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An EPQ Model for Deteriorating Items with Time Dependent Demand with Reliability and Flexibility in a Fuzzy Environment

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ABSTRACT: An EPQ model is formulated for deteriorating items with time dependent demand rate and time dependent production rate. To retain the confidence of the buyers the machine reliability, flexibility and packaging cost are considered. Shortages are allowed and it is completely backlogged. Mathematical model has been presented to find the optimal order quantity and total cost. The holding cost, setup cost, deterioration cost, labour cost for packing, material cost for packing, shortage cost, production cost involved in this model are taken as triangular fuzzy numbers. To validate the optimal solution, numerical example is provided. To analyze the effect of variations in the optimal solution with respect to the change in one parameter at a time, sensitivity analysis is carried out.

KEYWORDS: Production, inventory, optimality, time dependent production rate, time dependent demand rate.

I. INTRODUCTION

Maximize the total inventory profit or to minimize the total inventory cost is the aim of the inventory management. The EOQ (Economic Order Quantity) and EPQ (Economic Production Quantity) are used for finding the optimal order quantity. In this paper demand rate and production rate both considered to be time-dependent. Shortages are considered which are completely backlogged an d the holding cost, setup cost, deterioration cost, labour cost for packing, material cost for packing, shortage cost, production cost involved in this model are taken as triangular fuzzy numbers. Promoting the production of the product serves as the ultimate target of the manufacturer. To make the rate of production fast, the machines are subjected to continuous wear and tear which results in depreciation. To withstand these circumstances the manufacturers have to maintain high degree of reliability and flexibility. This paper elucidates about packaging, comprises the formulation of an EPQ model with the inclusion of package cost. Objective of this work is to investigate the effect of total cost and optimal order quantity with the variation of setup cost, holding cost and production cost.

II. LITERATURE REVIEW

Harries [1]studied EOQ (Economic Order Quantity) model and derived some useful results. After that Taft[2]presented EPQ (Economic Production Quantity) formula. Many inventory models in the inventory literature were presented considering time-dependent demand. Silver and Meal[3] were first presented an EOQ model for the case of the time varying demand. Donaldson[4] developed an inventory model with linearly time dependent demand. Teng*et al.*[5]established inventory model under trade credit financing with increasing demand. Khanra*etal.*[6]presented an inventory model for deteriorating item with time-dependent demand. Teng*etal.* [7] developed an EOQ model for increasing demand in asupply chain with trade credits. Large number of research papers presented by authors like Daveand Patel [8],Jalanand Chauduri[9], Jalan*et al.* [10], Mitra*et al.*[11]in this direction.

SamantaandRoy [12] established a continuous production inventory model of deteriorating itemwithshortagesandconsideredthatthe productionrate ischangedtoanother ata time whentheinventory level reaches a

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prefixed level. Hou[13]established an EPQ model with imperfectproduction process ,inwhich thesetup costisafunction ofcapitalexpenditure. Cardenas-Barron[14]presentedanEconomicProductionQuantity(EPQ)inventorymodel withplannedbackordersfordeterioratingtheEconomicProductionQuantity forasingle product. Cardenas-Barron [15] established an EPQ model with production capacity limitationandbreakdownwithimmediaterework.Krishnamoorthi*etal.*[16]established a single stage productionprocesswhere defectiveitemproduced arereworkedandtwomodels of rework processareconsidered,anEOQmodelfor withandwithoutshortages.

Recently, Liuetal. [17]establishedthe problemof a productionsystemthatcanproduce multiple productsbutalsosubjecttopreventivemaintenanceatthesetuptimes ofsomeproducts. Taleizadeh*etal.*[18]developedavendormanagedinventory fortwolevelsupplychain comprisedofonevendorandseveralnon-competing retailers,inwhichboththerawmaterial andthefinished producthavedifferentdeteriorationrates.Ghiamiand Williams [19] establishedaproduction-inventory modelinwhichamanufacturerisdeliveringa deterioratingproduct to retailers. Dr.NeerajAgarwal [19] has developed an EPQ model with time dependent demand and reduction delivery policy. Parvathi et al [20] have developed EPQ model with reliability and flexibility under fuzzy environment. This paper is based on [19] and [20] along with packaging cost under fuzzy environment. We have taken holding cost, setup cost, shortage cost, production cost, deterioration cost, labour cost for packing material cost for pack as triangular fuzzy number. We defuzzify the above cost using signed distance method.

III. PRELIMINARYCONCEPTS

3.1Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions, is a generalized fuzzy number $\tilde{A}$.

(i) μ_A is a continuous mapping from R to the closed interval [0, 1],

(ii) $\mu_A = 0$, $-\infty < x \le a_1$

(iii) $\mu_A = L(x)$ is strictly increasing on $[a_1, a_2]$

(iv) $\mu_A = w_A$, $a_2 \le x \le a_3$

(v) $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$

(vi) $\mu_A = 0$, $a_4 \le x \le \infty$

Where $0 < w_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$; when $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$.

3.2 Triangular fuzzy number

The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 < a_2 < a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_A = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

3. 3 The Function Principle

The function principle was introduced by Chen [6] to treat fuzzy arithmetical operations. This principle is used for addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

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(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}\times\tilde{B}=(c_1,c_2,c_3)$ where $T=(a_1b_1,a_1b_3,a_3b_1,a_3b_3)$, $c_1=\min T, c_2=a_2b_2, c_3=\max T$ if a_1, a_2, a_3, b_1, b_2, b_3 are all non zero positive real numbers, then $\tilde{A}\times\tilde{B}=(a_1b_1,a_2b_2,a_3b_3)$.

(iii) $-\tilde{B}=(-b_3,-b_2,-b_1)$ then the subtraction of $\tilde{B}$ from $\tilde{A}$ is $\tilde{A}-\tilde{B}=(a_1-b_3,a_2-b_2,a_3-b_1)$ where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(iv) $\frac{1}{\tilde{B}}=\tilde{B}^{-1}=(\frac{1}{b_3},\frac{1}{b_2},\frac{1}{b_1})$ where b_1, b_2, b_3 are all non-zero positive real numbers, then the division of $\tilde{A}$ and $\tilde{B}$ is $\frac{\tilde{A}}{\tilde{B}}=(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1})$

(v) For any real number K, $K\tilde{A}=(Ka_1,Ka_2,Ka_3) if K>0$
$K\tilde{A}=(Ka_3,Ka_2,Ka_1) if K<0$

3.4. Signed Distance Method

Defuzzificationof $\tilde{A}$ can be found by signed distance method. If $\tilde{A}$ is a triangular fuzzy number and is fullydetermined by (a_1, a_2, a_3), the signed distance from $\tilde{A}$ to 0 is defined as

$$d(\tilde{A},\tilde{0})=\int_0^1 d\left(\left[A_L(\alpha),A_R(\alpha)\right],\tilde{0}\right)d\alpha=\frac{(a_1+4a_2+a_3)}{4}$$

IV. NOTATIONS AND ASSUMPTIONS

The following notations and assumptions have been used to develop the fuzzy mathematical model.

4.1 NOTATIONS

I(t)	inventorylevel at anytime '*t*'
$I_1(t)$	Inventory during the interval (0,T_1)
$I_2(t)$	Inventory during the interval (T_1,T_2)
$I_3(t)$	Inventory during the interval (T_2,T)
P	production rate in units perunittime
D	demand rate in units perunittime
X	Demand during shortage period per unit per unit time
Q1	on hand inventorylevel during[*0,T1*]
Q	production quantity
*Q**	optimal production quantity
T	cycletime
T1	production time per unit per unit time
T2	The time at which the shortage begins
$\tilde{\gamma}$	Production cost per unit per unit time
$\tilde{h}$	Fuzzy Holdingcost perunit per unit time
$\tilde{A}$	Fuzzy Setup cost per unit per unit time
p_i	Proportion of cost of the machine m_i, i=1,2,…n.
c_i	Cost of the machine m_i i= 1,2,…n.
f_i	Maintenance cost of the machine m_i i= 1,2,…n

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θ	Deterioration rate
$\tilde{\alpha}$	Fuzzy deterioration cost per unit per unit time
M_i	Volume of investment in marketing method i=1,2,…k per unit per unit time
$\tilde{L}$	Fuzzy Labour cost for packing per parcel
$\tilde{K}$	Fuzzy Cost of material used for packing per packing
$\tilde{s}$	Fuzzy Shortage cost per unit per unit time
r	Number of parcels
$\widetilde{TC}$	Fuzzy Total cost
$\widetilde{TC}^*$	Fuzzy Optimal total cost
TC_{DG}	Defuzzified value of the total cost
Q^*_{DG}	Defuzzified value of optimal production cost

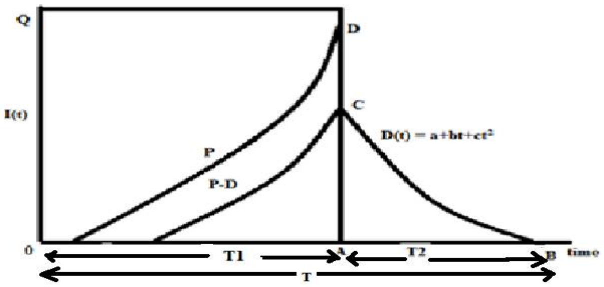
4.2 ASSUMPTIONS

The following assumptions are made throughout the manuscript:

1. Items are produced and added to the inventory.
2. Shortages are allowed and completely backlogged.
3. Production rate is proportional to demand rate.
4. The demand rate is time dependent. D(t)=$a+bt+ct^2$ where $a > 0, 0 < b < 1, < 0 < c < 1$.
5. The packing cost per parcel includes the labour cost and material cost used for packaging.

V. MODEL DEVELOPMENT

The production starts from O and finished at C. During t=0 to t=T_1, production rate is P and demand rate is D. Inventory reduces due to demand and deterioration. In between t=0 to t=T_1 the inventory accumulates at a rate P-D . The consumption starts from t=T_1 and finished at t=T_2 and the shortage starts at t=T_2. During the shortage period there will be some demand and it may be assumed as X.



The rate of change of inventory between [0, T] is given by

$$\frac{dI(t)}{dt} = \begin{cases} P - D - \theta I(t), & 0 \le t \le T_1 \\ -D, & T_1 \le t \le T_2 \\ -X, & T_2 \le t \le T \end{cases} \quad \ldots\ldots\ldots\ldots\ldots(I)$$

Here D= D(t)= a+bt+c t^2, P= λD and λ>1

Solve (I) with the condition I(0) = 0, I(T_1) = Q_1, I(T_2) = 0 I(T) = 0 and it is given by

$$I_1(t) = \frac{(\lambda - 1)}{\theta^2}[bt\theta + c\theta t^2 - 2ct]$$

$$Q_1 = (\lambda - 1)\left[a(T_1 - t) + \frac{b({T_1}^2 - t^2)}{2} + \frac{c({T_1}^3 - t^3)}{3}\right]$$

$$I_2(t) = a(T_2 - t) + \frac{b}{2}(T_2^2 - t^2) + \frac{c}{3}(T_2^3 - t^3)$$

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At t=0 the order quantity is $Q = aT + \frac{bT^2}{2} + \frac{cT^2}{3}$

$$I_3(t) = X(T - t)$$

The total cost is calculated by considering setup cost, production cost, inventory holding cost ,Depreciation cost, Maintaining cost, Marketing cost, deterioration cost, packaging cost and shortage cost.

5.1 The Depreciation cost per cycle

Suppose that the manufacturers owns machine m_i(i=1,....n) to execute sequentially the various tasks involved in the production process. It is quite naturals for the machine to get degraded due to continuous wear and tear, so the value of the machines gets depreciated .The proportion (p_i) of costs of machine m_i(i=1,....n) depreciated is determined by various methods and it varies from one machine to another.

The depreciated cost per cycle is $= \sum_{i=1}^{n} p_i c_i$

5.2The Maintenance cost per cycle

To maintain the degree of reliability and to prevent machine break down the machine have to be properly taken care. Since each machine serves purposes the pattern of its maintenance also differs.

The maintenance per cycle is $= \sum_{i=1}^{n} f_i$

5.3 The Marketing cost per cycle

To propagate the product in the marker rapidly the manufacturers may prefer one or more marketing strategies, (i.e) for example the manufacturer may advertise his products via two modes viz. Newspaper and Television for which separate cost are incurred. The marketing cost per cycle is $= \sum_{i=1}^{k} M_i$

5.4 Packaging cost

Packing cost per parcel includes both the labour costs and the material costs.

Packaging cost per cycle =$(\tilde{L} + \tilde{K})$r

5.5 Setup cost

$$S\tilde{C} = \frac{\tilde{A}}{T} = \frac{b\tilde{A}}{\sqrt{a^2 + 2bQ} - a}$$

5.6 Productioncost

$$\tilde{P}C = \frac{\tilde{\gamma}bQ}{\sqrt{a^2 + 2bQ - a}}$$

$$= \frac{\tilde{\gamma}bQ\sqrt{a^2 + 2bQ} + a}{\sqrt{a^2 + 2bQ} - a^2}$$

$$= \frac{\tilde{\gamma}}{2}(a + \sqrt{a^2 + 2bQ})$$

5.7Inventory holding cost

$$\tilde{H}C = \frac{\tilde{h}}{T}\left[\int_0^{T_1} \frac{(\lambda - 1)}{\theta^2}\{b\theta t + c\theta t^2 - 2ct\}dt + \int_{T_1}^{T_2}\left\{a(T_2 - t) + \frac{b}{2}(T_2^{\ 2} - t^2) + \frac{c}{3}(T_2^{\ 3} - t^3)\right\}dt\right]$$

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$$= \frac{\tilde{h}b}{\sqrt{a^2+2bQ}-a}\left[\left\{\frac{(\lambda-1)}{\theta^2}\left(\frac{b\theta T_1^{\,2}}{2}+\frac{c\theta T_1^{\,3}}{3}-cT_1^{\,2}\right)\right\}+\left\{\frac{aT_2^{\,2}}{2}+\frac{bT_2^{\,3}}{3}+\frac{cT_2^{\,4}}{4}\right\}-\left\{a\left(T_2T_1-\frac{T_1^{\,2}}{2}\right)+\frac{b}{2}\left(T_2^{\,2}T_1-\frac{T_1^{\,3}}{3}\right)+\frac{c}{3}\left(T_2^{\,2}T_2-\frac{T_2^{\,4}}{4}\right)\right\}\right]$$

5.8Shortage cost

$$\left[\tilde{S}\tilde{H}C=\frac{\tilde{s}}{T}\int_{T_2}^{T}X(T-t)dt\right]=\frac{\tilde{s}X}{T}\left\{\left(\frac{T^2}{2}\right)-\left(TT_2-\frac{T_2^2}{2}\right)\right\}=\frac{\tilde{s}Xb}{\sqrt{a^2+2bQ}-a}\left\{\left(\frac{T^2}{2}\right)-\left(TT_2-\frac{T_2^2}{2}\right)\right\}$$

5.9 Deterioration cost

$$D\tilde{T}C=\tilde{\alpha}\theta\frac{b}{\sqrt{a^2+2bQ}-a}\int_0^{T_1}I_1(t)dt$$

$$=\frac{\tilde{\alpha}(\lambda-1)b}{\theta\sqrt{a^2+2bQ}-a}\left\{\left(\frac{b\theta T_1^{\,2}}{2}+\frac{c\theta T_1^{\,3}}{3}-cT_1^{\,2}\right)\right\}$$

Total cost $T\tilde{C}=S\tilde{C}+P\tilde{C}+H\tilde{C}+SH\tilde{C}+D\tilde{T}C+$ Depreciation cost + Maintenance cost + Marketing cost + Packaging cost

$$\tilde{T}C=\frac{b}{\sqrt{a^2+2bQ}-a}\left[\begin{array}{l}\tilde{A}+\tilde{h}\left[\left\{\frac{(\lambda-1)}{\theta^2}\left(\frac{b\theta T_1^{\,2}}{2}+\frac{c\theta T_1^{\,3}}{3}-cT_1^{\,2}\right)\right\}+\left\{\frac{aT_2^{\,2}}{2}+\frac{bT_2^{\,3}}{3}+\frac{cT_2^{\,4}}{4}\right\}-\left\{a\left(T_2T_1-\frac{T_1^{\,2}}{2}\right)+\frac{b}{2}\left(T_2^{\,2}T_1-\frac{T_1^{\,3}}{3}\right)+\frac{c}{3}\left(T_2^{\,2}T_2-\frac{T_2^{\,4}}{4}\right)\right\}\right]+\\ \tilde{s}X\left\{\left(\frac{T^2}{2}\right)-\left(TT_2-\frac{T_2^2}{2}\right)\right\}+\tilde{\alpha}\left\{\frac{(\lambda-1)}{\theta}\left(\frac{b\theta T_1^{\,2}}{2}+\frac{c\theta T_1^{\,3}}{3}-cT_1^{\,2}\right)\right\}+\\ +\sum_{i=1}^{n}p_ic_i+\sum_{i=1}^{n}f_i+\\ \sum_{i=1}^{k}M_i+(\tilde{L}+\tilde{K})\,r\end{array}\right]$$

$$\frac{\tilde{\gamma}}{2}(a+\sqrt{a^2+2bQ})$$

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5.10 Defuzzification of the Total cost

$\tilde{A}=(A_1,A_2,A_3)$ $\tilde{h}=(h_1,h_2,h_3)$, $\tilde{s}=(s_1,s_2,s_3)$, $\tilde{\alpha}=(\alpha_1,\alpha_2,\alpha_3)$ $\tilde{L}=(L_1,L_2,L_3)$, $\tilde{K}=(K_1,K_2,K_3)$

$\tilde{\gamma}=(\gamma_1,\gamma_2,\gamma_3)$

$$TC_{DG}=\frac{b}{\sqrt{a^2+2bQ}-a}\left[\begin{array}{l} A+h\left[\begin{array}{r}\left\{\frac{(\lambda-1)}{\theta^2}\left(\frac{b\theta T_1^{\,2}}{2}+\frac{c\theta T_1^{\,3}}{3}-cT_1^{\,2}\right)\right\}+\left\{\frac{aT_2^{\,2}}{2}+\frac{bT_2^{\,3}}{3}+\frac{cT_2^{\,4}}{4}\right\}-\\ \left\{a\left(T_2T_1-\frac{T_1^{\,2}}{2}\right)+\frac{b}{2}\left(T_2^{\,2}T_1-\frac{T_1^{\,3}}{3}\right)+\frac{c}{3}\left(T_2^{\,2}T_2-\frac{T_2^{\,4}}{4}\right)\right\}\end{array}\right]+\\ sX\left\{\left(\frac{T^2}{2}\right)-\left(TT_2-\frac{T_2^2}{2}\right)\right\}+\frac{\alpha(\lambda-1)}{\theta}\left(\frac{b\theta T_1^{\,2}}{2}+\frac{c\theta T_1^{\,3}}{3}-cT_1^{\,2}\right)\\ +\sum_{i=1}^{n}p_ic_i+\sum_{i=1}^{n}f_i+\\ \sum_{i=1}^{k}M_i+(\mathrm{L}+\mathrm{K})\,\mathrm{r}\end{array}\right]+$$

$$\frac{\gamma}{2}\left(a+\sqrt{a^2+2bQ}\right)$$

5.11 Optimal solution

The optimal solution is obtained by differentiating TC with respect to Q, we get

$$\frac{dTC}{dQ}=\frac{-b^2}{2\left(\sqrt{a^2+2bQ}-a\right)^{\frac{3}{2}}\left(a^2+2bQ\right)^{\frac{1}{2}}}\left[\begin{array}{l}\tilde{A}+\tilde{h}\left[\begin{array}{l}\left\{\frac{(\lambda-1)}{\theta^2}\left(\frac{b\theta T_1^{\,2}}{2}+\frac{c\theta T_1^{\,3}}{3}-cT_1^{\,2}\right)\right\}+\\ \left\{\frac{aT_2^{\,2}}{2}+\frac{bT_2^{\,3}}{3}+\frac{cT_2^{\,4}}{4}\right\}-\\ \left\{\begin{array}{l}a\left(T_2T_1-\frac{T_1^{\,2}}{2}\right)+\\ \frac{b}{2}\left(T_2^{\,2}T_1-\frac{T_1^{\,3}}{3}\right)+\frac{c}{3}\left(T_2^{\,2}T_2-\frac{T_2^{\,4}}{4}\right)\end{array}\right\}\end{array}\right]+\\ \tilde{s}X\left\{\left(\frac{T^2}{2}\right)-\left(TT_2-\frac{T_2^2}{2}\right)\right\}+\tilde{\alpha}\left\{\frac{(\lambda-1)}{\theta^2}\left(\frac{b\theta T_1^{\,2}}{2}+\frac{c\theta T_1^{\,3}}{3}-cT_1^{\,2}\right)\right\}\\ +\sum_{i=1}^{n}p_iC_i+\sum_{i=1}^{n}f_i+\\ \sum_{i=1}^{k}M_i+(\tilde{\mathrm{L}}+\tilde{\mathrm{K}})\,\mathrm{r}\end{array}\right]$$

$$+\frac{\tilde{\gamma}b}{2\sqrt{a^2+2bQ}}$$

And $\frac{d^2TC}{dQ^2}>0$

The optimal (minimum) production quantity $Q=Q^*$ is obtained by solving $\frac{dTC}{dQ}=0$ we get

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$$Q = Q^{*} = \frac{1}{2b}\left[\left\{\frac{1}{\tilde{\gamma}}\right\}^{\frac{2}{3}}\left\{\begin{array}{l} \tilde{A}+\tilde{h}\left[\begin{array}{r}\left\{\frac{(\lambda-1)}{\theta^{2}}\left(\frac{b\theta T_1^{2}}{2}+\frac{c\theta T_1^{3}}{3}-cT_1^{2}\right)\right\}+\left\{\frac{aT_2^{2}}{2}+\frac{bT_2^{3}}{3}+\frac{cT_2^{4}}{4}\right\}- \\ \left\{a\left(T_2T_1-\frac{T_1^{2}}{2}\right)+\frac{b}{2}\left(T_2^{2}T_1-\frac{T_1^{3}}{3}\right)+\frac{c}{3}\left(T_2^{2}T_2-\frac{T_2^{4}}{4}\right)\right\}\end{array}\right]+ \\ \tilde{s}X\left\{\left(\frac{T^{2}}{2}\right)-\left(TT_2-\frac{T_2^{2}}{2}\right)\right\}+\tilde{\alpha}\left\{\frac{(\lambda-1)}{\theta}\left(\frac{b\theta T_1^{2}}{2}+\frac{c\theta T_1^{3}}{3}-cT_1^{2}\right)\right\}+ \\ +\sum_{i=1}^{n}p_ic_i+\sum_{i=1}^{n}f_i+ \\ \sum_{i=1}^{k}M_i+(\tilde{L}+\tilde{K})\,r \end{array}\right\}^{\frac{2}{3}}+a\right]^{2}-\frac{a^{2}}{2b} \qquad \mathbf{5.12}$$

Defuzzification of the optimal production quantity

$\tilde{A}=(A_1,A_2,A_3)$ $\tilde{h}=(h_1,h_2,h_3)$ $\tilde{s}=(s_1,s_2,s_3)$ $\tilde{\alpha}=(\alpha_1,\alpha_2,\alpha_3)$ $\tilde{L}=(L_1,L_2,L_3)$ $\tilde{K}=(K_1,K_2,K_3)$

$$Q_{DG}^{*} = \frac{1}{2b}\left[\left\{\frac{1}{\gamma}\right\}^{\frac{2}{3}}\left\{\begin{array}{l} A+h\left[\begin{array}{r}\left\{\frac{(\lambda-1)}{\theta^{2}}\left(\frac{b\theta T_1^{2}}{2}+\frac{c\theta T_1^{3}}{3}-cT_1^{2}\right)\right\}+\left\{\frac{aT_2^{2}}{2}+\frac{bT_2^{3}}{3}+\frac{cT_2^{4}}{4}\right\}- \\ \left\{a\left(T_2T_1-\frac{T_1^{2}}{2}\right)+\frac{b}{2}\left(T_2^{2}T_1-\frac{T_1^{3}}{3}\right)+\frac{c}{3}\left(T_2^{2}T_2-\frac{T_2^{4}}{4}\right)\right\}\end{array}\right]+ \\ sX\left\{\left(\frac{T^{2}}{2}\right)-\left(TT_2-\frac{T_2^{2}}{2}\right)\right\}+\alpha\left\{\frac{(\lambda-1)}{\theta}\left(\frac{b\theta T_1^{2}}{2}+\frac{c\theta T_1^{3}}{3}-cT_1^{2}\right)\right\}+ \\ +\sum_{i=1}^{n}p_ic_i+\sum_{i=1}^{n}f_i+ \\ \sum_{i=1}^{k}M_i+(L+K)\,r \end{array}\right\}^{\frac{2}{3}}+a\right]^{2}-\frac{a^{2}}{2b}$$

VI. NUMERICAL ILLUSTRATIONS

Example1: To illustrate the model a particular EPQ problem is taken,
$n=3, k=3, C_1=10{,}000, C_2=15{,}000, C_3=20{,}000, p_1=0.5\%, p_2=0.3\%, p_3=1\%,$
$f_1=50/unit/unittime, f_2=20/unit/unittime, f_3=10/unit/unittime, M_1=50/unit/unittime$
$M_2=40/unit/unittime, \tilde{A}=(90,100,110), \tilde{h}=(1,1.5,2), \tilde{\gamma}=(50,60,70), a=250, b=0.2$
$c=0.03, \tilde{L}=(1,2,3), \tilde{K}=(0.5,0.6,0.8), \lambda=0.8, \tilde{s}=(30,40,50), X=60, T_1=0.02, T_2=0.04$
$T_3=0.08, \theta=0.56, r=1, \alpha=1.175$

In this case $Q_{DG}^{*}=6471$, $TC_{DG}=22{,}771$

VII. SENSITIVITY ANALYSIS

Sensitivity analysis is carried out with the variation of different parameters. Taking all the numerical values as mentioned in the above numerical example.

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Table 1: Variation of production cost γ , setup cost A, and holding cost h on the optimal solution.

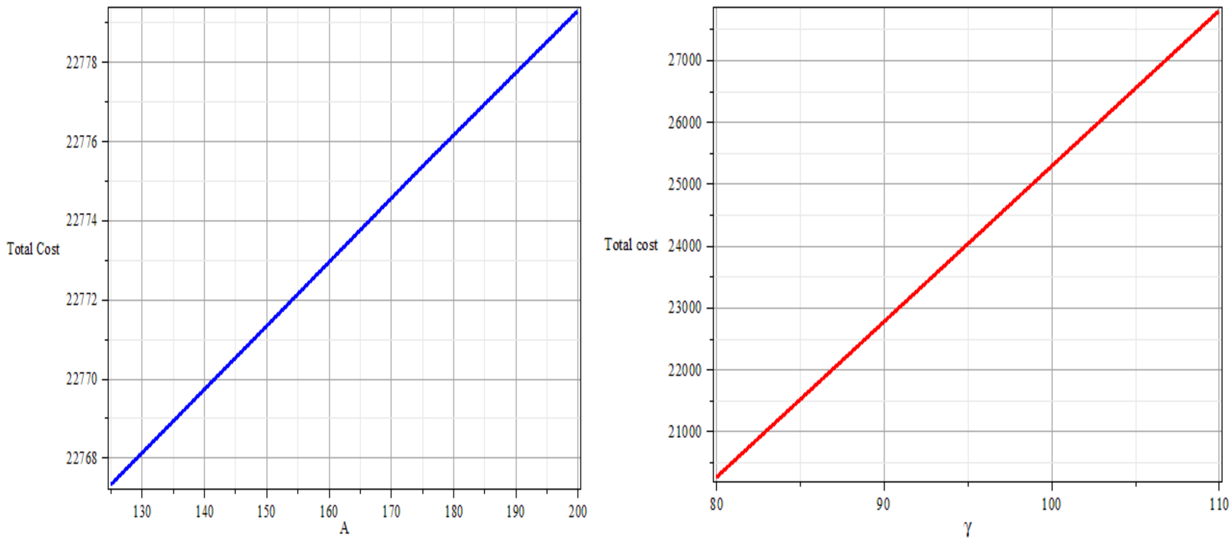
A	TC_{DG}	Q^*_{DG}	**γ**	TC_{DG}	Q^*_{DG}	**h**	TC_{DG}	Q^*_{DG}
125	22767.32	6366.29	80	20259.39	7005.45	2.25	22771.34	6471.05
150	22771.34	6471.05	90	22771.34	6471.05	3.75	22771.35	6473.36
175	22775.33	6575.01	100	25282.55	6027.97	5.25	22771.36	6487.68
200	22779.29	6678.19	110	27793.13	5653.57	6.75	22771.38	65014.99

All the above observations can be sum up as follows:

- Increase in setup cost leads increase in total cost and increase in production quantity.
- Increase in production cost leads increase in total cost and decrease in production quantity.
- Increase in holding cost leads slight increase in total cost and increase in production quantity.

We can also represent this situation pictorially.

Fig 1. Effect of Aand γ on Total cost function



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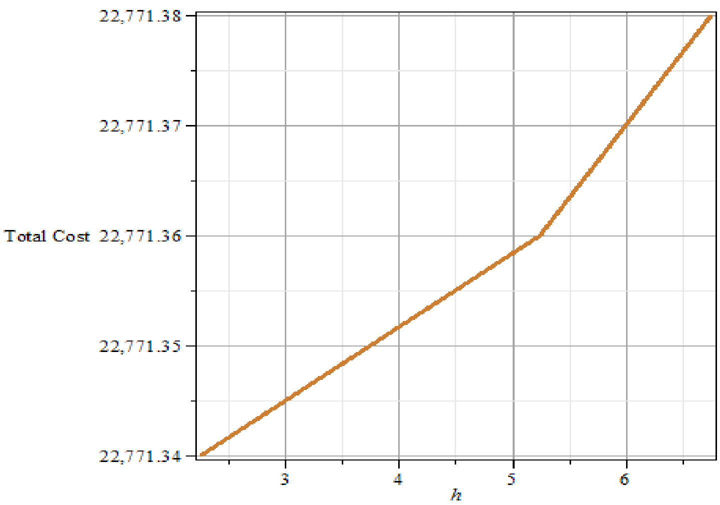

Fig3. Effect of h on Total cost function

VIII. CONCLUSIONS

Anfuzzy inventory model with time dependent demand is developed in this paper. This model is highly beneficial to the manufactures as the EPQ model is focused on quantity, reliability and flexibility. Mathematical formulation is presented for finding optimal order quantity and total cost. The total cost is convex function with respect to order quantity. The applicability of the proposed model is demonstrated by a numerical example. A sensitivity analysis shows that the changes are quiet sensitive with the change of parameters.

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