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Study of Counter-Current Flow Using Finite Difference Method

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ABSTRACT: The present paper deals with the counter-current flow arising in two immiscible phase flow through homogeneous porous media. Such a flow occurs due to the different wetting abilities of the fluid. When another fluid is injected into the porous medium, where the medium is filled with some fluid which preferentially wets the medium, then there is a spontaneous flow of the resident fluid from the media, which is called counter-current Imbibition. Governing equation of this problem is a partial differential equation. Solution is obtained by a Successive over relaxation method.

KEYWORDS: Counter-current, Porous media, Imbibition, ,Immiscible, S.O.R.

I. Introduction

Consider a finite cylindrical piece of homogenous porous matrix of length L. It is fully saturated with a liquid (native fluid N). Also it is completely surrounded by an impermeable surface except for one end which is exposed to an adjacent formation of injected liquid (I). It is assumed that injected fluid is preferentially more wetting than that of native liquid. Phenomenon of linear counter-current Imbibition occurs due to this arrangement. There is a spontaneous linear flow of injected fluid into the medium and a counter flow of the resident fluid from the medium.

The counter-current flow takes place in two immiscible phase flow through homogeneous porous media [1]. Water is considered as an injected fluid and oil is considered as native fluid. Verma[2] discussed the existence and uniqueness of similarity transformation. Verma [3] has used perturbation method to obtain the solution for cracked porous medium.Verma and Rammohan [4] obtained existence and uniqueness of similarity of imbibition equation.Rangel, E.R., and Kovscek, A.R.[5] discussed analytical study of multidimensional imbibition in fractured porous media.Li K.W.[6] have discussed this phenomena by including wettabilityMehta[7] discussed the solution by group invariant method of instability phenomenon with power law nonlinearity.Rose[8] discussed the modelling forced versus spontaneous capillary imbibition processes commonly occurring in porous sediments .Sharma [9] discussed analytic solution.

II. STATEMENT OF THE PROBLEM

A finite cylindrical piece of homogenous porous matrix of length L which is assumed to be fully saturated with a resident fluid. Its boundaries are completely surrounded by an impervious surface except for one end. This end is uncovered to an adjacent formation of injected water. Injected water is assumed to be more wetting than that of native liquid . Due to this arrangement there is a unstructured linear flow of injected fluid into the medium. Also there is a counter flow of the resident fluid from the medium, which is known as the phenomenon of linear counter-current imbibition. Here water is considered as injected fluid and oil is considered as native fluid.

The governing equations to this phenomenon is a partial differential equation. The solution is obtained by Successive over Relaxation Method.

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III. MATHEMATICAL FORMULATION

Using Darcy's law for seepage velocity of flowing fluids

$$V_w = -\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \quad (1)$$

$$V_o = -\frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x} \quad (2)$$

Where V_w and V_o are seepage velocity of water and oil respectively, k is the permeability of the homogeneous medium, k_w and k_o are relative permabilities of water and oil respectively, P_w and P_o are the pressures and μ_o and μ_w are viscosities of water and oil respectively.

The equations of continuity for the flowing phase are:

$$P\frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \quad (3)$$

$$P\frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \quad (4)$$

Where P is the porosity of the medium and S_w and S_o are saturation of injected and oil respectively.

For imbibition phenomenon an analytic condition is given by

$$V_w = -V_o \quad (5)$$

$$P_c = P_o - P_w \quad (6)$$

From equations (1),(2) and (5) , we have

$$\frac{k_o}{\mu_o}\frac{\partial P_o}{\partial x} + \frac{k_w}{\mu_w}\frac{\partial P_w}{\partial x} = 0 \quad (7)$$

From (6) and (7),we have

$$\frac{k_o}{\mu_o}\left\{\frac{\partial P_c}{\partial x} + \frac{\partial P_w}{\partial x}\right\} + \frac{k_w}{\mu_w}\frac{\partial P_w}{\partial x} = 0$$

$$\therefore \frac{\partial P_w}{\partial x} = \frac{-k_o/\mu_o}{\left\{\frac{k_o}{\mu_o}+\frac{k_w}{\mu_w}\right\}}\frac{\partial P_c}{\partial x} \quad (8)$$

From (1) and (3) , we get

$$P\frac{\partial s_w}{\partial t} - \frac{\partial}{\partial x}\left\{\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x}\right\} = 0 \quad (9)$$

From equations (8) and (9), we get

$$P\frac{\partial s_w}{\partial t} + \frac{\partial}{\partial x}\left\{k\frac{k_o k_w}{k_o\mu_w + k_w\mu_o}\frac{dP_c}{dS_w}\frac{\partial S_w}{\partial x}\right\} = 0 \quad (10)$$

Setting $D(S_w) = \frac{k_o k_w}{k_o\mu_w + k_w\mu_o} = D$

D is called co-efficient of saturation which is assumed to be a constant.

Using the relation between capillary pressure and phase saturation as

$$P_c = -\beta S_w \quad (11)$$

Equation (10) can be written as

$$P\frac{\partial s_w}{\partial t} - \frac{\partial}{\partial x}\left[kD\beta\frac{\partial s_w}{\partial x}\right] = 0 \quad (12)$$

With $s_w(0,t) = s_0$, $s_w(L,t) = s_1$, $s_w(L,t) = 0, 0 \le x \le L$

$$\frac{\partial s_w}{\partial t} - kD\beta\frac{\partial^2 s_w}{\partial x^2} = 0 \quad (13)$$

Let $X = \frac{x}{L}$, $T = \frac{kD}{L^2}t$

$$\frac{\partial s_w}{\partial T} - \beta\frac{\partial^2 s_w}{\partial X^2} = 0 \quad (14)$$

With $s_w(X,0) = s_0$, $s_w(0,T) = s_1$, $s_w(1,T) = 0, 0 \le X \le 1$

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IV. MATHEMATICAL SOLUTION OF THE PROBLEM

Using S.O.R. method [10], we have

$$s_{w_{i,j+1}} = s_{w_{i,j}} + \frac{\beta k}{2h^2}\left(s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}} + s_{w_{i+1,j+1}} - 2s_{w_{i,j+1}} + s_{w_{i-1,j+1}}\right)$$

Let $\boldsymbol{r} = \frac{\boldsymbol{k}}{\boldsymbol{h^2}}$

$$(1+\beta r)s_{w_{i,j+1}} = s_{w_{i,j}} + \frac{\beta r}{2}\left(s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}} + s_{w_{i+1,j+1}} + s_{w_{i-1,j+1}}\right)$$

$$\boldsymbol{\lambda_i} = s_{w_{i,j}} + \frac{\beta r}{2}\left(s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}}\right)$$

$$s_{w_{i,j+1}} = (1-\omega)s_{w_{i,j}} + \omega\left[\frac{\beta r}{2(1+\beta r)}\left(s_{w_{i+1,i}} + s_{w_{i-1,j+1}}\right) + \frac{\boldsymbol{\lambda_i}}{(1+\beta r)}\right]$$

Choose k = 0.01, h=0.1, $\omega = 1.48$, $s_0 = 1$, $s_1 = 0$, $\beta = 56$

$$s_{w_{i,j+1}} = -0.48 s_{w_{i,j}} + 1.48\left[\left(\frac{28}{57}\right)\left(s_{w_{i+1,j}} + s_{w_{i-1,j+1}}\right) + \frac{\lambda_i}{57}\right]$$

Where $\boldsymbol{\lambda_i} = s_{w_{i,j}} + 28\left(s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}}\right)$

Numerical calculations for different values of saturation at different time and different length are shown in the following table.

T→	T=0.01	T=0.02	T=0.003	T=0.04	T=0.05
$X\downarrow$	Sw				
0	1	1	1	1	1
0.1	0.727017544	0.835371397	0.857708312	0.879384202	0.867977237
0.2	0.528554509	0.686104814	0.73032403	0.750923363	0.763585322
0.3	0.384268401	0.556081154	0.61710676	0.636874159	0.665620226
0.4	0.279369869	0.445917717	0.517484347	0.536263833	0.57556228
0.5	0.203106796	0.354460805	0.430777363	0.448307639	0.494076225
0.6	0.147662204	0.279706628	0.356126674	0.372179169	0.421296654
0.7	0.107353013	0.219351394	0.292515949	0.30694653	0.222952098
0.8	0.078047524	0.171104425	0.238829731	0.159388707	0.390261803
0.9	0.056741919	0.132852669	0.130508639	0.212609066	0.051645734
1	0.041252371	0.037702203	0.073182307	0.081999027	0.204324832

V. CONCLUSION

Solution is obtained by assuming linear relation between capillary pressure and phase saturation . It is observed that the saturation decreases as length increases and as time increases saturation increases.

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