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# Performance Analysis of Multi-Antenna Receiver Over Correlated Α-μ Fading Channel

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**ABSTRACT:** In this paper fading correlation of α-μ distribution considering multi-antenna receiver is discussed. Diversity combining techniques are briefly outlined. Correlation coefficient and joint probability density function for alpha-mu fading is shown. Generalized correlation coefficient for two alpha-mu fading variates is defined. Simulation results of outage and Bit Error Rate (BER) for different α and μ parameters and correlation coefficient is shown and performance is discussed.

**KEYWORDS**: α-μ Distribution, Bit Error Rate, Correlation, Diversity Combining, Outage Probability.

## I. Introduction

The multipath fading in wireless communication is modeled by several distribution such as Rayleigh, Ricean, Weibull, Nakagami. In the reference [1] alpha-mu (α-μ) fading model is proposed to describe the mobile radio signal considering two important phenomenon of radio propagation non-linearity and clustering. The α-μ represents a generalized fading distribution for small-scale variation of the fading signal in a non line-of-sight fading condition. The α-μ distribution is flexible, general and has easy mathematical tractability. In fact, the Generalized Gamma Distribution (GGD) also known as Stacy distribution has been renamed as α-μ distribution in [2]. As given in its name, alpha-mu distribution is written in terms of two physical parameters, namely α and μ. The power parameter ($\alpha > 0$) is related to the non-linearity of the environment i.e. propagation medium, whereas the parameter ($\mu > 0$) is associated to the number of multipath clusters.

In the α-μ fading distribution and its probability density function has been described. The correlation is a physical phenomenon among channels which affects the performance of a wireless communication system and it can not be neglected in a realistic fading scenarios. Formulation for the α-μ multivariate joint density function and its corresponding distribution function are provided in [3]. BER and outage probability for the selection combining technique in a correlated α-μ fading channel is obtained in [4]. The performance analysis of dual selection combining diversity receiver over correlated α-μ fading channels with arbitrary parameters is shown in [5], fading between the diversity branches and interferers is correlated and distributed with α-μ distribution. Performance of a dual-branch switched and stay combining diversity receiver, operating over correlated α-μ fading in the presence of co-channel interference have been analyzed in [6]. Outage and Bit error rate for i.i.d. α-μ fading channel with a maximal ratio combining receiver is evaluated in [7].

The paper is organized as follows. In Section 2, the alpha-mu fading model and the Probability density function is briefly discussed. In Section 3, diversity combining techniques for α-μ distribution, maximal ratio combining (MRC), equal gain combining (EGC), and selection combining (SC) is discussed. In Section 4, fading correlation in alpha-mu is explained, expression for correlation coefficient and constant & exponential correlation for alpha mu is shown. Monte-carlo simulation results for outage and BER analysis for various diversity multi-antenna systems are presented in Section 5. The paper is concluded by Section 6.

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## II. THE α-μ FADING MODEL

In the $\alpha$-$\mu$ distribution, it is considered that a signal is composed of clusters of multipath waves propagating in a non-homogenous environment. In any one of the cluster, the phases of the scattered waves are random and have similar delay times. Further, the delay-time spreads of different clusters is generally relatively large. It is assumed that the clusters of multipath waves have the scattered waves with identical powers. As a result, the obtained envelope, is a non-linear function of the modulus of the sum of the multipath components. The $\alpha$-$\mu$ probability density function (PDF), $f_R(r)$ of envelope R is given in [1] as

$$f_R(r) = \frac{\alpha\ \mu^{\mu}\ \mathrm{r}^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\ \Gamma(\mu)}\ \exp\left[-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right] ;\ \text{where}\ \hat{r} = \sqrt[\alpha]{E(R^{\alpha})} = \sqrt[\alpha]{2\mu\sigma^2} \qquad (1)$$

where $\alpha > 0$ is the power parameter, and α-root mean value of $R^{\alpha}$ is given as

and $\mu \geq 0$, is the inverse of the normalized variance of $\alpha$-$\mu$ envelope $R^{\alpha}$, and

$$\Gamma(z) = \int_0^{\infty} t^{z-1}\, \mathrm{e}^{(-t)}\, dt$$ is the Gamma function.

Outage of $\alpha$-$\mu$ channel given in [8] as,
$$P_{out} = \frac{\gamma_{thr}\left(\mu,\ \mu\left(\frac{\gamma_{thr}}{\gamma}\right)^{\alpha/2}\right)}{\Gamma(u)} \qquad (2)$$

The expression of BER for α-μ fading channel is obtained in [9], through MGF approach.

## III. DIVERSITY COMBINING

Diversity technique provides multiple copies of the same signal on different branches, which undergo independent fading. If one branch undergoes a deep fade, the another branch may have strong signal. In space diversity fading are minimized by the simultaneous use of two or more physically separated antennas. Thus having more than one path to select the SNR at receiver may be improved by selecting appropriate combining technique. Diversity combining methods for uncorrelated fading channels are enumerated below:

### A. *Selection Combining (SC)*

Selection combining is based on the principle of selecting the best signal among all the signals received from different branches at the receiving end. In this method, the receiver monitors the SNR of the incoming signal using switch logic. The branch with highest instantaneous SNR is connected to demodulator.

SNR of selection combining is given as

$$\gamma_{SC} = \mathrm{Max}\left(R_1^2, R_2^2, \ldots\ldots\ldots, R_L^2\right) \qquad (3)$$

Where $R_1, R_2, \ldots, R_L$ represent the fading envelope for multi-channels seen by L number of antennas.

### B. *Maximal Ratio Combining (MRC)*

This is the most complex scheme in which all branches are optimally combined at the receiver. MRC requires scaling and co-phasing of individual branch. In this all the signals are weighted according to their individual signal voltage to noise power ratios and then summed. Thus MRC produces an output SNR, which is equal to the sum of the individual SNRs. Best statistical reduction of fading is achieved by this method.

SNR of MRC is given as

$$\gamma_{MRC} = R_1^{\ 2} + R_2^{\ 2} + \ldots\ldots + R_L^2 \qquad (4)$$

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C. *Equal Gain Combining (EGC)*

A variant of MRC is equal gain combining (EGC), where signals from each branch are co-phased and their weights have equal magnitude. This method also has possibility like MRC of producing an acceptable output signal from a number of unacceptable input signals. SNR improvement of EGC is better than selection combining but not better than MRC. SNR of EGC is given as

$$\gamma_{EGC} = \left(R_1 + R_2 + \dots\dots + R_L\right)^2 \tag{5}$$

## IV. FADING CORRELATION IN α-μ

While studying the performance of diversity techniques, it is assumed that the signals considered for combining are independent of one another. The assumption of independence of channels facilitates the analysis but restricts the understanding of correlation phenomenon.

As per statistics theory variables are said to be correlated, when increase or decrease in one variable is accompanied by increase or decrease in other variable. Correlation is said to be positive when either both variables increase or both variables decrease. Correlation is said to be negative when one variable increases and other variable decreases. The degree of relationship between two correlated variables is measured as Correlation Coefficient ($\rho$).

$$\rho = \frac{cov(x,y)}{var(x)var(y)} = \frac{cov(x,y)}{\sigma_x\ \sigma_y} \tag{6}$$

In fact the assumption of channel independence is not valid due to reasons such as insufficient antenna spacing in smaller mobile units. Let us consider a α-μ random variable $R_1$ with variance $\sigma^2$. In order to generate a new α-μ random variable $R_2$ with the same variance of $R_1$ and a correlation factor $\rho$ between them, the equation for $R_2$ can be given as

$$R_2 = \rho\ \mathrm{R}_1 + \left(1 - \rho^2\right)^{\frac{1}{2}} v \tag{7}$$

where $v$ is a α-μ random variable with variance $\sigma^2$. Since correlation parameter $\rho$ is $0 \le \rho \le 1$, therefore when $\rho$ is 0; then $R_2 = v$, i.e. $R_2$ is a complex Gaussian random variable but uncorrelated with $R_1$. When $\rho$ is 1; then $R_2 = R_1$, i.e. $R_2$ is fully correlated with $R_1$. Taking mean values of $R_1$ and $v$ as zero and since both have the same variance $\sigma^2$, then the variance of $R_2$ is also $\sigma^2$. Then from eq. (6) generalized correlation coefficient $\rho$ for two α-μ fading variates $R_1$ and $R_2$ is defined as

$$\rho = \frac{cov(R_1, R_2)}{\sqrt{\sigma_{R_1}{}^2 \times \sigma_{R_2}{}^2}} = \frac{cov(R_1, R_2)}{\sigma_{R_1} \times \sigma_{R_2}} \tag{8}$$

For wireless system performance and design study, characterization of the correlation properties of the signals is of utmost importance. Therefore determination of the joint probability density function (joint PDF) is significant. Let $R_1$ and $R_2$ be two α-μ fading variates whose marginal statistics are respectively described as below:

$$R_1 \in \{\alpha_1, \mu_1, \hat{r}_1\} \text{ and } R_2 \in \{\alpha_2, \mu_2, \hat{r}_2\} \tag{9}$$

and $0 \le \rho \le 1$ be a correlation parameter. Due to insufficient antenna spacing, the signal envelopes $R_1$ and $R_2$ experience correlative α-μ fading with joint distribution given [10] as:

$$f_{R_1,R_2}(R_1,R_2) = f_{R_1}(R_1)\, f_{R_2}(R_2) \sum_{l=0}^{\infty} \frac{l!\,\Gamma(\mu_1)}{\Gamma(\mu_1 + l)} \rho_{12}' \times L_l^{\mu_1 - 1}\left(\frac{\mu_1 R_1^{\alpha_1}}{\hat{R}_1^{\alpha_1}}\right) L_l^{\mu_1 - 1}\left(\frac{\mu_1 R_2^{\alpha_2}}{\hat{R}_2^{\alpha_2}}\right) \tag{10}$$

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A. *Constant Correlation*

This corresponds to identically distributed α-μ fading channels. That means all channels are assumed to have the same average SNR per symbol and same fading parameter. This model assumes that the power correlation coefficient $\rho$ is the same between all the channel pairs ($d$, $d'$= 1,2,…..,D) and is given [11] as

$$\rho = \rho_{dd'} = \frac{\operatorname{cov}\left(r_d^2, r_{d'}^2\right)}{\sqrt{\operatorname{var}\left(r_d^2\right)\operatorname{var}\left(r_{d'}^2\right)}} \quad d \neq d' \quad 0 \leq \rho < 1 \tag{11}$$

This model corresponds to the scenario of multichannel reception from closely spaced diversity antennas or three antennas placed on an equilateral triangle.

B. *Exponential Correlation:*

This corresponds to identically distributed α-μ fading channels. That means all channels are assumed to have the same average SNR per symbol and same fading parameter. This model assumes that an exponential power correlation coefficient $\rho_{dd'}$ between any pair of channels ($d$, $d'$= 1,2,…..,D) is given [11] by

$$\rho = \rho_{dd'} = \frac{\operatorname{cov}\left(r_d^2, r_{d'}^2\right)}{\sqrt{\operatorname{var}\left(r_d^2\right)\operatorname{var}\left(r_{d'}^2\right)}} = \rho^{|d-d'|} \qquad 0 \leq \rho < 1 \tag{12}$$

This model may corresponds to the scenario of multichannel reception from equi-spaced diversity antennas in which the correlation between the pairs of combined signals decays as the spacing between antennas increases.

## V. Simulation Results and Discussions

Outage performance and BER for different combining techniques for correlated α-μ fading channel for multi-antenna system obtained by Monte-Carlo simulation are shown in Fig. 1 to Fig.12. In these simulation 1000000 samples have been considered for each curve shown.

As observed in Fig. 1 to Fig. 3, α =7/4, μ=2, ρ=0.99, and when number of receiving antennas Rx is varied from 3 to 5, the outage probability decreases i.e. performance of system improves for desired average SNR. It is seen in Fig. 4 to Fig. 6, α =7/4, μ=1, ρ=0.5, and by varying number of receiving antennas Rx from 3 to 5, the bit error rate decreases i.e. performance of system improves for desired average SNR.

In Fig.7 to Fig.9, where α =7/4, μ=1, and ρ=0.99, when number of receiving antennas Rx is varied from 3 to 5, the outage probability again decreases as desired for a given average SNR. It is worth noting that in this case (μ=1) outage is higher than the corresponding values of previous (μ=2) case. In Fig. 3 (μ=2) for 1dB average SNR, for MRC exponential correlation outage is $2.5\times10^{-3}$, whereas from Fig. 9 (μ=1) for 1dB average SNR, the respective outage is $1.5\times10^{-1}$, which is a poor performance comparatively. Thus multipath cluster parameter μ, plays an important role in outage performance of wireless network.

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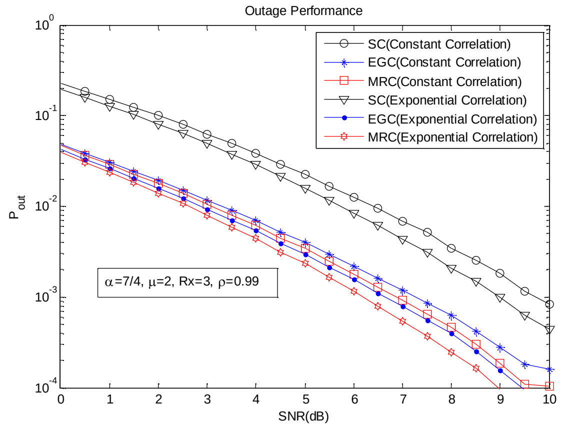


Fig. 1. Outage for α=7/4, μ=2, Rx =3 and ρ= 0.99 for different combining.

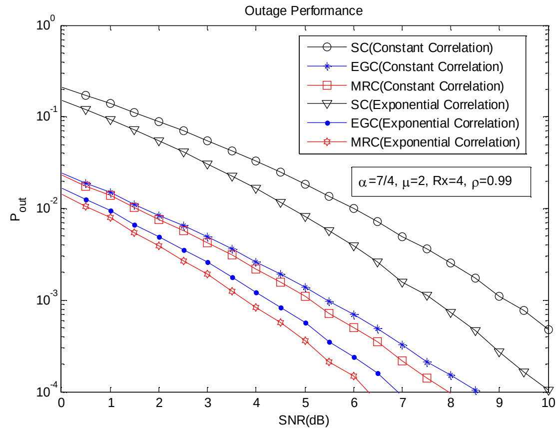


Fig. 2. Outage for α=7/4, μ=2, Rx =4 and ρ= 0.99 for different combining.

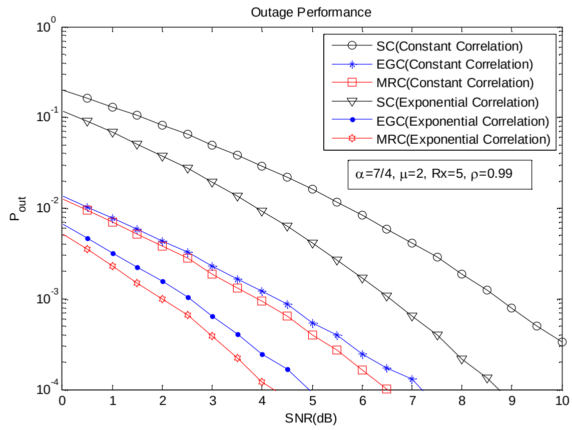


Fig. 3. Outage for α=7/4, μ=2, Rx =5 and ρ= 0.99 for different combining.

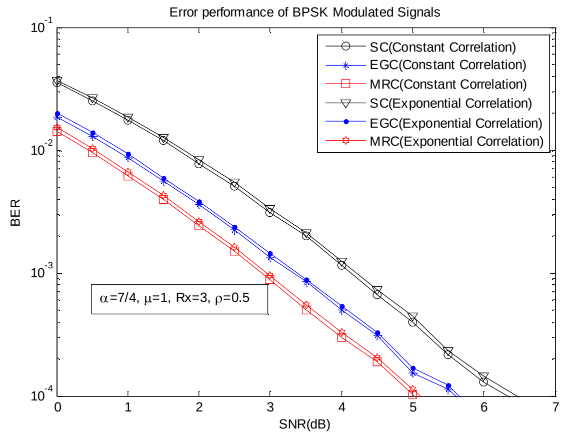


Fig. 4. BER for α=7/4, μ=1, Rx =3 and ρ= 0.5 for different combining.

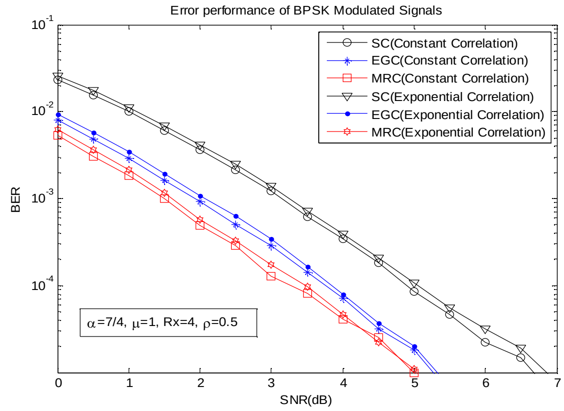


Fig. 5. BER for α=7/4, μ=1, Rx =4 and ρ= 0.5 for different combining.

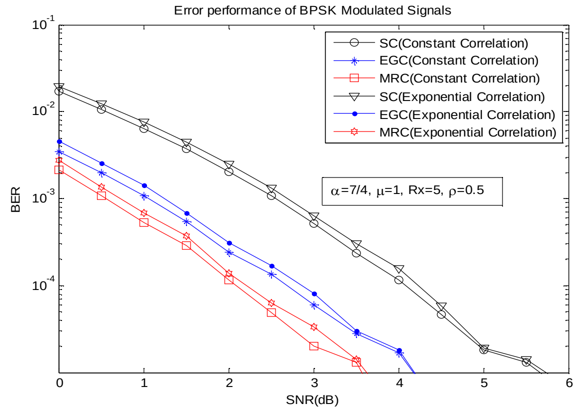


Fig. 6. BER for α=7/4, μ=1, Rx =5 and ρ= 0.5 for different combining.

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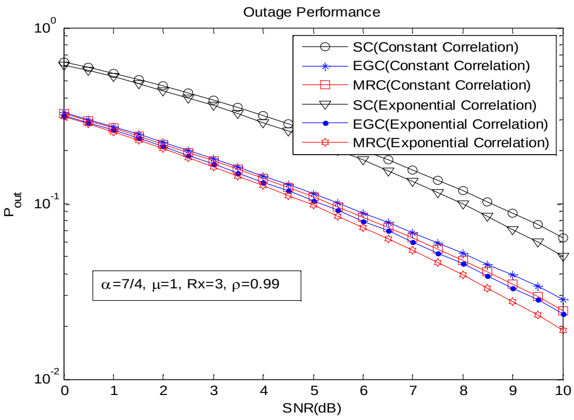


Fig. 7. Outage for α=7/4, μ=1, Rx =3 and ρ= 0.99 for different combining.

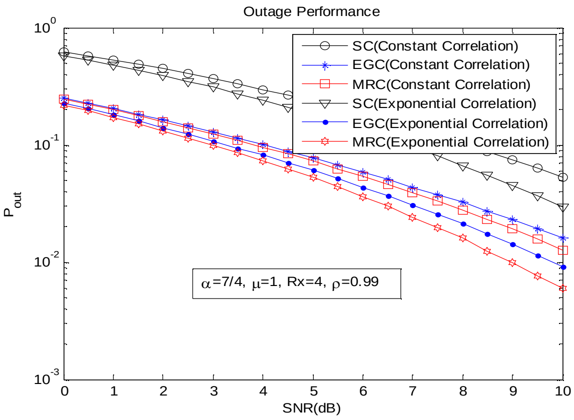


Fig. 8. Outage for α=7/4, μ=1, Rx =4 and ρ= 0.99 for different combining.

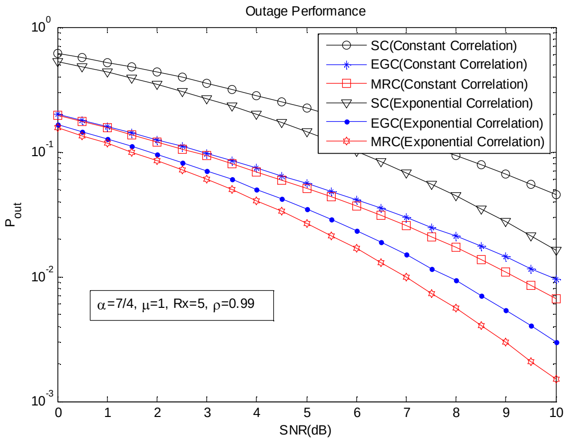


Fig. 9. Outage for α=7/4, μ=1, Rx =5 and ρ= 0.99 for different combining.

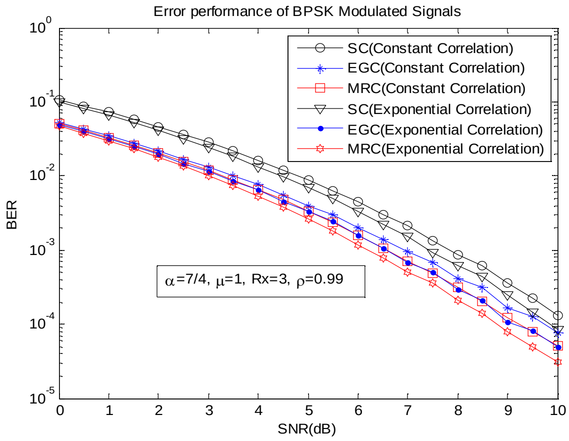


Fig. 10. BER for α=7/4, μ=1, Rx =3 and ρ= 0.99 for different combining.

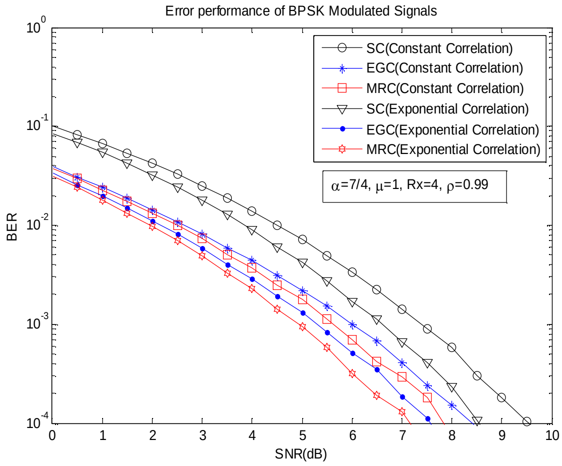


Fig. 11. BER for α=7/4, μ=1, Rx =4 and ρ= 0.99 for different combining.

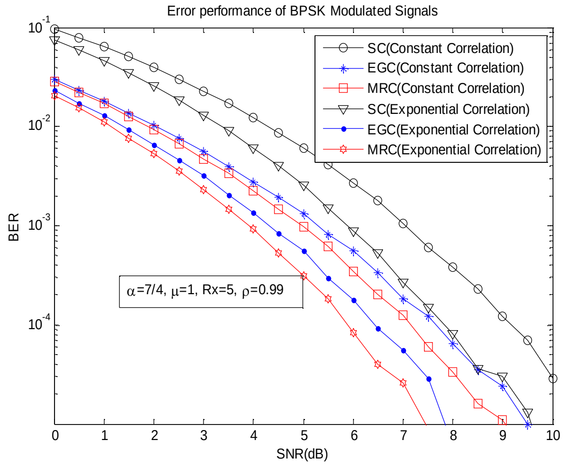


Fig. 12. BER for α=7/4, μ=1, Rx =5 and ρ= 0.99 for different combining.

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In a similar way regarding BER for $\alpha$ =7/4, $\mu$=1, and $\rho$=0.99 shown in Fig. 10 to Fig. 12, by varying number of receiving antennas Rx from 3 to 5, the bit error rate (BER) decreases i.e. performance of system improves for desired average SNR. But again it can be seen that BER decrease is not as it was for $\mu$=2 case. Regarding correlation coefficient, we notice from Fig.6 ($\alpha$ =7/4, $\mu$=1, Rx=5, $\rho$=0.5) at 2dB average SNR, BER is $1.5\times10^{-4}$ for MRC exponential and from Fig.12 ($\alpha$ =7/4, $\mu$=1, Rx=5, $\rho$=0.99) at 2dB average SNR, corresponding BER is $6\times10^{-3}$. Thus BER is higher for high value of correlation. It is also observed that in multiple antenna diversity correlated fading performance of MRC is better among all and then the EGC and SC follows the performance subsequently. Also, performance of exponential correlation is better than constant correlation. The MRC is optimal combining scheme but it is at the expense of complexity. On the other hand the SC combiner chooses the branch with highest SNR i.e. output is equal to the signal on only one of the branches, hence it does not require knowledge of the signal phases on each branch as in the case of MRC or EGC.

## VI. Conclusion and Future Work

In this paper $\alpha$-$\mu$ fading model and probability density function have been briefly discussed. The simulated and analytical results of performance metrics such as outage and BER for $\alpha$-$\mu$ fading correlation multi-antenna system have been illustrated. The effect of number of antenna variation and constant & exponential correlation coefficient variation on BER and outage is brought out. The result obtained in this paper will help researcher to explore correlation concept in multi-antenna $\alpha$-$\mu$ generalized fading model.

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