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# Window Function Analysis in OFDM System for PAPR Reduction

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**ABSTRACT:** Orthogonal Frequency Division Multiplexing (OFDM) is an efficient method of data transmission for high speed communication systems. However, the main drawback of OFDM system is the high Peak to Average Power Ratio (PAPR) of the transmitted signals. OFDM consist of large number of independent subcarriers, as a result of which the amplitude of such a signal can have high peak values. In multicarrier modulation the orthogonality between closely spaced subcarrier is necessary in order to overcome inter-carrier interference (ICI).This ICI reduces the bit error rate (BER) performance of the system. In this paper through MATLAB coding, several windows are simulated and compared on a single graph. The comparison results are tabulated through which an accurate conclusion for appropriate window to be used in OFDM systems can be decided.

**KEYWORDS**: Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), MATLAB, BER

## I. INTRODUCTION TO OFDM

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation (MCM) technique which seems to be an attractive candidate for fourth generation (4G) wireless communication systems. OFDM offer high spectral efficiency, immune to the multipath delay, low inter-symbol interference (ISI), immunity to frequency selective fading and high power efficiency. Due to these merits OFDM is chosen as high data rate communication systems such as Digital Video Broadcasting (DVB) and based mobile worldwide interoperability for microwave access (mobile Wi-MAX). However OFDM system suffers from serious problem of high PAPR. In OFDM system output is superposition of multiple sub-carriers. In this case some instantaneous power output might increase greatly and become far higher than the mean power of system. To transmit signals with such high PAPR, it requires power amplifiers with very high power scope. These kinds of amplifiers are very expensive and have low efficiency-cost. If the peak power is too high, it could be out of the scope of the linear power amplifier. This gives rise to non-linear distortion which changes the superposition of the signal spectrum resulting in performance degradation. If no measure is taken to reduce the high PAPR, MIMO-OFDM system could face serious restriction for practical applications [1]-[4].

The usage of windows smooths the observed signal over the edges of measured time interval and gradually reduces it to zero. This prevents glitches in the “assumed” signal repetitively reconstructed by the Fourier Transform. This phenomenon reduces the spectral leakage.

The process of multiplying the measured signal with a smoothly ending function (that gradually reduces to zero) is called “windowing” technique and the function is called “window” function. Frequency domain characteristics of window functions are often analyzed to determine their suitability for specific application/system limitations. Spectral characteristics of window functions are used to underpin the answer to the most commonly occurring dilemma of making a choice between “detection” and “resolution”.

“Detection” means detecting a desired signal in the presence of broadband noise. Resolution refers to the ability of distinguishing narrowband spectral components. Available are plethora of parameters that would help us to select the window functions which would suit our needs.

## II. OFDM MODEL

The binary information bits are mapped to complex-valued MQAM symbols in a 2-dimensional signal constellation. The output of the mapper is serial-to-parallel converted and processed using an N -point complex inverse fast Fourier transform (IFFT). The N complex-valued time domain signals are then followed by a guard interval (GI), which contains the number of last L −1 samples (N > L). The GI consists of a partial repetition of an OFDM symbol so it does not affect the PAPR. Therefore, we do not take the GI into consideration here. Passing through a PAPR reduction block such as peak windowing, the signals undergoes a digital-to-analog conversion and are transmitted after high power amplifier. At the receiver, the received signals can be demodulated by the reverse process of the transmitter.

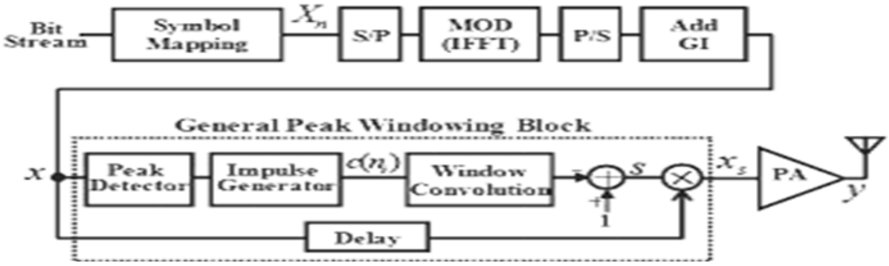

Fig1: OFDM Transmitter with Peak Windowing

## III. PAPR IN OFDM SIGNALS

### *A. PAPR*

In general, the PAPR [3] of OFDM signals x(t) is defined as the ratio between the maximum instantaneous power and its average power

$$\mathrm{PAPR}[x(t)] = \frac{P_{\mathrm{PEAK}}}{P_{\mathrm{AVERAGE}}} = 10\log_{10}\frac{\max[|X(n)|^2]}{E[|x_n|^2]}$$

Where PPEAK represents peak output power, PAVERAGE means average output power. E[ •] denotes the expected value, xn represents the transmitted OFDM signals which are obtained by taking IFFT operation on modulated input symbols Xk [7].

xn is expressed as:

$$x_n = \frac{1}{\sqrt{N}}\sum_{K=0}^{N-1} X_k {W_N}^{nk}$$

The instantaneous output of an OFDM system often has large fluctuations compared to traditional single-carrier systems. This requires that system devices, such as power amplifiers, A/D converters and D/A converters, must have large linear dynamic ranges. If this is not satisfied, a series of undesirable interference is encountered when the peak signal goes into the non-linear region of devices at the transmitter, such as high out of band radiation and inter-modulation distortion. PAPR reduction techniques are therefore of great importance for OFDM systems. Also due to the large fluctuations in power output the HPA (high power amplifier) should have large dynamic range. This results in poor power efficiency.

## IV. PROPOSED TECHNIQUE

In this section, we outline the conventional peak windowing method and propose a new PAPR reduction technique. The proposed method overcomes the drawback of the conventional method when successive peaks emerge within a half of the window size. The clipping method is the simplest way to reduce PAPR. However, it distorts signals nonlinearly and significantly increases the out-of-band radiation. A different approach is to multiply large signal peaks with a certain window function. In order to maintain the out-ofband radiation within a certain level, it is benefit to increase the window length. On the other hand, the window should not be too long, because a long widow length implies that many signal samples are affected, which degrades the BER performance.

**A. List of Window Functions**:

***1. Bartlett-Hann Window***

$$w(n) = a_0 - a_1 \left|\frac{n}{N-1} - \frac{1}{2}\right| - a_2 \cos\left(\frac{2\pi n}{N-1}\right)$$
$$a_0 = 0.62; \quad a_1 = 0.48; \quad a_2 = 0.38$$

Like Bartlett, Hann, and Hamming windows, this window has a main lobe at the origin and asymptotically decaying side lobes on both sides. It is a linear combination of weighted Bartlett and Hann windows with near side lobes lower than both Bartlett and Hann and with far side lobes lower than both Bartlett and Hamming windows. The main lobe width of the modified Bartlett-Hann window is not increased relative to either Bartlett or Hann window main lobes.

***2. Rectangular window*****:**

$$w(n) = 1$$

The rectangular window (sometimes known as the boxcar or Dirichlet window) is the simplest window, equivalent to replacing all but N values of a data sequence by zeros, making it appear as though the waveform suddenly turns on and off. w = rectwin(L) returns a rectangular window of length L in the column vector w. This function is provided for completeness; a rectangular window is equivalent to no window at all.

***3. Hann- Window:***

$$w(n) = 0.5\left(1 - \cos\left(\frac{2\pi n}{N-1}\right)\right) = \operatorname{hav}\left(\frac{2\pi n}{N-1}\right)$$

The Hann window named after Julius von Hann and also known as the Hanning (for being similar in name and form to the Hamming window), von Hann and the raised cosine window is defined by (with hav for the have rsine function).

***4. Gaussian Window:***

$$w(n) = e^{-\frac{1}{2}\left(\frac{n-(N-1)/2}{\sigma(N-1)/2}\right)^2}$$

The Fourier transform of a Gaussian is also a Gaussian (it is an Eigen function of the Fourier Transform). Since the Gaussian function extends to infinity, it must either be truncated at the ends of the window, or itself windowed with another zero-ended window. Since the log of a Gaussian produces a parabola, this can be used for nearly exact quadratic interpolation in frequency estimation.

***5. Flat top Window:***

$$w(n) = a_0 - a_1 \cos\left(\frac{2\pi n}{N-1}\right) + a_2 \cos\left(\frac{4\pi n}{N-1}\right) - a_3 \cos\left(\frac{6\pi n}{N-1}\right) + a_4 \cos\left(\frac{8\pi n}{N-1}\right)$$

$$a_0 = 1; \quad a_1 = 1.93; \quad a_2 = 1.29; \quad a_3 = 0.388; \quad a_4 = 0.028$$

Flat top window is a partially negative-valued window that has a flat top in the frequency domain. Such windows have been made available in spectrum analyzers for the measurement of amplitudes of sinusoidal frequency components. They have a low amplitude measurement error suitable for this purpose, achieved by the spreading of the energy of a sine wave over multiple bins in the spectrum. This ensures that the unattenuated amplitude of the sinusoid can be found on at least one of the neighbouring bins. The drawback of the broad bandwidth is poor frequency resolution. To compensate, a longer window length may be chosen.

## V. COMPARISION OF WINDOWS

### *1. Commonly used Parameters for Window Selection:*

**Main Lobe Width**: Main Lobe Width is directly linked to “Frequency resolution”. Narrower the main lobe, more will be its frequency resolution (Ability of selecting finest frequency components). Main Lobe Width is the measure of the main lobe width at -3dB or -6dB below main lobe peak. When the main lobe width decreases, the remaining energy spreads out to side lobes thereby increasing spectral leakage/decreasing amplitude accuracy (decreasing “detection” ability). A compromise is needed to strike a balance between detection and resolution that would suit the application at hand.

**Side Lobe Level**: Side lobes occur at either side of the main lobe and they approach zero at integral multiples of Fs/N (where Fs=Sampling Frequency; N is the length of N-point FFT/DFT). Side lobes directly affect the extent to which adjacent frequency components leak into adjacent frequency bins. Side Lobe Level is usually measure for that side lobe which has the maximum peak (compared to other side lobes). It is measured in decibels relative to the peak of the main lobe. Lesser the side lobe level of a window more will be its detection ability. The side lobe roll-off rate is the asymptotic decay rate in decibels per decade of frequency of the peaks of the side lobes.
For an “Ideal” Window:
**1)** Main lobe width (*MWL*) is very small (High Frequency Resolution).
**2)** Side Lobe level(*SLL*) is very high (Good noise suppression – High detection ability).
**3)** Side Lobes fall-offs rapidly.

**Normalized Equivalent Noise Bandwidth**: The windowing of a signal in time also affects the resolution bandwidth that can be achieved. When calculating the spectrum of a signal segment the resolution bandwidth achieved with a window is always lower (worse) than the resolution bandwidth achieved without a window. The normalized equivalent noise bandwidth (NENBW) of a window is one measure of how much it reduces the resolution bandwidth that can be achieved. To compensate for this a longer signal segment should be processed. Equivalent noise bandwidth (ENBW) compares the window to an ideal, rectangular filter. It is the equivalent width of a rectangular filter that passes the same amount of white noise as the window. The normalized ENBW (NENBW) is the ENBW multiplied by the time duration of the signal being windowed. NENBW values for windows available in Spectrum Analyzer ResBW are listed in the following table. For our computation we assume that the time duration is of unit duration ,therefore ENBW and NENBW will be equal.

***Calculation of Equivalent Noise Bandwidth***: Equivalent Noise Bandwidth is calculated as a ratio of inherent power gain over coherent power gain. The equation is particularly useful for simulations as we do not have to bother about the transfer functions of each window functions in frequency domain.

## VI. SIMULATION RESULTS

In the simulation, we assume that the OFDM system consists of 1024 subcarriers with 16-QAM. We also assume that the oversampling factor is four. We also assume that the guard interval is one by four. If we oversample the OFDM signal by a factor of four, the PAPR of the discrete signal is almost the same as that of continuous signal.

| Window Function | Side Lobe Level | 3dB Bandwidth (Bins) | 6dB Bandwidth (Bins) | ENBW (Bins) |
|---|---|---|---|---|
| Rectangular | -13.26 | 0.896 | 1.248 | 1 |
| Hanning | -32.17 | 1.472 | 2.048 | 1.52 |
| Hamming | -43.5 | 1.344 | 1.856 | 1.37 |
| Bartlett | -26.5 | 1.312 | 1.792 | 1.33 |
| Blackman | -58.1 | 1.664 | 2.336 | 1.75 |
| Kaiser | -69.5 | 1.728 | 2.432 | 1.82 |
| Gaussian | -32.3 | 1.152 | 1.664 | 1.24 |
| Flattop | -68.75 | 3.84 | 4.672 | 3.83 |

Table1. Measured Window Parameters

To characterize the side lobes of the window, the maximum side lobe level and side lobe roll-off rates are defined. The maximum side lobe level is the level in decibels relative to the main lobe peak gain, of the maximum side lobe. The side lobe roll-off rate is the asymptotic decay rate, in decibels per decade of frequency, of the peaks of the side lobes. Table 1 lists the characteristics of several window functions and their effects on spectral leakage and resolution.

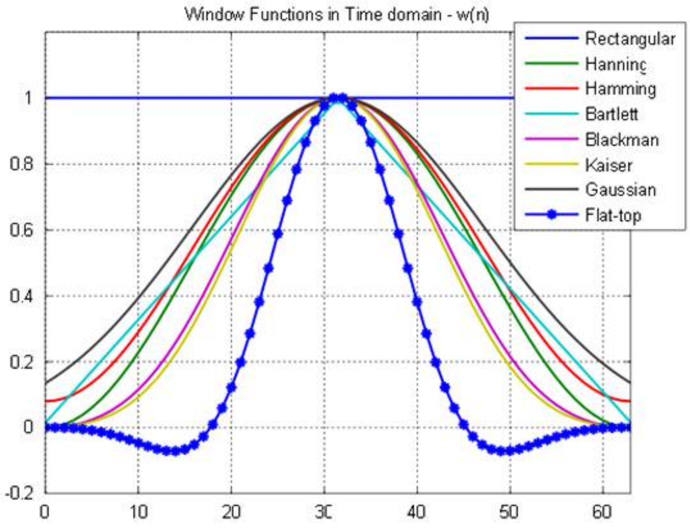

Fig2. Window functions in time domain

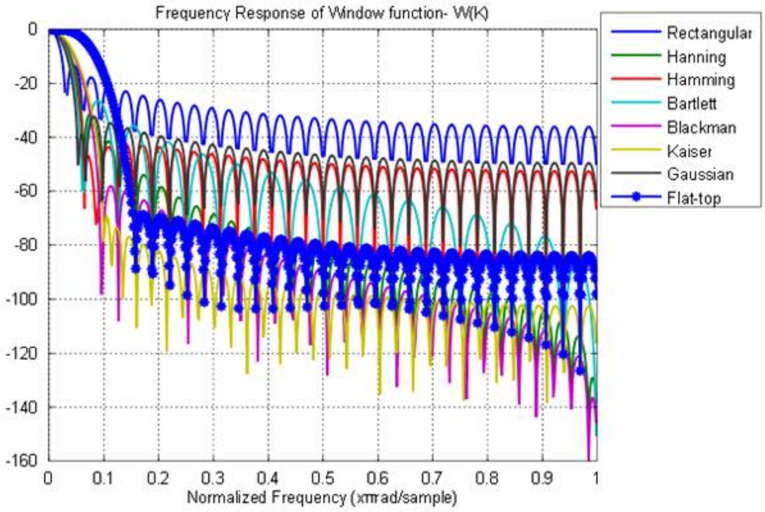

Fig3. Window functions in frequency domain

Since the OFDM signal is multiplied with several of these windows the resulting spectrum is a convolution of the original OFDM spectrum with the spectrum of the applied window. So, ideally the window should be as narrow band as possible. On the other hand, the window should not be too long in the time domain, because that implies that many signal samples are affected, which increases the bit error ratio.

When selecting an appropriate window function for an application, this comparison graph may be useful. The frequency axis has units of FFT "bins" when the window of length N is applied to data and a transform of length N is computed. For instance, the value at frequency ½ "bin" is the response that would be measured in bins k and k+1 to a sinusoidal signal at frequency k+½. It is relative to the maximum possible response, which occurs when the signal frequency is an integer number of bins. The value at frequency ½ is referred to as the maximum *scalloping loss* of the window, which is one metric used to compare windows. The rectangular window is noticeably worse than the others in terms of that metric.

Other metrics that can be seen are the width of the main lobe and the peak level of the side lobes, which respectively determine the ability to resolve comparable strength signals and disparate strength signals. The rectangular window (for instance) is the best choice for the former and the worst choice for the latter. What cannot be seen from the graphs is that the rectangular window has the best noise bandwidth, which makes it a good candidate for detecting low-level sinusoids in an otherwise white noise environment. Interpolation techniques, such as zero-padding and frequency-shifting, are available to mitigate its potential scalloping loss.

Selecting a window function is not a simple task. In fact, there is no universal approach for doing so. However, Table 2 helps in initial choice. On comparing the performance of different window functions, the best window for the application can be found.

| Signal Content | Window |
|---|---|
| Sine wave or combination of sine waves | Hann |
| Sine wave (amplitude accuracy is important) | Flat Top |
| Narrowband random signal (vibration data) | Hann |
| Broadband random (white noise) | Uniform |
| Closely spaced sine waves | Uniform, Hamming |
| Excitation signals (Hammer blow) | Force |
| Response signals | Exponential |
| Unknown content | Hann |

Table 2. Initial Window Choice Based on Signal Content

## VII. CONCLUSION & FUTURE SCOPE

OFDM is a very attractive technique for wireless communications due to its spectrum efficiency and channel robustness. One of the serious drawbacks of OFDM systems is that the composite transmit signal can exhibit a very high PAPR when the input sequences are highly correlated. In this paper, several important aspects are described as well as mathematical analysis is provided, including the distribution of the PAPR used in OFDM systems. The usage of windows (like Bartlett-Hann, Hann- Window, Gaussian Window, Flat top Window, except for rectangular window) smooths the observed signal over the edges of measured time interval and gradually reduces it to zero. This prevents glitches in the “assumed” signal repetitively reconstructed by the Fourier Transform. This phenomenon reduces the spectral leakage.

Each window has its own characteristics, and different windows are used for different applications. To choose a spectral window, you must guess the signal frequency content. If the signal contains strong interfering frequency components distant from the frequency of interest, choose a window with a high side lobe roll-off rate. If there are strong interfering signals near the frequency of interest, choose a window with a low maximum side lobe level.In general, the Hanning window is satisfactory in 95 % of cases because it has good frequency resolution and reduced spectral leakage.

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