \ or\ \ Count < 2$

$goto\ step\ 7$

$else\ goto\ 21.$

$Step\ 21\ :\ End.$

Theorem 1. Let G be a circular-arc graph corresponding to a circular-arc family A. Let $D = \{u, v, w\}$ be a minimum dominating set which is obtained by MDS algorithm. If the number of arcs in D such that there is no arc in $nbd^{+}(i)$, which is adjacent to an arc in $nbd^{-}(i)$ are atleast 2, then D is also split dominating set of A .where $i \in D$

Proof : Let G be a circular-arc graph corresponding to a circular-arc family A.

Let $D = \{u, v, w\}$ be a minimum dominating set which is obtained by MDS algorithm.

Suppose **that** the number of arcs in D such that there is no arc in $nbd^{+}(i)$, which is adjacent to an arc in $nbd^{-}(i)$ are atleast 2.

We have to show that minimum dominating set D is also a split dominating set.

Suppose **that** the number of arcs in D such that there is no arc in $nbd^{+}(i)$, which is adjacent to an arc in $nbd^{-}(i)$ are 2, say u, v ,where $i \in D$.

There is no arc which intersects both $nbd^{+}(u)$ and $nbd^{-}(u)$ and also there is no arc which both $nbd^{+}(v)$ and $nbd^{-}(v)$.

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Since there is no arc which intersect both $nbd^{+}(u)$ and $nbd^{-}(u)$, the vertices in $nbd^{+}(u)$ and $nbd^{-}(u)$ are non-adjacent in the induced subgraph $< V - D >$.

Therefore the induced subgraph $< V - D >$ connected interval graph.

IV. CONCLUSION

This paper presents an algorithm for finding an efficient dominating set of an interval family if it exists and Also an algorithm for finding a non- split dominating set of circular-arc graph.

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