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A Framelet Domain Approach for ECG Signal Denoising

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ABSTRACT: Current denoising techniques for signals are mostly conducted in the wavelet transform domain. In this paper, a new denoising approach for denoising the signals in the Framelet domain is proposed. Medical signals are initially decomposed using framelets. After the decomposition, they are denoised using a new thresholding approach using hard and soft thresholding. The recomposed signal is then obtained and Signal to Noise ratio value is calculated. The results are compared with that obtained by wavelet technique.

KEYWORDS*:* Signal Denoising, Framelets, Analysis Filter, Synthesis filter, dwt, hard and soft thresholding

I. INTRODUCTION

The morphology of ECG signal has been used for recognizing much variability's of heart activity, so it is very important to get the parameters of ECG signal clear without noise. This step gives a full picture and detailed information about the electrophysiology of the heart diseases and the ischemic changes that may occur like the myocardial infarction, conduction defects and arrhythmia. In order to support clinical decision making, reasoning tool to the ECG signal must be clearly represented and filtered, to remove out all noises and artifacts from the signal. ECG signal is one of the biosignals that is considered as a non-stationary signal and needs a hard work to denoising. The noises that affect ECG signals are power line interference, baseline wanders, physiological noise like muscle contraction.

A decomposition technique called framelets can be used to remove the artifacts from these noisy ECG signals effectively. Thresholding is used in framelet domain to smooth out or to remove some coefficients of framelet transform subsignals of the measured signal. The noise content of the signal is reduced, effectively, under the nonstationary environment.

The outline of this paper is as follows. In section II, we present an Existing system. Section III ,we give our Proposed system. Section IV & V consists of proposed system algorithm and experimental results respectively. Section VI & VII provides the performance evaluation and conclusion respectively.

II. THE WAVELET TRANSFORM

The special structure of wavelet bases may be appreciated by considering generation of an orthonormal wavelet basis for function $g \in l^2(R^2)$(the space of square integrable real functions). The approach of Daubechies (1992) is the most often adopted in applications of wavelets in statistics, mutually orthonormal, functions or parent wavelets: the scaling function, ϕ (sometimes referred to as the father wavelet), and the mother wavelet, ψ. Other wavelets in the basis are then generated by translation of scaling function ϕ, and dilations and translations of mother wavelet ψ using the relationships

$$\Phi_{j0k}(t)=2^{j0/2}\,\psi(2^{\,j0}t-k),$$

$$\psi_{jk}(t)=2^{j/2}\,\psi(2^{\,j}t-k), \qquad (1)$$

$$j=j_{0+}\,j_0+1,\ldots;\ k \in Z$$

for some fixed $j_0 \in Z$**,** where Z is set of integers. The $2^{j/2}$ term maintains unity norm of the basis function at various scales and j and k are the scaling and translation parameters, respectively. A unit increase in j in (1) has no effect on scaling function (ϕ_{j0k} has a fixed width), but packs oscillations of ψ_{jk} into half the width (doubles its scale or resolution). A unit increase in k in (1) shifts the location of both ϕ_{j0k} and ψ_{jk}, the former by a fixed amount (2^{-j0}) and the latter by an amount proportional to its width (2^{-j}). Given the wavelet basis, a function $g \in l^2(R^2)$ is then represented in a corresponding wavelet series as

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$$g(t) = \sum_{k \in z} c_{j_0 k}\, \Phi_{j0k}(t) + \sum_{j=j_0} \sum_{k \in z} w_{jk}\psi_{jk}(t) \qquad (2)$$

with $c_{j0k} = (g, \phi j_0 k)$ and $w_{jk} = (g, \psi jk)$(where $(\cdot, \cdot)$ is the standard l^2-inner product of two functions: $(g1, g2)=_R g1(t)g2(t)dt$). The wavelet expansion (2) represents the function **g** as a series of successive approximations. Given a vector of function value $\mathbf{g} = [g(t1), g(t2), \ldots, g(tn)]T$ of equally spaced points ti, the DWT of **g** is given by

$$d = Wg, \qquad (3)$$

where d is an $n\times1$ vector comprising both discrete scaling coefficients $u_{j0,k}$ and discrete wavelet coefficients $d_{j,k}$ and **W** is an orthogonal $n \times n$ matrix associated with orthonormal wavelet basis chosen. Both $u_{j0,k}$ and $d_{j,k}$ are related to their continuous counterparts $c_{j0,k}$ and $w_{j,k}$ via the relationships $c_{j0,k} \approx u_{j0,k}/\sqrt{n}$ and $w_{jk} \approx d_{j,k}/\sqrt{n}$ .The factor $\sqrt{n}$ arises because of the difference between continuous and discrete orthonormality conditions. Note that, because of the orthogonality of W, the inverse DWT (IDWT) is simply given by

$$g = W'd, \qquad (4)$$

where W' denotes the transpose of W.

In 2011, Nagendra [1] used wavelet transform as a tool for processing non-stationary signals like ECG signals. The variations of the wavelet techniques like Multi resolution DWT, Fast Wavelet Transform, Lifting Wavelets, Multi wavelet Transform, Stationary Wavelet Transform, Wavelet Packet Decomposition, Fractional wavelet transform were used. The three different thresholding methods namely universal (sqtwolog), minimax and heursure thresholding methods were used for denoising the ECG signal[1]. The major disadvantage is that it achieves higher shift frequency and has poor time frequency localization[4].

The proposed method is based on decomposing the signal into mulite levels using Framelet Transform .

III. PROPOSED SYSTEM

3. 1 FRAMELET TRANSFORM

Framelet are very similar to wavelets but have some important differences. In particular, whereas wavelets have an associated scaling function $\Phi(t)$ and wavelet function $\psi(t)$, framelets have one scaling function $\Phi(t)$ and two wavelet functions $\psi1(t)$ and$\psi2(t)$ .The scaling function $\Phi(t)$ and the wavelets $\psi1(t)$
and $\psi2(t)$ are defined through these equations by the low-pass (scaling) filter $h_0(n)$ and the two high-pass (wavelet) filters $h_1(n)$ and $h_2(n)$.
Let

$$\Phi(t)=\sqrt{2} \sum h_0\Phi(\square\, 2t\text{-}n) \qquad (5)$$
$$\psi_i(t)=\sqrt{2}h_1(n)\, \Phi(2t-n), \quad i=1,2.$$

Any function f(t) could be written as a series expansion in terms of the scaling function and wavelets by[4]:

$$f(t)= \sqrt{2} \sum_{k=-\infty}^{\infty} c(k)\, \Phi_\kappa(t) + \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} d_1(j,k)\, \psi_{i,j,k}(t)+ d_2(j,k)\, \psi_{i,j,k}(t) \qquad (6)$$

where $c(k)=\int f(t)\, \Phi_\kappa(t)dt$
$d_1(j,k)= \int f(t)\, \psi_{i,j,k}(t)\, dt, \quad i=1,2.$

In this expansion, the first summation gives a function that is a low resolution or coarse approximation of f(t) at scale j = 0. For each increasing j in the second summation, a higher or finer resolution function is added, which adds increasing details. The filters $h_i(n)$ and $h_i(-n)$ should satisfy the perfect reconstruction (PR) conditions. From the basic multirate identities, the PR conditions are the following[4]:

$$H_0(Z).\ H_0(1/Z)+ H_1(Z).\ H_1(1/Z)+ H_2(Z).\ H_2(1/Z) =2 \qquad (7)$$
$$H_0(-Z).\ H_0(1/Z)+ H_1(-Z).\ H_1(1/Z)+ H_2(-Z).\ H_2(1/Z) = 0$$

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Let K_0 denote the number of zeros $H_0(e_{jw})$ has at $w = \pi$. For i = 1,2, let K_i denote the number of zeros $H_i(e_{jw})$ has at w = 0. Then the Z-transform of each $h_i(n)$ factors as follows:

$$H_0 (z) = Q_0 (Z) (Z+1)^{K0} \quad (8)$$

$$H_1 (z) = Q_1 (Z) (Z+1)^{K1} \quad (9)$$

$$H_2 (z) = Q_2 (Z) (Z+1)^{K2} \quad (10)$$

K_0 denotes the degree of polynomials representable by integer translates of $\Phi(t)$ and is related to the smoothness of $\Phi(t)$. K_1 and K_2 denote the number of zero moments of the wavelets filters $h_1(n)$ and $h_2(n)$, provided $K_0>K_1$ and $K_0\geq K_2$. If it is desired for a given class of signals that the wavelets have two zero moments (for example), then the remaining degrees of freedom can be used to achieve a higher degree of smoothness by making K_0 greater than K_1 and K_2. Although the values K_i need not all be equal, there is still the constraint:

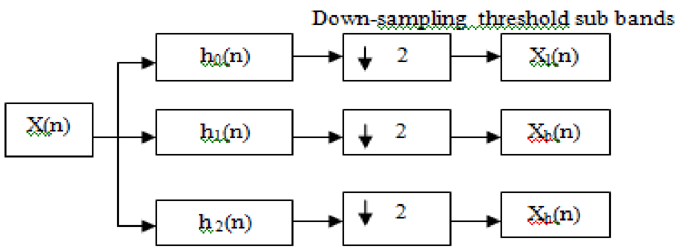
$$\text{Length } h_0 \geq K_0+\min(K_1, K_2) \quad (11)$$

So the minimum length of h_0 is $K_0+\min(K_1, K_2)$.
In the orthonormal case $K_0 = K_1$ and $K2 = \infty$ (as $h_2 = 0$),
which gives the minimum length of h_0 to be $2K_0$, which is consistent with Daubechies orthonormal filters.

The framelet transform is implemented on discrete-time signals using the over sampled analysis and synthesis filter bank .The analysis filter bank consists of three analysis filters- one low pass filter denoted by $h_0(n)$ and two distinct high pass filters denoted by $h_1(n)$ and $h_2(n)$. As the input signal X(N) travels through the system, the analysis filter bank decomposes it into three sub bands, each of which is then down-sampled by 2. From this process $X_L(N/2)$, $X_{H1}(N/2)$ and $X_{H2}(N/2)$ are generated, which represent the low frequency (or coarse) subband and the two high frequency (or detail) sub bands, respectively. The up sampled signals are filtered by the corresponding synthesis low pass $h_0*(n)$ and two high pass $h_1*(n)$ and $h_2*(n)$ filters and then added to reconstruct the original signal. Note that the filters in the synthesis stage, are not necessary the same as those in the analysis stage. For an orthogonal filter bank, $h_i*(n)$ are just the time reversals of $h_i(n)$. Wavelet frames, having the form described above, have twice as many wavelets than is necessary. However, if the filter bank is iterated a single time on its lowpass branch (h_0), the total oversampling rate will be 7/4. For a three-stage filter bank, the oversampling rate will be 15/8. When this filter bank is iterated on its lowpass branch indefinitely, the total oversampling rate increases toward 2, which is consistent with the redundancy of the frame for $L_2(R)$.

3.2 THE FRAMELET FILTERS

The analysis and the synthesis filters are stored as cell array. Because the frame is a tight frame, the synthesis filters are the time-reversed versions of the analysis filters. The analysis filter bank consists of three analysis filters-one low pass filter and two high pass filters. The analysis filter bank decomposes the signal into three sub bands, each of which is then down-sampled by 2. The representation of analysis filter and its filter coefficients are shown in Fig(1) and Fig(2). The advantage of the framelets is that it achieves low frequency than dwt because at each scale, it has twice as many wavelets as the dwt and has better time frequency localization.



Fig(1) Analysis filter (X(n)- Input signal, $h_0(n)$- low pass , $h_1(n)$- band pass , $h_2(n)$- high pass)

n	$h_0(n)$	$h_1(n)$	$h_2(n)$
0	0.00069616789827	0.000142030174443	0.00014203017443
1	-	0.00549320005590	-0.00549320005590
2	0.02692519074183	0.01098019299363	-0.00927404236573
3	-	0.136449097656612	0.07046152309968
4	0.04145457368920	-0.21696226276259	0.013542356651691
5	0.19056483888763	0.33707999754362	0.645783549940472
6	0.58422553883167	0.33707999754362	0.645783549940472

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7	0.58422553883167	0.21696226276259-	-0.13542356651691
8	0.19056483888763	-0.13644909765612	-0.07046152309968
9	-	0.01098019299363	0.00927404236573
10	0.04145457368920	0.00549320005590	0.00549320005590
11	-	-0.00014203017443	-0.00014203017443
	0.02692519074183		
	0.00069616789827		
	0		
	0		

Fig(2) Analysis filter coefficients

3.3 DENOISING IN FRAMELET ENVIRONMENT

Initially, healthy ECG signal is selected and noise is added to the signal.

3.3.1 NOISE ADDITON

A noise (Gaussian) is added to the original signal. Gaussian noise is signal independent and each sample will be changed from the normal by a small amount. The signals amplitude fluctuates randomly that results in the noise.
The PDF of a Gaussian random variable x, is given by

$$P(x)= 1/(\sigma\sqrt{2\pi})\{e^{-(x-\mu)2/(2\sigma 2)}\} \qquad (12)$$

x is the gray level, μ is the mean, σ is the standard deviation and σ^2 is the variance. Mathematically defined:

$$S(n) + V(n) = V_s(n) \qquad (13)$$

Where S(n) is the noise, V(n) is noise free ECG and $V_s(n)$ is the noisy ECG signal.

To denoise the signal, thresholding calculation is required. This is explained in the next section.

3.3.2 CALCULATION OF THRESHOLD

For each level of decomposition, a different threshold (T) value is calculated using the formula given in Equation 14.

$$T= \sigma\sqrt{2\log N} \qquad (14)$$

Where σ- median of framelt coefficients in each sub band
N- length of the signal in each sub band.

This method yields an overly smoothened signal. This threshold value is used to threshold the noisy Framelet coefficients. In thresholding, hard and soft threshold techniques are used. These techniques are explained in the next section.

3.3.3 HARD THRESHOLD

In this method , if framelet coefficients are greater than the threshold ,then the coefficients are retained else the coefficients are made zero. The formula is given by eqn(14) and it is represented in Fig (3).

$$A' = \begin{cases} A, & |A| \geq \sigma \\ 0, & |A| < \sigma \end{cases} \qquad (15)$$

where σ-threshold and A- the framelet coefficients.

3.3.4 SOFT THRESHOLD

In this method, if framelet coefficients are greater than or equal to the threshold , then the coefficients are subtracted from threshold and if coefficients are less than or equal to threshold,the coefficients are added to the threshold else the coefficients are made zero. The formula is given by equation(15) and it is represented in the Fig(4).

$$A^{|} = \begin{cases} A-T, & \text{if } A \geq T \\ A+T, & \text{if } A \leq -T \\ 0, & \text{if } A < -T \end{cases} \qquad (16)$$

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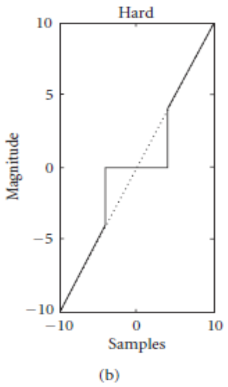
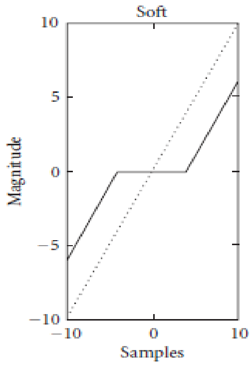


Fig(3) Hard Threshold

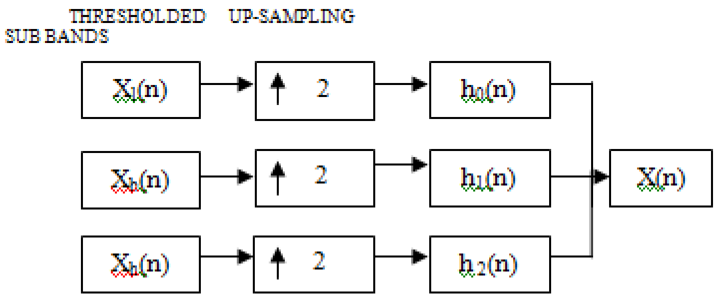


Fig(4) Soft Threshold

The thresholded framlet coefficients are passed to the synthesis filters for recomposition of signals.

3.4 INVERSE FRAMELET TRANSFORM

To reconstruct the original signal from the discrete framelet transformed signal, inverse fast discrete framelet transform should be used. The synthesis filter bank recomposes the three thresholded sub bands using Inverse Framelet transform. In this, each signal is up-sampled by 2 and it is represented in Fig(5).



Fig(5) Synthesis Filter (X(n)- denoised signal)

The overall algorithm of the proposed system is given in the next section.

IV. PROPOSED ALGORITHM

The algorithm for the proposed system is as follows:

Step1 : Read the ECG signal.
Step2 : Add noise to the signal.
Step3 : Apply framelet transform for decomposition
Step5 : Calculate threshold using visushrink method
Step6 : Framelet coefficients are then passed into thresholding function.

IF HARD THRESHOLD
Step6.1.1 : Unique value for each sub band is calculated.
Step61.2 : if coeff>thr, then coeff=0.
Step6.1.3 : else coeff are retained.

IF SOFT THRESHOLD
Step 6.1.1: Unique value for each sub band is calculated.
Step6.1.2 : if coeff≥thr, then coeff are added to the threshold.
else if coeff≤thr, then coeff are subtracted from the threshold.

Step6.1.3 : else coeff are retained.

Step7 : Apply inverse framelet transform to the thresholded sub bands.
Step8 : Display the denoised signal.

The flowchart of this algorithm is represented in fig(7) and fig(8) where Fr.coeff- framelet coefficients.

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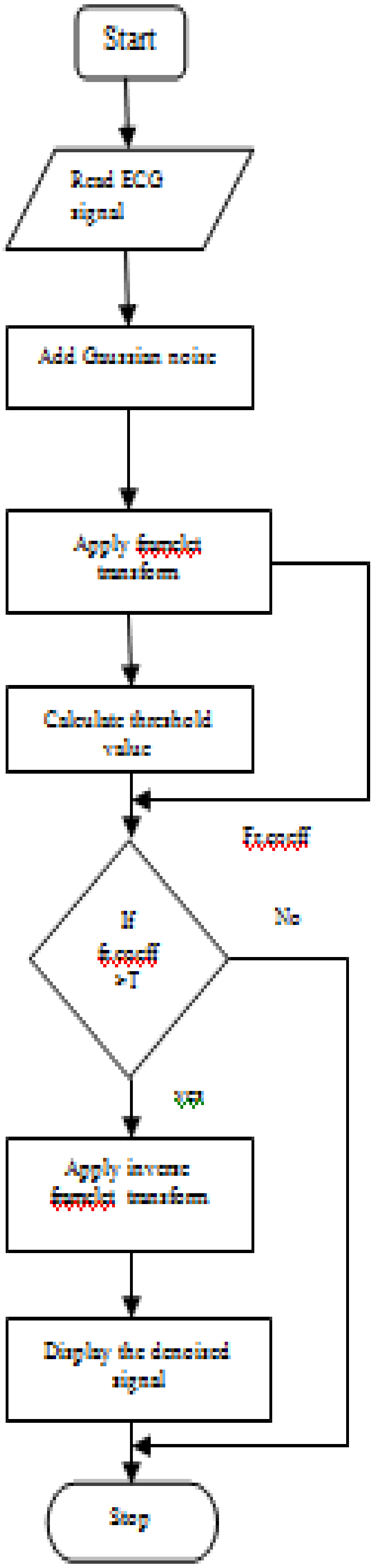

Fig (7). Flow chart for hard threshold

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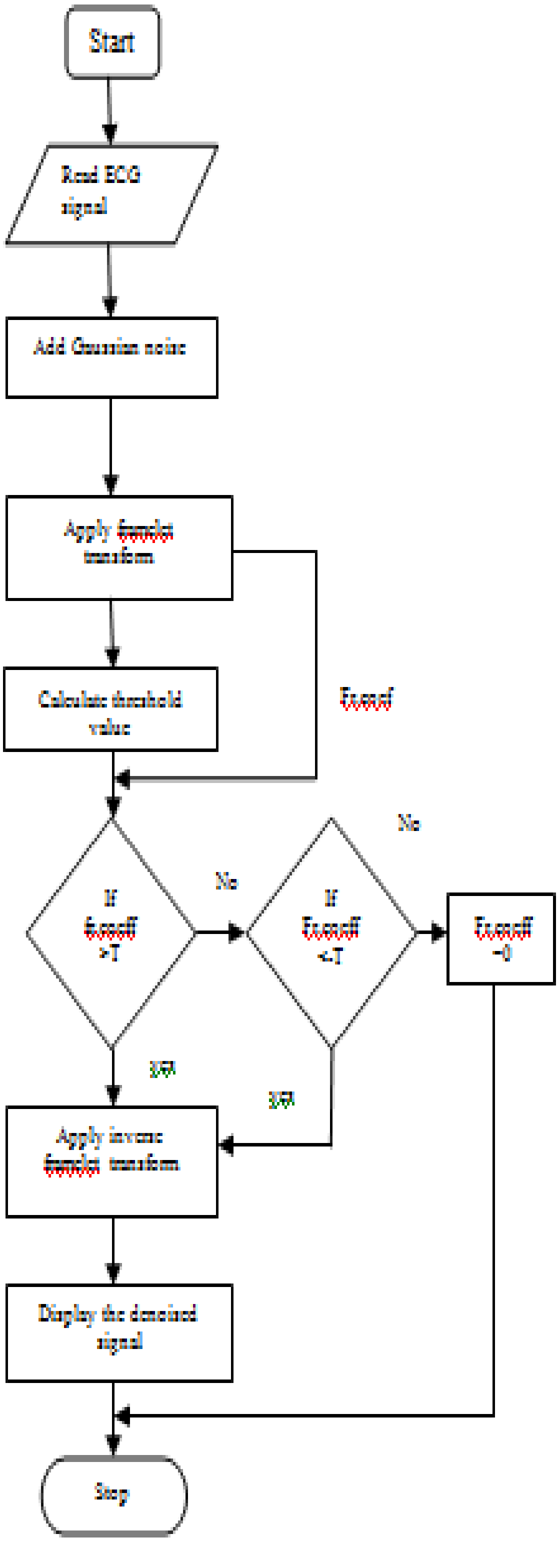

Fig.(8). Flow chart for soft threshold

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V. EXPERIMENTAL RESULTS

For the evaluation of the method, ten ECG signals are downloaded from the PhysioBank[8]. Each signal is having duration of 1 min. A known percentage of Gaussian noise is added to the noise free ECG signal. Experiments were conducted for various noise levels from 0%-50%.

Initially, the ECG signals are decomposed using single level framelet transform. Then denoising is conducted using Visushrink and proposed thresholding method. The denoised signal had poor visual quality of the signal but better SNR. Then, multilevel decomposition (level=2,3,4) using framelet transform is conducted and the denoising procedure is repeated. It is observed that the proposed thresholding method gives better SNR as well as good visual quality of the signal.

Table 1 details the detailed comparison of SNR (dB) of various ECG signals for different noise levels. The results of wavelet functions and Framelet transform are illustrated in Fig(10). The visual representation of denoised ECG for various noise levels using proposed framelet transform is illustrated in Fig 10 [(e),(f)].

For implementing the framelet transform, we used Framelet ToolBox [7]. The simulation was carried out using in Matlab 7 environment..

VI. PERFORMANCE EVALUATION

The performance of the reconstructed signal is measured by the calculation of important quality metrics. i.e, Signal-to-Noise ratio (SNR)[6].

The formula to calculate the SNR is as follows:

$$SNR= 20*\log10(\sigma/\sigma_n) \quad (16)$$

Where σ - Std dev of denoised signal.
σ_n - Std dev of noise.

Table 1 Comparison of SNR of various ECG signals for different noise levels

SIGNALS	NOISE LEVEL	WAVELETS		FRAMELETS	
		HARD THRESHOLD	SOFT THRESHOLD	HARD THRESHOLD	SOFT THRESHOLD
		SNR(dB)	SNR(dB)	SNR(dB)	SNR(dB)
a04m	20	8.0700	8.0415	6.7788	9.6893
	30	3.8719	3.1212	3.4868	5.8379
	40	-0.4398	0.0537	1.1871	2.9104
a05m	20	9.9568	10.4509	7.7635	11.6813
	30	5.6188	5.8705	4.7486	7.9287
	40	2.5726	2.3156	2.0301	5.2459
a07m	20	12.3860	12.3379	10.4902	13.6283
	30	8.1033	7.5376	6.5900	9.7609
	40	4.4922	4.7543	4.3741	6.7358

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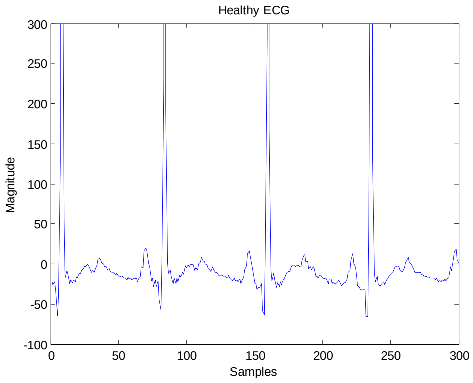
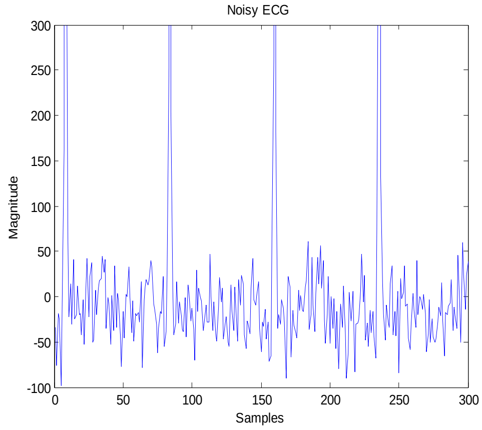
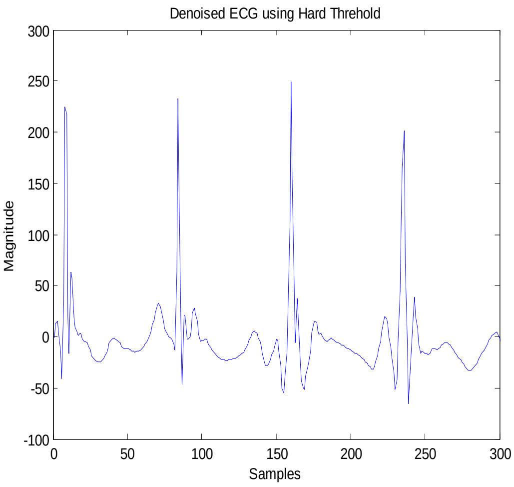
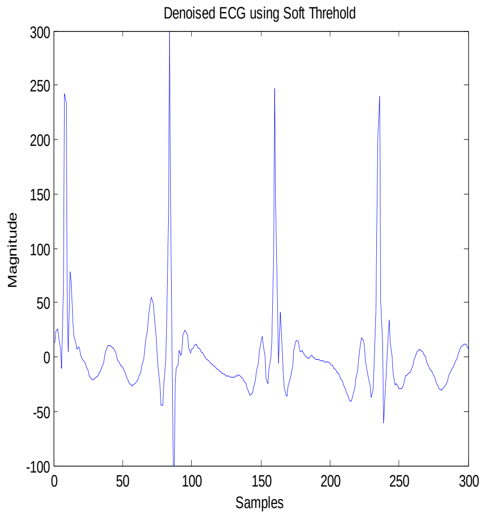


(a)



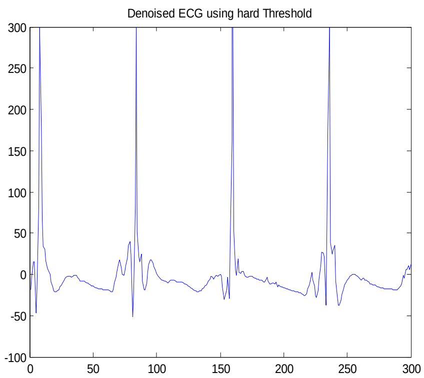
(b)

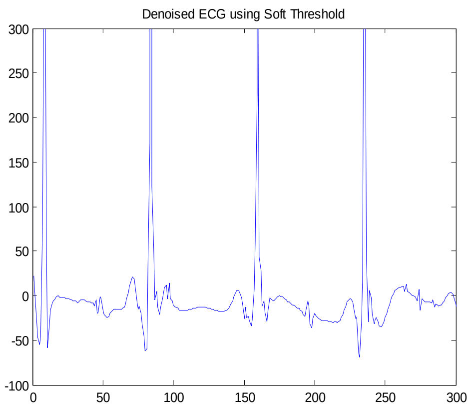




(d)

(c)





(f)

(e)

Fig 10. Noise level=30% (a) Healthy ECG (b) Noisy ECG (c) Denoised ECG(dwt-hard threshold) (d) Denoised ECG (dwt- soft threshold) (e) Denoised ECG (framelet-hard threshold) (f) Denoised ECG (framelet-soft threshold)

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VII. CONCLUSION

An approach for denoising of the ECG signal in the Framelet domain is proposed. The noisy ECG signal is decomposed using framelet transform. The decomposed signal is then denosied using a new thresholding technique. Performance evaluation shows that the results are better than Visushrink algorithm. This technique also shows that multilevel framelet decomposition with soft thresholding gives better results than hard thresholding. Experimental results further show that the technique is able to achieve better SNR than the wavelet based denoising techniques.

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