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Fuzzy Inventory Model for Deteriorating Items with Weibull Demand and Time Varying Holding Cost under Trade Credit

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ABSTRACT: This paper develops a fuzzy inventory model for variable deteriorating items with time dependent two parameter Weibull demand rate. Shortages are allowed and partially backlogged. Each cycle has shortages, which have been partially backlogged to suit present day competition in the market. This inventory system follows a time varying holding cost. Here, the retailer is allowed a trade credit offer by the supplier to buy more items. The deteriorating cost, shortage cost and opportunity cost are assumed as a trapezoidal fuzzy numbers. Signed distance method is used to defuzzify the model. Optimal solution for the model is derived and the trade credits on the optimal replenishment policy are studied with the help of numerical example. A sensitivity analysis is also given to show the effect of the costs.

KEYWORDS: Inventory, partial backlogging, Weibull demand, time varying holding cost, trade credit , fuzzy cost.

I. INTRODUCTION

In many inventory models uncertainty is due to fuzziness and fuzziness is the closed possible approach to reality. In recent years some researchers gave their attention towards a time dependent rate because the demand of newly launched products such as fashionable garments, electronic items, and mobiles etc. increases with time and later it becomes constant. F. Harris (1915) developed first inventory model. Lotfi A. Zadeh (1965) introduced the concept of fuzzy set theory in inventory modelling. L. A. Zadeh and R. E. Bellman (1970) considered an inventory model on decision making in fuzzy environment. H. J. Zimmerman (1983) tried to use fuzzy sets in operational research. K. S. Park (1987) define the fuzzy set theoretical interpretation of an EOQ problem. M. Vujosevic, D. Petrovic and R. Petrovic (1996) developed an EOQ formula by assuming inventory cost as a fuzzy number. J. S. Yao and H. M. Lee (1999) developed a fuzzy inventory model by considering backorder as a trapezoidal fuzzy number.

Singh and Singh (2008) developed the fuzzy inventory model for finite rate of replenishment using the signed distance method. Halim et al. (2008) developed a fuzzy inventory model for perishable items with stochastic demand, partial backlogging and fuzzy deterioration rate. The model is further extended to consider fuzzy partial backlogging factor. Goni and Maheswari (2010) discussed the retailer's ordering policy under two levels of delay payments considering the demand and the selling price as triangular fuzzy numbers. They used graded mean integration representation method for defuzzification. Halim et al. (2010) addressed the lot sizing problem in an unreliable production system with stochastic machine breakdown and fuzzy repair time. They defuzzified the cost per unit time using the signed distance method.

Deterioration is defined as damage, decay or spoilage of the items that are stored for future use always loose part of their value with passage of time, so deterioration cannot be avoided in any business scenarios. Deterioration is applicable to many inventories in practice like blood, fashion goods, agricultural products and medicine, highly volatile liquids such as gasoline; alcohol and turpentine undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film, grain, etc. deteriorate through a gradual loss

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of potential or utility with the passage of time. So decay or deterioration of physical goods in stock is a very realistic feature and inventory researchers felt the necessity to use this factory into consideration.

Demand is a major factor in inventory management. In inventory models, four types of demand are basically assumed, i.e., constant demand, time-dependent demand, probabilistic demand and stock-dependent demand. Many authors attempted to study such models. Silver and Meal (1969), were the first to suggest a simple modification of the EOQ formula for the case of varying demand. Later, Silver and Mean (1973) developed an approximate solution procedure, Recently Tripathy and Mishra (2010) developed the model for Weibull time-dependent demand rate with complete backlogging.

In the EOQ model, we assumed that the supplier must be paid for the items as soon as the items are received. However, in practice, this may not true. In today's business transactions, it is more and more to see that a supplier will allow a certain fixed period for setting the amount owed to him for the items supplied. Usually there is no charge if the outstanding amount is settled within the permitted fixed settlement period. Beyond this period, interest is charged. Recently Haley and Higgins (1973), Kingsman (1983), Chapman et al. (1985) examined the effect of the trade credit on the optimal inventory policy. Furthermore, Goyal (1985) explored a single item economic order quantity model under conditions of permissible delay in payments , Aggarwal and Jaggi (1995) extended Goyal's model to the case of deterioration, Kumar, M et al. (2008) developed an EOQ model for time varying demand rate under trade credits, Kumar, M et al. (2009) presented an inventory model for power demand rate incremental holding cost under permissible delay in payments and Kumar et al. (2009) developed an inventory model for quadratic demand rate, inflation with permissible delay in payments.

In this paper developed an inventory model for deteriorating items with demand as two parameter Weibull distribution rate, time varying holding cost, partially backlogged and delay period is allowed. The deterioration cost, shortage cost and opportunity cost taken as trapezoidal fuzzy number. Signed distance method is applied for defuzzification. The total inventory costs are obtained in crisp and as well as fuzzy sense with the help of Signed distance method. Numerical examples have been given to illustrate the model. Sensitivity analysis has also been carried out to observe the effects on the optimal solution.

II. NOTATIONS AND ASSUMPTIONS

The proposed inventory model having following notations and assumptions:

2.1 Notations

- $I(t)$ - Inventory level at time t
- $\theta(t)$ - θt is the deterioration rate per unit time
- A - Ordering cost per cycle in $
- C_1 - Deteriorating cost per unit per unit time
- C_2 - Shortage cost, per unit per unit time
- C_3 - Opportunity cost due to last sales, $ per unit
- T - Length of the replenishment cycle
- I_e - Interest which can be earned $ per unit time
- I_c - Interest charges which invested in inventory, $ per unit time
- q - The maximum inventory level at t = 0
- BI - The maximum backordered units during stock-out period.
- Q - The economic order quantity for the inventory cycle.
- M - Permissible delay in settling the amounts
- T_1 - Time when inventory level comes down to zero, $0 < T_1 < T$
- TC_1 - Total inventory cost when $M < T_1$

TC_2 - Total inventory cost when $M > T_1$
$\tilde{C}_1$ - Fuzzy deteriorating cost
$\tilde{C}_2$ - Fuzzy shortage cost, per unit per year
$\tilde{C}_3$ - Fuzzy opportunity cost due to last sales, $ per unit
$T\tilde{C}_1$ - Fuzzy total inventory cost when $M < T_1$
$T\tilde{C}_2$ - Fuzzy total inventory cost when $M > T_1$

$TC_{1(ds)}(T_1,T)$ - defuzzify value of $T\tilde{C}_1$
$TC_{2(ds)}(T_1,T)$ - defuzzify value of $T\tilde{C}_2$

2.2 Assumptions

1. The inventory system involves only one item
2. Replenishment occurs instantaneously on ordering i.e. lead time is zero and it takes place at an infinite rate.
3. The holding cost is linear with time dependent $h(t) = (h + rt)$ where $h > 0$ and $r > 0$ is the inventory holding cost per unit time in $.
4. The demand rate function $D(t) = \alpha\beta t^{\beta-1}$; $\alpha > 0,\ \beta > 0$, follows two parameter Weibull distribution in time t.
5. Shortages are allowed and unsatisfied demand is backlogged at the rate of $\frac{1}{1+\delta(T-t)}$ the backlogging parameter δ is a positive constant, $T_1 < t < T$

III. FUZZY PRELIMINARIES

A fuzzy set $\tilde{A}$ on the given universal set X is a set of ordered pairs $\tilde{A} = \{x, \mu_A(x) : x \in X\}$

Where: $\mu_{\tilde{A}} : X \to [0,1]$, is called membership function. The α − cut of $\tilde{A}$, is defined by

$$A_\alpha = \{X : \mu_A(x) = \alpha, \alpha \geq 0\}$$

If R is the real line, then a fuzzy number is a fuzzy set $\tilde{A}$ with membership function $\mu_{\tilde{A}} : X \to [0,1]$, having the following properties:

(i) $\tilde{A}$ is normal, i.e., there exists $x \epsilon R$ such that $\mu_A(x) = 1$
(ii) $\tilde{A}$ is piece-wise continuous
(iii) Supp $(\tilde{A}) = cl\{x \in R : \mu_A(x) > 0\}$, where cl represents the closure of a set
(iv) $\tilde{A}$ is a convex fuzzy set.

Definition 3.1: A trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d) is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(X) = \begin{cases} L(X) = \dfrac{x-a}{b-a}, & \text{when } a \leq x \leq b \\ 1 & , \text{when } b \leq x \leq c \\ R(X) = \dfrac{d-x}{d-c}, & \text{when } c \leq x \leq d \\ 0 & , \text{otherwise} \end{cases}$$

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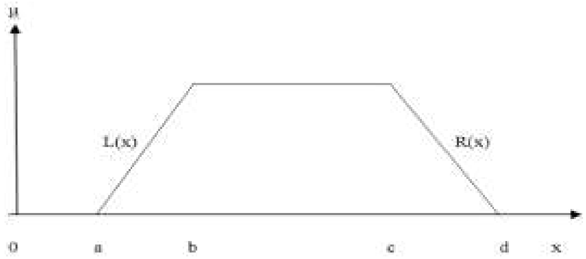

Fig.1: Trapezoidal Fuzzy Number

Definition 3.2: A fuzzy set is called in *LR*-Form, if there exist reference functions *L* (for left), *R* (for right), and scalars m > 0 and n > 0 with membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{\sigma - x}{m}\right), \text{for } x \le \sigma \\ 1 \qquad\qquad, \text{for } \sigma \le x \le \gamma \\ R\left(\dfrac{x - \gamma}{n}\right), \text{for } x \ge \gamma \end{cases}$$

Where σ is a real number called the mean value of $\tilde{A}$, m and n are called the left and right spreads, respectively. The functions *L* and *R* map $R^{+} \to [0,1]$, and are decreasing. A *LR*-Type fuzzy number can be represented as $\tilde{A} = (\sigma, \gamma, m, n)_{LR}$

Definition 3.3: Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then arithmetical operations are defined as:

(i) Addition: $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(ii) Subtraction: $\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

(iii) Multiplication: $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

(iv) Division: $\dfrac{\tilde{A}}{\tilde{B}} = (\dfrac{a_1}{b_4}, \dfrac{a_2}{b_3}, \dfrac{a_3}{b_2}, \dfrac{a_4}{b_4})$

(v) Scalar Multiplication: For any real number α ,

$$\alpha\tilde{A} = \begin{Bmatrix} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), \alpha \ge 0 \\ (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1), \alpha < 0 \end{Bmatrix}$$

Definition 3.4: Let $\tilde{A}$ be a fuzzy set defined on R. Then the signed distance of $\tilde{A}$ is defined as:

$$d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)] d\alpha$$

where $A_\alpha = [A_L(\alpha), A_R(\alpha)]$

$$= [a + (b - a)\alpha, d - (d - c)\alpha], \ \alpha \in [0,1]$$

is $\alpha -$ cut of fuzzy set $\tilde{A}$, which is a close interval.

Remark: The signed distance *d(a, 0) = a*, for all *a, 0* ∈ *R*. The meaning of Definition 3.4 is as the follows, if *0< a* then the distance between *a* and *0* is *d(a,0) = a*. If *a < 0* then the distance between *a* and *0* is − *d(a, 0)* = −*a*. Therefore, we call *d(a, 0) = a* is the signed distance between *a* and *0.*

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IV. MODEL FORMULATION

4.1 (CRISP MODEL)

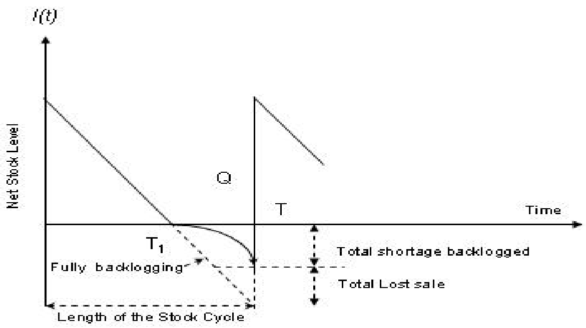

Fig.1 Graphical representation of the inventory system

In this model two parameter weibull type of demand is considered with two parameter exponential distribution deterioration rates. Depletion of the inventory occurs due to demand (supply) as well as due to inventory i.e. during the period (0, T) and shortages occurs during period (T_1, T).The differential equation of inventory level I (t) w.r.t. time is given by

$$\frac{dI(t)}{dt} + \theta t I(t) = -\alpha\beta t^{\beta-1} \quad ; \quad 0 < t \le T_1 \qquad \text{-------------- (1)}$$

$$\frac{dI(t)}{dt} = \frac{-\alpha\beta t^{\beta-1}}{1+\delta(T-t)} \qquad ; \quad T_1 \le t \le T \qquad \text{-------------- (2)}$$

With boundary condition $I(T_1) = 0$ and I (0) = q. The solution of equation (1) and (2) is

$$I(t) = \left\{\alpha\left(T_1^{\beta} - t^{\beta}\right) + \frac{\alpha\beta\theta}{2(\beta+2)}\left(T_1^{\beta+2} - t^{\beta+2}\right) - \frac{\alpha\theta}{2}\left(T_1^{\beta}t^2 - t^{\beta+2}\right) - \frac{\alpha\beta\theta^2}{4(\beta+2)}\left(T_1^{\beta+2}t^2 - t^{\beta+4}\right)\right\} \qquad \text{---- (3)}$$

$$I(t) = \alpha t^{\beta}(\delta T - 1) - \frac{\alpha\beta\delta t^{\beta+1}}{\beta+1} \qquad \text{-------------- (4)}$$

I (0) = q in (3) we get

$$q = \alpha\left(T_1^{\beta}\right) + \frac{\alpha\beta\theta}{2(\beta+2)}\left(T_1^{\beta+2}\right)$$

The maximum backordered inventory BI is obtained at t = T , then from (4)

$$BI = -I(T) = -\alpha T^{\beta}(\delta T - 1) + \frac{\alpha\beta\delta T^{\beta+1}}{\beta+1}$$

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Economic order quantity

$$Q = q + BI = \alpha\left(T_1^{\beta}\right) + \frac{\alpha\beta\theta}{2(\beta+2)}\left(T_1^{\beta+2}\right) - \alpha T^{\beta}(\delta T - 1) + \frac{\alpha\beta\delta T^{\beta+1}}{\beta+1} \qquad \text{-------------- (5)}$$

Inventory holding cost:

$$HC = \int_0^{T_1} hI(t)dt$$
$$= \int_0^{T_1} (h+rt)I(t)dt$$
$$= \left\{\frac{h\alpha\beta T_1^{\beta+1}}{\beta+1} + \frac{r\alpha\beta T_1^{\beta+2}}{2(\beta+2)} + \frac{h\alpha\beta\theta T_1^{\beta+3}}{3(\beta+3)} + \frac{r\alpha\beta\theta T_1^{\beta+4}}{8(\beta+4)} - \frac{h\alpha\beta\theta^2 T_1^{\beta+5}}{12(\beta+5)} - \frac{r\alpha\beta\theta^2 T_1^{\beta+6}}{16(\beta+6)}\right\} \text{------- (6)}$$

Shortage Cost:

$$SC = -C_2\int_{T_1}^{T} I(t)dt$$
$$= C_2\left[\frac{\alpha(\delta T - 1)(T_1^{\beta+1} - T^{\beta+1})}{(\beta+1)} + \frac{\alpha\beta\delta(T^{\beta+2} - T_1^{\beta+2})}{(\beta+1)(\beta+2)}\right] \qquad \text{-------------- (7)}$$

Opportunity Cost:

$$LC = C_3\int_{T_1}^{T}\left[1 - \frac{\alpha\beta t^{\beta-1}}{1+\delta(T-t)}\right]dt$$
$$= C_3\left[\frac{\delta\alpha T^{\beta+1}}{\beta+1} - \delta\alpha T_1^{\beta}\left(T - \frac{\beta T_1}{\beta+1}\right)\right] \qquad \text{-------------- (8)}$$

Deterioration cost:

$$DC = C_1\left[Q - \int_0^{T_1} D(t)dt\right]$$
$$= C_1\left[\frac{\alpha\beta\theta T_1^{\beta+2}}{2(\beta+2)} - \alpha T^{\beta}(\delta T - 1) + \frac{\alpha\beta\delta T^{\beta+1}}{\beta+1}\right] \qquad \text{-------------- (9)}$$

Setup cost:

$$SC = A \qquad \text{-------------- (10)}$$

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CRISP MODEL (CASE- I): $M < T_1$

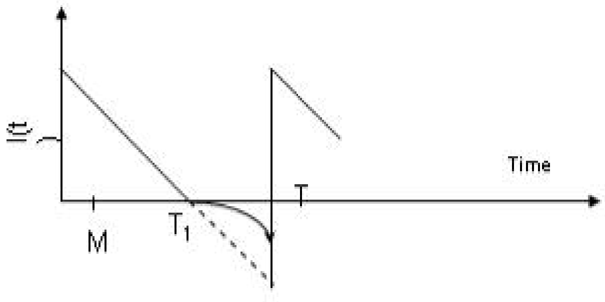

Fig.2. Inventory level as a function of time for case I: $M < T_1$

In this situation since the length of period with positive stock is larger than the credit period, the buyer can use the sale revenue to earn interest at an annual rate I_e in $(0, T_1)$

The interest earned IE_1, is

$$IE_1 = PI_e \int_0^M D(t)t dt = \frac{PI_e \alpha\beta M^{\beta+1}}{(\beta+1)} \qquad \text{-------------- (11)}$$

However, beyond the credit period, the unsold stock is supposed to be financed with an annual rate I_c and the interest charged IC_1 is

$$IC_1 = PI_c \int_M^{T_1} I(t)dt$$

$$= PI_c \left\{ \begin{aligned} & T_1^{\beta}\left(\frac{\alpha\theta M^3 - 6\alpha M}{6}\right) + T_1^{\beta+1}\left(\frac{\alpha\beta}{\beta+1}\right) + T_1^{\beta+2}\left(\frac{\alpha\beta\theta^2 M^3 - 6\alpha\beta\theta}{12(\beta+2)}\right) \\ & + T_1^{\beta+3}\left(\frac{\alpha\beta\theta}{3(\beta+3)}\right) - T_1^{\beta+5}\left(\frac{\alpha\beta\theta^2}{30(\beta+2)}\right) + \frac{\alpha M^{\beta+1}}{\beta+1} - \frac{\alpha\theta M^{\beta+3}}{(\beta+2)(\beta+3)} - \frac{\alpha\beta\theta^2 M^{\beta+5}}{20(\beta+2)} \end{aligned} \right\} \qquad \text{----- (12)}$$

Total Inventory cost per unit time is given by

$$TC_1 = \frac{A + HC + SC + LC + DC + IC_1 - IE_1}{T}$$

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$$TC_1(T_1,T)=\left\{\begin{array}{l} \dfrac{A}{T}+\dfrac{1}{T}\left\{\dfrac{h\alpha\beta T_1^{\beta+1}}{\beta+1}+\dfrac{r\alpha\beta T_1^{\beta+2}}{2(\beta+2)}+\dfrac{h\alpha\beta\theta T_1^{\beta+3}}{3(\beta+3)}+\dfrac{r\alpha\beta\theta T_1^{\beta+4}}{8(\beta+4)}-\dfrac{h\alpha\beta\theta^2 T_1^{\beta+5}}{12(\beta+5)}-\dfrac{r\alpha\beta\theta^2 T_1^{\beta+6}}{16(\beta+6)}\right\} \\ +\dfrac{C_2}{T}\left[\dfrac{\alpha(\delta T-1)(T_1^{\beta+1}-T^{\beta+1})}{(\beta+1)}+\dfrac{\alpha\beta\delta(T^{\beta+2}-T_1^{\beta+2})}{(\beta+1)(\beta+2)}\right]+\dfrac{C_3}{T}\left[\dfrac{\delta\alpha T^{\beta+1}}{\beta+1}-\delta\alpha T_1^{\beta}\left(T-\dfrac{\beta T_1}{\beta+1}\right)\right] \\ +\dfrac{PI_c}{T}\left\{\begin{array}{l} T_1^{\beta}\left(\dfrac{\alpha\theta M^3-6\alpha M}{6}\right)+T_1^{\beta+1}\left(\dfrac{\alpha\beta}{\beta+1}\right)+T_1^{\beta+2}\left(\dfrac{\alpha\beta\theta^2 M^3-6\alpha\beta\theta}{12(\beta+2)}\right)+T_1^{\beta+3}\left(\dfrac{\alpha\beta\theta}{3(\beta+3)}\right) \\ -T_1^{\beta+5}\left(\dfrac{\alpha\beta\theta^2}{30(\beta+2)}\right)+\dfrac{\alpha M^{\beta+1}}{\beta+1}-\dfrac{\alpha\theta M^{\beta+3}}{(\beta+2)(\beta+3)}-\dfrac{\alpha\beta\theta^2 M^{\beta+5}}{20(\beta+2)} \end{array}\right\} \\ +\dfrac{C_1}{T}\left[\dfrac{\alpha\beta\theta T_1^{\beta+2}}{2(\beta+2)}-\alpha T^{\beta}(\delta T-1)+\dfrac{\alpha\beta\delta T^{\beta+1}}{\beta+1}\right]-\dfrac{PI_e\alpha\beta M^{\beta+1}}{T(\beta+1)} \end{array}\right\}$$

-------------- (13)

The necessary condition for $TC_1(T_1,T)$ to be minimize is that $\frac{\partial TC_1(T_1,T)}{\partial T}=0$ and $\frac{\partial TC_1(T_1,T)}{\partial T_1}=0$

Solving these equations we find the optimum values of T_1 and T say T_1^* and T^* for which cost is minimum and the sufficient condition is $\left(\frac{\partial^2 TC_1(T_1,T)}{\partial T^2}\right)\left(\frac{\partial^2 TC_1(T_1,T)}{\partial T_1^2}\right)-\left(\frac{\partial^2 TC_1(T_1,T)}{\partial T\partial T_1}\right)^2>0$,

$$\frac{\partial^2 TC_1(T_1,T)}{\partial T_1^2}>0,\ \frac{\partial^2 TC_1(T_1,T)}{\partial T^2}>0$$

The optimal solution of the equations (13) can be obtained by using appropriate software. This has been illustrated by the following numerical example.

Crisp Model (Case II) : $M > T_1$

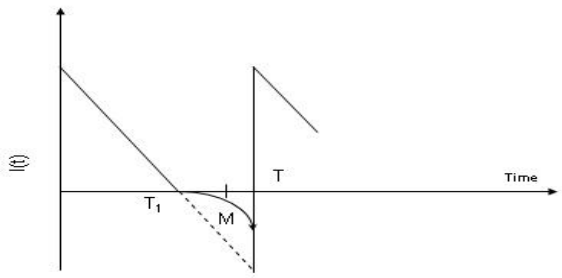

Fig.3. Inventory level as a function of time for case II: $M > T_1$

In this situation interest charged $IC_2=0$ and the interest earned per time unit is

$$IE_2=PI_e\left\{\int_0^{T_1}D(t)tdt+(M-T_1)T_1D(T)\right\}$$

$$=PI_e\left\{\alpha\beta MT_1^{\beta}-\frac{\alpha\beta^2T_1^{\beta+1}}{\beta+1}\right\}$$ -------------- (14)

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Total Inventory cost per unit time is given by

$$TC_2 = \frac{A + HC + SC + LC + DC - IE_2}{T}$$

$$TC_2 = \begin{cases} \frac{A}{T} + \frac{1}{T}\left\{ \frac{h\alpha\beta T_1^{\beta+1}}{\beta+1} + \frac{r\alpha\beta T_1^{\beta+2}}{2(\beta+2)} + \frac{h\alpha\beta\theta T_1^{\beta+3}}{3(\beta+3)} + \frac{r\alpha\beta\theta T_1^{\beta+4}}{8(\beta+4)} - \frac{h\alpha\beta\theta^2 T_1^{\beta+5}}{12(\beta+5)} - \frac{r\alpha\beta\theta^2 T_1^{\beta+6}}{16(\beta+6)} \right\} \\ + \frac{C_2}{T}\left[\frac{\alpha(\delta T - 1)(T_1^{\beta+1} - T^{\beta+1})}{(\beta+1)} + \frac{\alpha\beta\delta(T^{\beta+2} - T_1^{\beta+2})}{(\beta+1)(\beta+2)} \right] \\ + \frac{C_3}{T}\left[\frac{\delta\alpha T^{\beta+1}}{\beta+1} - \delta\alpha T_1^{\beta}\left(T - \frac{\beta T_1}{\beta+1} \right) \right] + \frac{C_1}{T}\left[\frac{\alpha\beta\theta T_1^{\beta+2}}{2(\beta+2)} - \alpha T^{\beta}(\delta T - 1) + \frac{\alpha\beta\delta T^{\beta+1}}{\beta+1} \right] \\ - \frac{PI_e}{T}\left\{ \alpha\beta M T_1^{\beta} - \frac{\alpha\beta^2 T_1^{\beta+1}}{\beta+1} \right\} \end{cases}$$

-------------- (15)

The necessary condition for $TC_2(T_1, T)$ to be minimize is that $\frac{\partial TC_2(T_1,T)}{\partial T} = 0$ and $\frac{\partial TC_2(T_1,T)}{\partial T_1} = 0$ Solving these equations we find the optimum values of T_1 and T say T_1^* and T^* for which cost is minimum and the sufficient condition is $\left(\frac{\partial^2 TC_2(T_1,T)}{\partial T^2}\right)\left(\frac{\partial^2 TC_2(T_1,T)}{\partial T_1^2}\right) - \left(\frac{\partial^2 TC_2(T_1,T)}{\partial T \partial T_1}\right)^2 > 0$

$$\frac{\partial^2 TC_2(T_1,T)}{\partial T^2} > 0, \quad \frac{\partial^2 TC_2(T_1,T)}{\partial T_1^2} > 0$$

The optimal solution of the equations (15) can be obtained by using appropriate software. This has been illustrated by the following numerical example.

4.2. FUZZY MODEL:

Let us consider the inventory model in fuzzy environment due to deterioration cost, shortage cost and opportunity cost. Let $\tilde{C}_1 = (C_{11}, C_{12}, C_{13}, C_{14})$, $\tilde{C}_2 = (C_{21}, C_{22}, C_{23}, C_{24})$ and $\tilde{C}_3 = (C_{31}, C_{32}, C_{33}, C_{34})$ are trapezoidal fuzzy numbers then the total cost per unit time in fuzzy sense is

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Fuzzy model: Case- I

$$T\tilde{C}_1(T_1,T)=\left\{\begin{array}{l} \frac{A}{T}+\frac{1}{T}\left\{\frac{h\alpha\beta T_1^{\beta+1}}{\beta+1}+\frac{r\alpha\beta T_1^{\beta+2}}{2(\beta+2)}+\frac{h\alpha\beta\theta T_1^{\beta+3}}{3(\beta+3)}+\frac{r\alpha\beta\theta T_1^{\beta+4}}{8(\beta+4)}-\frac{h\alpha\beta\theta^2 T_1^{\beta+5}}{12(\beta+5)}-\frac{r\alpha\beta\theta^2 T_1^{\beta+6}}{16(\beta+6)}\right\} \\ +\frac{\tilde{C}_2}{T}\left[\frac{\alpha(\delta T-1)(T_1^{\beta+1}-T^{\beta+1})}{(\beta+1)}+\frac{\alpha\beta\delta(T^{\beta+2}-T_1^{\beta+2})}{(\beta+1)(\beta+2)}\right]+\frac{\tilde{C}_3}{T}\left[\frac{\delta\alpha T^{\beta+1}}{\beta+1}-\delta\alpha T_1^{\beta}\left(T-\frac{\beta T_1}{\beta+1}\right)\right] \\ +\frac{PI_c}{T}\left\{\begin{array}{l} T_1^{\beta}\left(\frac{\alpha\theta M^3-6\alpha M}{6}\right)+T_1^{\beta+1}\left(\frac{\alpha\beta}{\beta+1}\right)+T_1^{\beta+2}\left(\frac{\alpha\beta\theta^2 M^3-6\alpha\beta\theta}{12(\beta+2)}\right) \\ +T_1^{\beta+3}\left(\frac{\alpha\beta\theta}{3(\beta+3)}\right)-T_1^{\beta+5}\left(\frac{\alpha\beta\theta^2}{30(\beta+2)}\right)+\frac{\alpha M^{\beta+1}}{\beta+1}-\frac{\alpha\theta M^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{\alpha\beta\theta^2 M^{\beta+5}}{20(\beta+2)} \end{array}\right\} \\ +\frac{\tilde{C}_1}{T}\left[\frac{\alpha\beta\theta T_1^{\beta+2}}{2(\beta+2)}-\alpha T^{\beta}(\delta T-1)+\frac{\alpha\beta\delta T^{\beta+1}}{\beta+1}\right]-\frac{PI_e\alpha\beta M^{\beta+1}}{T(\beta+1)} \end{array}\right\}$$

-------------- (16)

Now defuzzify the total cost $T\tilde{C}_1(T_1,T)$ by using Signed distance method , we have

$$TC_{1(ds)}(T_1,T)=\frac{1}{4}\left\{TC_{1(ds_1)}(T_1,T)+TC_{1(ds_2)}(T_1,T)+TC_{1(ds_3)}(T_1,T)+TC_{1(ds_4)}(T_1,T)\right\}$$

-------------- (17)

The necessary condition for $TC_{1(ds)}(T_1,T)$ to be minimize is that $\frac{\partial TC_{1(ds)}(T_1,T)}{\partial T}=0$ and $\frac{\partial TC_{1(ds)}(T_1,T)}{\partial T_1}=0$

Solving these equations we find the optimum values of T_1 and T say T_1^* and T^* for which cost is minimum and the sufficient condition is $\left(\frac{\partial^2 TC_{1(ds)}(T_1,T)}{\partial T^2}\right)\left(\frac{\partial^2 TC_{1(ds)}(T_1,T)}{\partial T_1^2}\right)-\left(\frac{\partial^2 TC_{1(ds)}(T_1,T)}{\partial T\partial T_1}\right)^2>0,$

$$\frac{\partial^2 TC_{1(ds)}(T_1,T)}{\partial T_1^2}>0,\quad \frac{\partial^2 TC_{1(ds)}(T_1,T)}{\partial T^2}>0$$

The optimal solution of the equations (17) can be obtained by using appropriate software. This has been illustrated by the following numerical example.

Fuzzy model: Case- II

$$T\tilde{C}_2(T_1,T)=\left\{\begin{array}{l} \frac{A}{T}+\frac{1}{T}\left\{\frac{h\alpha\beta T_1^{\beta+1}}{\beta+1}+\frac{r\alpha\beta T_1^{\beta+2}}{2(\beta+2)}+\frac{h\alpha\beta\theta T_1^{\beta+3}}{3(\beta+3)}+\frac{r\alpha\beta\theta T_1^{\beta+4}}{8(\beta+4)}-\frac{h\alpha\beta\theta^2 T_1^{\beta+5}}{12(\beta+5)}-\frac{r\alpha\beta\theta^2 T_1^{\beta+6}}{16(\beta+6)}\right\} \\ +\frac{\tilde{C}_2}{T}\left[\frac{\alpha(\delta T-1)(T_1^{\beta+1}-T^{\beta+1})}{(\beta+1)}+\frac{\alpha\beta\delta(T^{\beta+2}-T_1^{\beta+2})}{(\beta+1)(\beta+2)}\right] \\ +\frac{\tilde{C}_3}{T}\left[\frac{\delta\alpha T^{\beta+1}}{\beta+1}-\delta\alpha T_1^{\beta}\left(T-\frac{\beta T_1}{\beta+1}\right)\right] \\ +\frac{\tilde{C}_1}{T}\left[\frac{\alpha\beta\theta T_1^{\beta+2}}{2(\beta+2)}-\alpha T^{\beta}(\delta T-1)+\frac{\alpha\beta\delta T^{\beta+1}}{\beta+1}\right]-\frac{PI_e}{T}\left\{\alpha\beta M T_1^{\beta}-\frac{\alpha\beta^2 T_1^{\beta+1}}{\beta+1}\right\} \end{array}\right\},$$

-------------- (18)

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Now defuzzify the total cost $T\tilde{C}_2(T_1,T)$ by using Signed distance method , we have

$$TC_{2(ds)}(T_1,T)=\frac{1}{4}\left\{TC_{2(ds_1)}(T_1,T)+TC_{2(ds_2)}(T_1,T)+TC_{2(ds_3)}(T_1,T)+TC_{2(ds_4)}(T_1,T)\right\}$$

-------------- (19)

The necessary condition for $TC_{2(ds)}(T_1,T)$ to be minimize is that $\frac{\partial TC_{2(ds)}(T_1,T)}{\partial T}=0$ and $\frac{\partial TC_{2(ds)}(T_1,T)}{\partial T_1}=0$

Solving these equations we find the optimum values of T_1 and T say T_1^* and T^* for which cost is minimum and the sufficient condition is $\left(\frac{\partial^2 TC_{2(ds)}(T_1,T)}{\partial T^2}\right)\left(\frac{\partial^2 TC_{2(ds)}(T_1,T)}{\partial T_1^2}\right)-\left(\frac{\partial^2 TC_{2(ds)}(T_1,T)}{\partial T\partial T_1}\right)^2>0,$

$$\frac{\partial^2 TC_{2(ds)}(T_1,T)}{\partial T_1^2}>0,\ \frac{\partial^2 TC_{2(ds)}(T_1,T)}{\partial T^2}>0$$

The optimal solution of the equations (19) can be obtained by using appropriate software. This has been illustrated by the following numerical example.

V. NUMERICAL EXAMPLE

Case I: $M<T_1$

For Crisp Model:

A = 400; h =10; r = 0.4; P =30; α = 0.98; β = 5 ; C_1 = 35 ; C_2 = 10 ; C_3 = 11 ; θ = 0.01, I_e =0.12 ; I_c = 0.15; δ = 0.03 ; M=0.0274 in appropriate units. The solution of crisp model is : T = 1.2227; T_1 = 0.0408; TC_1 = 407.901

For Fuzzy Model:

When $\tilde{C}_1,\tilde{C}_2,\tilde{C}_3$ are trapezoidal fuzzy numbers then, $\tilde{C}_1=(32,34,36,38)$, $\tilde{C}_2=(7,9,11,13)$ $\tilde{C}_3=(8,10,12,14)$. The solution of fuzzy model is : T = 1.2226 ; T_1 = 0.0407 ; $TC_{1(ds)}(T_1,T)$ = 407.719

Case II: $M>T_1$

For Crisp Model:

A = 400; h = 10; r = 0.4; P = 30; α = 0.98; β = 5 ; C_1 = 35 ; C_2 = 10 ; C_3 = 11 ; θ = 0.01, I_e = 0.12 ; I_c = 0.15; δ = 0.03; M = 0.9863 in appropriate units. The solution of crisp model is : T = 1.2225; T_1 = 0.6815; TC_2 = 407.542

For Fuzzy Model:

When $\tilde{C}_1,\tilde{C}_2,\tilde{C}_3$ are trapezoidal fuzzy numbers then, $\tilde{C}_1=(32,34,36,38)$, $\tilde{C}_2=(7,9,11,13)$ $\tilde{C}_3=(8,10,12,14)$. The solution of fuzzy model is : T = 1.2224 ; T_1 = 0.6817 ; $TC_{2(ds)}(T_1,T)$ = 407.358

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VI. SENSITIVITY ANALYSIS

To study the effects of changes in the system parameters, the sensitivity is analyzed. The results are shown in below tables

Sensitivity Analysis on Parameter C_1

Table.1

Defuzzify Value of C_1	Fuzzify value of parameter $\tilde{C}_1$	T_1	T	$TC_{1(ds)}(T_1,T)$
36	(33,35,37,39)	0.0406	1.2164	410.06
37	(34,36,38,40)	0.0405	1.2103	412.17
38	(35,37,39,41)	0.0403	1.2043	414.24
39	(36,38,40,42)	0.0402	1.1986	416.27

Table.2

Defuzzify Value of C_1	Fuzzify value of parameter $\tilde{C}_1$	T_1	T	$TC_{2(ds)}(T_1,T)$
36	(33,35,37,39)	0.6814	1.2162	409.69
37	(34,36,38,40)	0.6812	1.2101	411.81
38	(35,37,39,41)	0.6811	1.2041	413.87
39	(36,38,40,42)	0.6809	1.1983	415.90

Sensitivity Analysis on Parameter C_2

Table.3

Defuzzify Value of C_2	Fuzzify value of parameter $\tilde{C}_2$	T_1	T	$TC_{1(ds)}(T_1,T)$
9	(6,8,10,12)	0.0402	1.2244	407.46
8	(5,7,9,11)	0.0397	1.2260	407.01
7	(4,6,8,10)	0.0391	1.2277	406.56
6	(3,5,7,9)	0.0386	1.2294	406.11

Table.4

Defuzzify Value of C_2	Fuzzify value of parameter $\tilde{C}_2$	T_1	T	$TC_{2(ds)}(T_1,T)$
9	(6,8,10,12)	0.6766	1.2242	407.11
8	(5,7,9,11)	0.6718	1.2258	406.68
7	(4,6,8,10)	0.6670	1.2275	406.24
6	(3,5,7,9)	0.6623	1.2292	405.80

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Sensitivity Analysis on Parameter C_3

Table.5

Defuzzify Value of C_3	Fuzzify value of parameter $\tilde{C}_3$	T_1	T	$TC_{1(ds)}(T_1,T)$
12	(9,11,13,15)	0.0435	1.2227	407.91
13	(10,12,14,16)	0.0462	1.2226	407.93
14	(11,13,15,17)	0.0489	1.2226	406.94
15	(12,14,16,18)	0.0516	1.2225	406.95

Table.6

Defuzzify Value of C_3	Fuzzify value of parameter $\tilde{C}_3$	T_1	T	$TC_{2(ds)}(T_1,T)$
12	(9,11,13,15)	0.6822	1.2225	407.55
13	(10,12,14,16)	0.6827	1.2224	407.56
14	(11,13,15,17)	0.6833	1.2224	407.57
15	(12,14,16,18)	0.6839	1.2223	407.59

VII. OBSERVATIONS

1) From Table 1, as we increase the parameter C_1, the optimum values of T_1 and T decrease. By this effect, the total cost $TC_{1(ds)}(T_1,T)$ increases.
2) From Table 2, as we increase the parameter C_1, the optimum values of T_1 and T decrease. By this effect, the total cost $TC_{2(ds)}(T_1,T)$ increases.
3) From Table 3, as we decrease the parameter C_2, the optimum values of T_1 and T increase. By this effect, the total cost $TC_{1(ds)}(T_1,T)$ decreases.
4) From Table 4, as we decrease the parameter C_2, the optimum values of T_1 and T increase. By this effect, the total cost $TC_{2(ds)}(T_1,T)$ decreases.
5) From Table 5, as we increase the parameter C_3, the optimum values of T_1 increase and T decrease. By this effect, the total cost $TC_{1(ds)}(T_1,T)$ increases.
6) From Table 6, as we increase the parameter C_3, the optimum values of T_1 increase and T decrease. By this effect, the total cost $TC_{2(ds)}(T_1,T)$ increases.

VIII. CONCLUSION

In this paper, we developed an EOQ model for variable deteriorating items with Weibull demand and time- varying holding cost in the fuzzy sense and when the supplier offer a trade credit period. The supplier offers credit period to the retailer who has the reserve money to make the payments, but decides to avail the benefits of credit limit. Shortages are allowed and partially backlogged. The deteriorating cost, shortage cost and opportunity cost are represented by trapezoidal fuzzy numbers . The optimum results of fuzzy model are defuzzified by signed distance method. Finally, numerical example and sensitivity analysis are provide to illustrate and inference the theoretical results. The proposed model can be extended for stock dependent demand and the effect of inflations in fuzzy environment.

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