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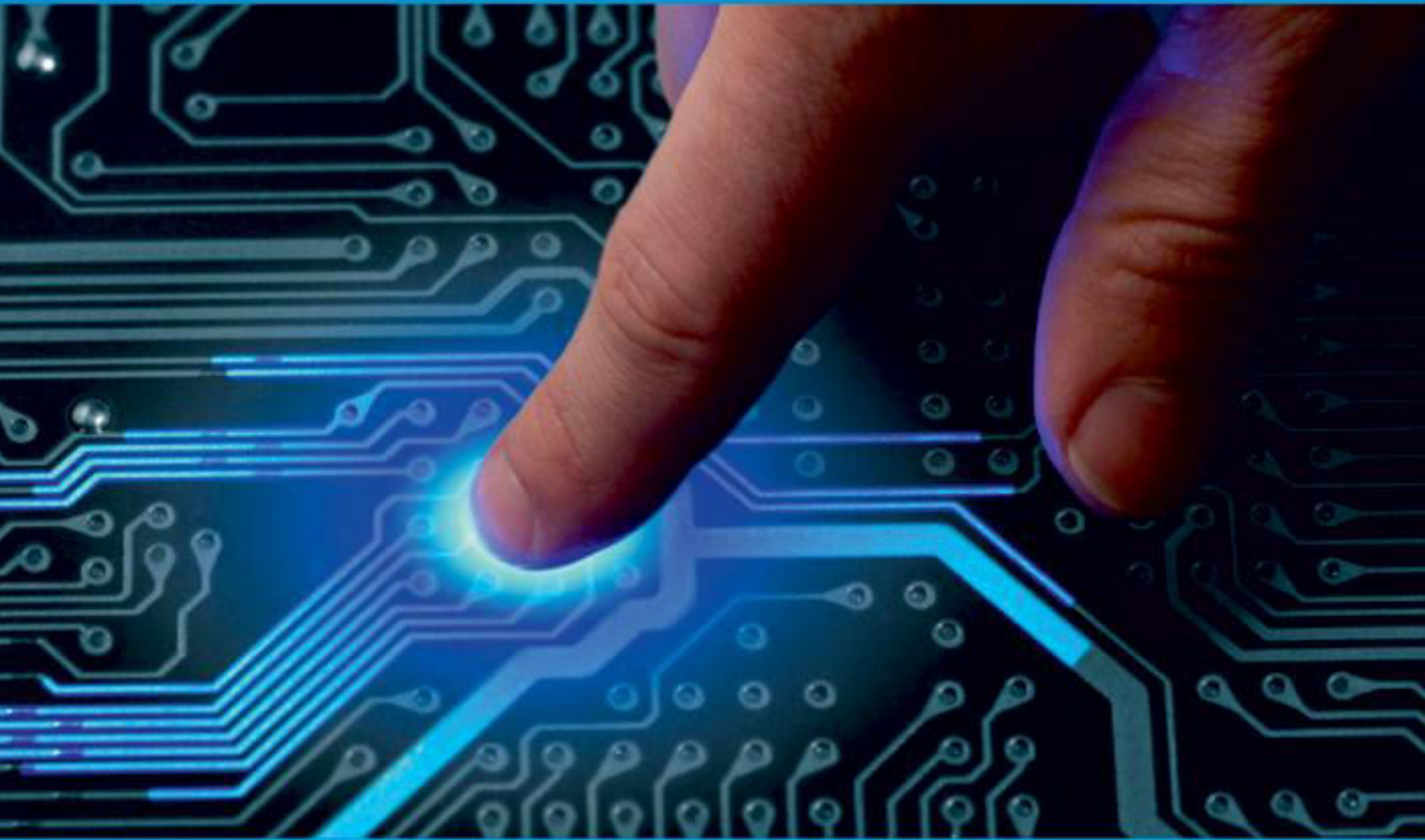

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QT Bézier Curves and Surfaces with Two Shape Parameters

Reenu Sharma

Department of Mathematics, Rani Durgawati Govt. P.G. College, Mandla, M.P., India

ABSTRACT: In this paper a new kind of QT (Quartic Trigonometric) Bézier curve with two shape parameters λ and μ is presented and the corresponding trigonometric Bézier surfaces are defined. These curves not only inherit most properties of the usual quartic Bézier curves with the Bernstein basis in the polynomial space, but also enjoy some other advantageous properties for shape modelling. The shape parameters λ and μ provide freedom in terms of design and shape control of the curve. Thus we can construct smooth curves of almost any shape. The shape of the curve can be adjusted by altering the values of shape parameters while the control polygon is kept unchanged. These curves can be used as an efficient model for geometric design in the fields of CAGD.

KEY WORDS: Trigonometric Bézier basis function, Trigonometric Bézier curve, Trigonometric Bézier surface, Shape parameters.

I. INTRODUCTION

Bézier curves and surfaces are the basic tools for modelling in Computer Aided Geometric Designing (CAGD) and Computer Graphics (CG). Bézier polynomial has several applications in the fields of engineering, science and technology such as highway or railway rout designing, networks, Computer aided design system, animation, environment design, robotics, communications and many other disciplines because it is easy to compute and is also very stable. The classical Bézier curves have some limitations that their shape and position are fixed relative to their control polygon. Thus people attempt to find a solution of the problem in the non-polynomial function space. During the last few years, a major research focus has been the use of trigonometric functions or the blending of polynomial and trigonometric functions.

Trigonometric B-splines were first presented in [1] and the recurrence relation for the trigonometric B-splines of arbitrary order was established in [2]. In recent years, several new trigonometric splines have been studied in the literature; see [3], [4] and [5]. In [6] cubic trigonometric Bézier curve with two shape parameters were presented. In [7], a novel generalization of Bézier curve and surface with n shape parameters are presented. In [8], the cubic trigonometric polynomial spline curve of G^3 continuity is constructed, which can be G^5 continuity under special condition. In [9], uniform T-B-spline basis function of $(n+1)^{th}$ order and its solution is presented. In [10], quartic splines with C^2 continuity are presented for a non-uniform knot vectors which are C^2 and G^3 continuous under special case. Algebraic-Trigonometric blended spline curves are presented in [11] which can represent some transcendental curves. Cubic trigonometric Bézier curve with two shape parameters is presented in [12]. Recently in [13], a quadratic trigonometric Bézier curve with shape parameter is constructed which is G^1 continuous. In [14], the generalized basis functions of degree n+1 with two shape parameters are presented. The cubic trigonometric polynomial spline curve of G^1 continuity is constructed in [15], which can be G^3 continuity under special condition. In [16], the cubic trigonometric polynomial curve similar to the cubic Bézier curves is constructed. In [17], the shape features of the cubic trigonometric polynomial curves with a shape parameter are analysed. An extension of the Bézier model is studied in [18]. In [19], [20], [21] and [22] quartic and cubic trigonometric Bézier curve respectively with shape parameter is presented and the effect of shape parameter is studied. A new rational cubic trigonometric Bézier curve with four shape parameters is defined in [23]. Quartic trigonometric Bézier curves and surfaces with shape parameter is presented in [24] which contains only one shape parameter. If we make any change in shape parameter or any control point then entire curve segment is changed. To overcome these shortcomings, a quartic trigonometric Bézier curve with two shape parameters is constructed in this paper. By introducing two shape parameters, we can modify the shape of the curve in the direction of first and last segment of the control polygon.

In this paper a new kind of QT (quartic trigonometric) Bézier Curve with two shape parameters is presented. The paper is organized as follows. In section 2, quartic trigonometric Bézier basis functions with two shape parameters are established and the properties of the basis functions are shown. In section 3, quartic trigonometric Bézier curves are

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given and some properties are discussed. By using shape parameters, shape control of the quartic trigonometric Bézier curves is studied. In section 4, the representation of quartic trigonometric Bézier surface has been shown. In section 5, the approximability of the quartic trigonometric Bézier curves and the quartic Bézier curves with Bernstein basis corresponding to their control polygon are studied. Conclusion is presented in Section 6.

II. QT Bézier Basis Functions With Two Shape Parameters

Firstly, the definition of QT Bézier basis functions with two shape parameters is given as follows.

The construction of the basis functions

Definition 1. For an arbitrarily selected real values of λ and μ where $\lambda,\ \mu \in$ [-20, 0], the following five functions of $t(t \in \left[0, \frac{\pi}{2}\right])$ are defined as QT Bézier basis functions with two shape parameters λ and μ:

$$\begin{cases} b_0(t) = (1-\sin t)^2\cos^2 t - \lambda(1-\sin t)^2(1-\cos t)^2 \\ b_1(t) = \sin t(1-\sin t) + \lambda(1-\sin t)^2(1-\cos t)^2 \\ b_2(t) = 1 - 2\sin^2 t\cos^2 t + (\sin t + \cos t)(2\sin t\cos t - 1) \\ b_3(t) = \cos t(1-\cos t) + \mu(1-\sin t)^2(1-\cos t)^2 \\ b_4(t) = (1-\cos t)^2\sin^2 t - \mu(1-\sin t)^2(1-\cos t)^2 \end{cases} \quad (1)$$

The properties of the basis functions

Theorem 1 The basis functions (1) have the following properties:

(a) ***Non-negativity***: $b_i(t) \geq 0$ for $i = 0, 1, 2, 3, 4.$

(b) ***Partition of unity***: $\sum_{i=0}^{4} b_i(t) = 1$

(c) ***Symmetry***: $b_i(t;\lambda,\mu) = b_{4-i}\left(\frac{\pi}{2} - t; \mu.\lambda\right)$, for $i = 0, 1, 2, 3, 4.$

(d) ***Monotonicity***: For a given parameter t, as the shape parameters λ and μ increases (or decreases), $b_0(t)$, $b_4(t)$ decreases (or increases) and $b_1(t)$, $b_3(t)$ increases (or decreases). $b_2(t)$ remains unchanged for any increase or decrease in the shape parameters λ and μ.

Proof (a) For t$\in [0, \frac{\pi}{2}]$ and $\lambda, \mu \in [-20, 0]$, then

$0 \leq (1-\sin t) \leq 1,\ 0 \leq (1-\cos t) \leq 1,\ 0 \leq \sin t \leq 1,\ 0 \leq \cos t \leq 1$

It is obvious that $b_i(t) \geq 0$ for $i = 0, 1, 2, 3, 4$.

(b) $$\sum_{i=0}^{4} b_i(t) = (1-\sin t)^2\cos^2 t - \lambda(1-\sin t)^2(1-\cos t)^2 + \sin t(1-\sin t) + \lambda(1-\sin t)^2(1-\cos t)^2 + 1 - 2\sin^2 t\cos^2 t + (\sin t + \cos t)(2\sin t\cos t - 1) + \cos t(1-\cos t) + \mu(1-\sin t)^2(1-\cos t)^2 + (1-\cos t)^2\sin^2 t - \mu(1-\sin t)^2(1-\cos t)^2 = 1$$

The remaining cases follow obviously. The curves of the QT Bézier basis functions with two shape parameters are shown in Figure 1 for $\lambda = -20,\ -20 \leq \mu \leq 0$ in (a), $\lambda = -10,\ -20 \leq \mu \leq 0$ in (b), $\lambda = 0,\ -20 \leq \mu \leq 0$ in (c), $\mu = -20,\ -20 \leq \lambda \leq 0$ in (d), $\mu = -10$ in, $-20 \leq \lambda \leq 0$ (e) and $\mu = 0,\ -20 \leq \lambda \leq 0$ in (f). Note that $b_2(t)$ (green line) remains unchanged for any increase or decrease in the shape parameters λ and μ.

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III. QT BÉZIER CURVES WITH TWO SHAPE PARAMETERS

The construction of the QT Bézier curve with two shape parameters

Definition 2. Given control points $P_i = 0,1,2,3,4$ in R^2 or R^3, then

$$r(t) = \sum_{i=0}^{4} b_i(t)P_i, \quad t \in [0,1]; \ \lambda, \ \mu \in [-20,0] \qquad (2)$$

is called a QT Bézier curve with two shape parameters λ and μ.

From the definition of the basis function, some properties of the QT Bézier curve can be obtained as follows:

Theorem 2 The QT Bézier curves (2) have the following properties:

a) *Terminal Properties*: $r(0) = P_0, \quad r\left(\frac{\pi}{2}\right) = P_4,$ (3)

$$r'(0) = -2P_0 + P_1 + P_2, \ r'\left(\frac{\pi}{2}\right) = 2P_4 - P_2 - P_3 \qquad (4)$$

b) *Symmetry*: P_0, P_1, P_2, P_3, P_4 and P_4, P_3, P_2, P_1, P_0 define the same QT Bézier curve in different parametrizations, i.e.,

$$r(t; \lambda, \mu; P_0, P_1, P_2, P_3, P_4) = r\left(\frac{\pi}{2} - t; \lambda, \mu; P_4, P_3, P_2, P_1, P_0\right); \quad t \in \left[0, \frac{\pi}{2}\right], \ \lambda, \mu \in [-20,0] \quad (5)$$

c) *Geometric invariance*: The shape of a QT Bézier curve is independent of the choice of coordinates, i.e. (2) satisfies the following two equations:

$$r(t; \lambda, \mu; P_0 + q, P_1 + q, P_2 + q, P_3 + q, P_4 + q) = r(t; \lambda, \mu; P_0, P_1, P_2, P_3, P_4) + q$$
$$r(t; \lambda, \mu; P_0 * T, P_1 * T, P_2 * T, P_3 * T, P_4 * T) = r(t; \lambda, \mu; P_0, P_1, P_2, P_3, P_4) * T \qquad (6)$$
$$t \in \left[0, \frac{\pi}{2}\right], \quad \lambda, \mu \in [-20, 0]$$

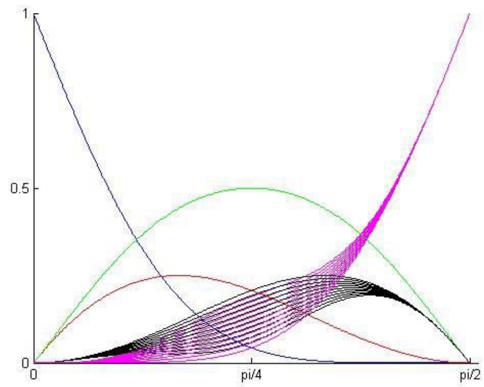
where q is arbitrary vector in R^2 or R^3, and T is an arbitrary $d \times d$ matrix, d=2 or 3.

d) *Convex hull property*: The entire QT Bézier curve segment lies inside its control polygon spanned by P_0, P_1, P_2, P_3, P_4.

IV. SHAPE CONTROL OF QT BÉZIER CURVE

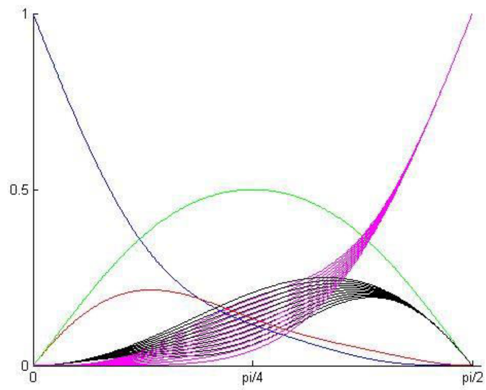
For $t \in \left[0, \frac{\pi}{2}\right]$, we rewrite (2) as follows:

$$r(t) = \sum_{i=0}^{4} P_i c_i(t) + \lambda(1 - \sin t)^2(1 - \cos t)^2(P_1 - P_0) + \mu(1 - \sin t)^2(1 - \cos t)^2(P_3 - P_4)] \quad (7)$$



λ =-20, $-20 \le \mu \le 0$

(a)



μ =-20, $-20 \le \lambda \le 0$

(d)

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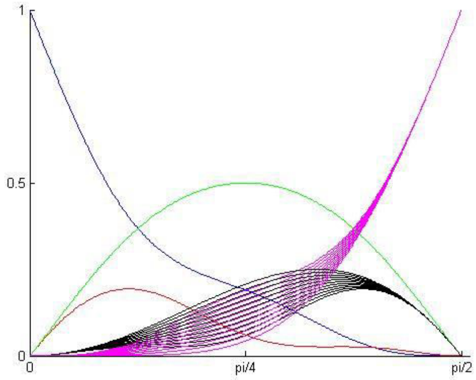
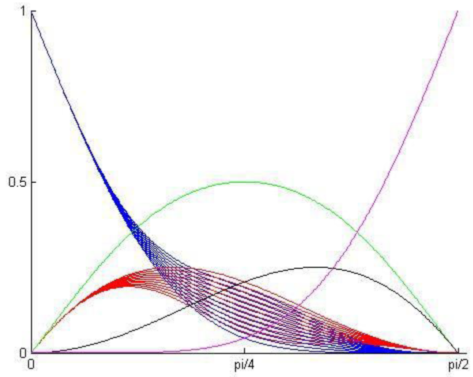
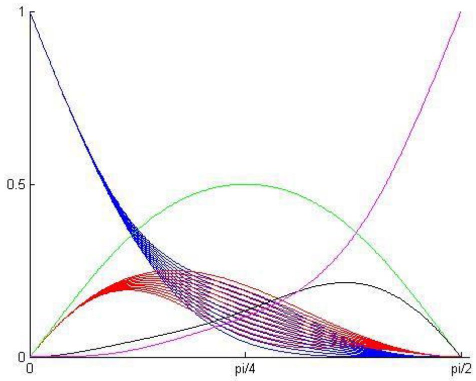


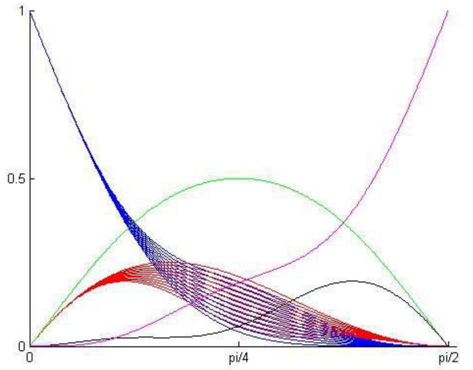
λ =-10, $-20 \leq \mu \leq 0$
(b)



μ =-10, $-20 \leq \lambda \leq 0$
(e)



λ =0, $-20 \leq \mu \leq 0$
(c)



μ =0, $-20 \leq \lambda \leq 0$
(f)

Fig. 1. QT Bézier basis functions for different values of λ and μ.

where $c_0(t) = (1 - \text{sin}t)^2 \cos^2 t$,

$$c_1(t) = \text{sin}t(1 - \text{sin}t),$$
$$c_2(t) = 1 - 2\sin^2 t \cos^2 t + (\text{sin}t + \text{cos}t)(2\sin t \cos t - 1),$$

$c_3(t) = \text{cos}t(1 - \text{cos}t)$,
$c_4(t) = (1 - \text{cos}t)^2 \sin^2 t$.

Obviously, the shape parameters λ and µ affects the curve on the control edges $(P_1 - P_0)$ and $(P_3 - P_4)$ respectively. The shape parameters λ and µ serve to effect local control in the curve: as λ increases, the curve moves in the direction of edge $(P_1 - P_0)$ and as λ decreases, the curve moves in the opposite direction to the edge $(P_1 - P_0)$. Similarly as µ increases, the curve moves in the direction of edge $(P_3 - P_4)$ and as µ decreases, the curve moves in the opposite direction to the edge $(P_3 - P_4)$.
Figure 2 shows a computed example of QT Bézier Curves with different values of shape parameters λ and μ. These curves are generated by setting for $\lambda = -20, -20 \leq \mu \leq 0$ in (a), $\lambda = -10, -20 \leq \mu \leq 0$ in (b), $\lambda = 0, -20 \leq \mu \leq 0$ in (c), $\mu = -20, -20 \leq \lambda \leq 0$ in (d), $\mu = -10, -20 \leq \lambda \leq 0$ in (e) and $\mu = 0, -20 \leq \lambda \leq 0$ in (f). In Figure 3, another example of shape modelling of QT Bézier Curves is presented for $\lambda, \mu = 0$.

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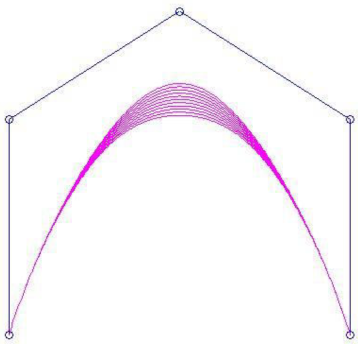
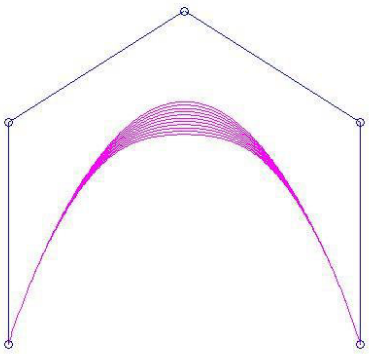
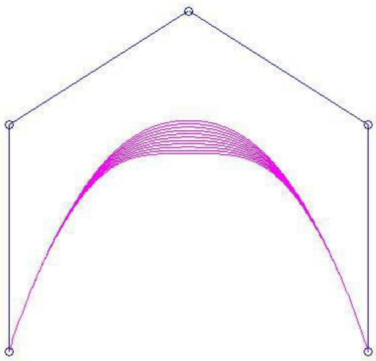


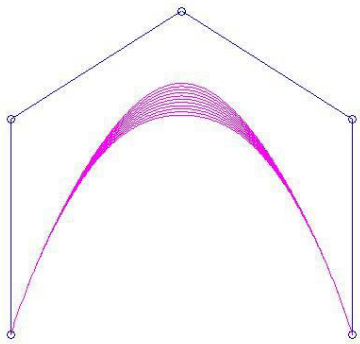
λ =-20, $-20 \le \mu \le 0$
(a)



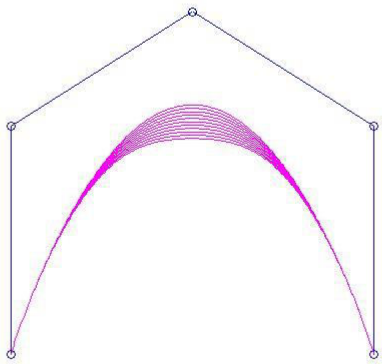
λ =-10, $-20 \le \mu \le 0$
(b)



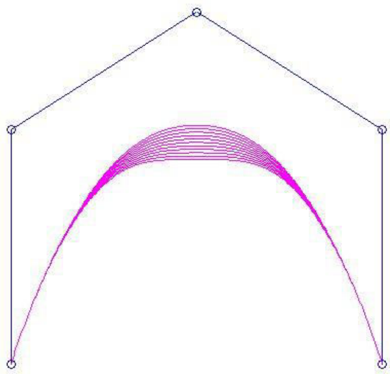
λ =0, $-20 \le \mu \le 0$
(c))



μ =-20, $-20 \le \lambda \le 0$
(d)



μ =-10, $-20 \le \lambda \le 0$
(e)



μ =0, $-20 \le \lambda \le 0$
(f)

Fig. 2: A computed example of QT Bézier curve with different values of shape parameters λ and μ

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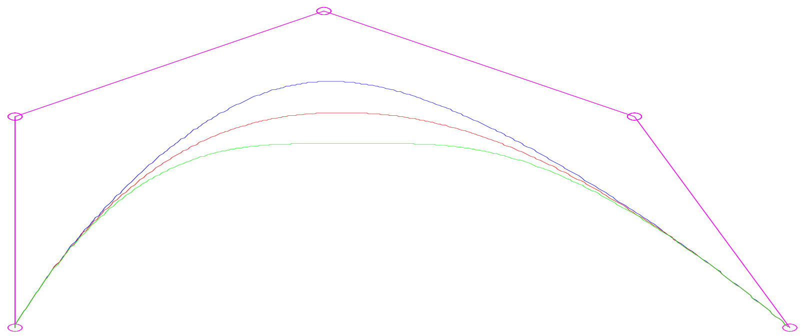

Fig. 3. QT Bézier Curves with different values of shape parameters λ and μ

V. THE QT BÉZIER SURFACES WITH TWO SHAPE PARAMETERS

Given control points $P_{rs}(r = i, \ldots, i+3; s = j, \ldots, j+3)$, $(i = 0,1, \ldots, n-1;\ j = 0,1, \ldots, m-1)$ using the tensor product method, we can construct the QT Bézier surface with two shape parameters λ and μ

$$T(u,v) = \sum_{r=i}^{i+3} \sum_{s=j}^{j+3} b_{r,4}(\lambda_1, \mu_1; u) b_{s,4}(\lambda_2, \mu_2; v) P_{rs}; \qquad (u,v) \in \left[0, \frac{\pi}{2}\right] \times \left[0, \frac{\pi}{2}\right]$$

where $b_{r,4}(\lambda_1, \mu_1; u)$ and $b_{s,4}(\lambda_2, \mu_2; v)$ are quartic trigonometric polynomial base functions for $\lambda_1, \mu_1, \lambda_2, \mu_2 \in$ [-20, 0]. Obviously these surfaces have properties similar to the corresponding QT Bézier curves. An example of QT Bézier curve is presented in Figure 4 and its corresponding QT Bézier surface is shown in Figure 5.

VI. APPROXIMABILITY

Suppose P_0, P_1, P_2, P_3 and P_4 are not collinear; the relationship between QT Bézier curve $r(t)$ and the quartic Bézier curve $B(t) = \sum_{i=0}^{4} P_i \binom{4}{i} (1-t)^{4-i} t^i$ with the same control points P_i is given by

$$r\left(\frac{\pi}{4}\right) - P_2 = \frac{(\sqrt{2}-1)^2}{4}\{P_0 - 2P_2 + P_4\} + \frac{(\sqrt{2}-1)}{2}\{P_1 - 2P_2 + P_3\} +$$

$$\frac{(\sqrt{2}-1)^4}{4}\{\lambda(P_1 - P)_0 + \mu(P_3 - P_4)\}$$

and

$$B\left(\tfrac{1}{2}\right) - P_2 = \tfrac{1}{16}[P_0 + 4P_1 - 10P_2 + 4P_3 + P_4] \tag{8}$$

These equations shows that QT Bézier curve can be made closer to the control polygon by altering the values of shape parameters λ and μ.

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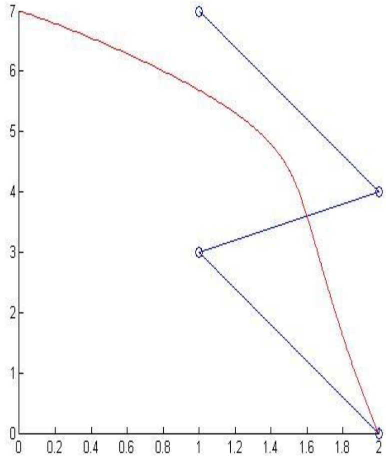

Fig. 4: QT Bézier curve

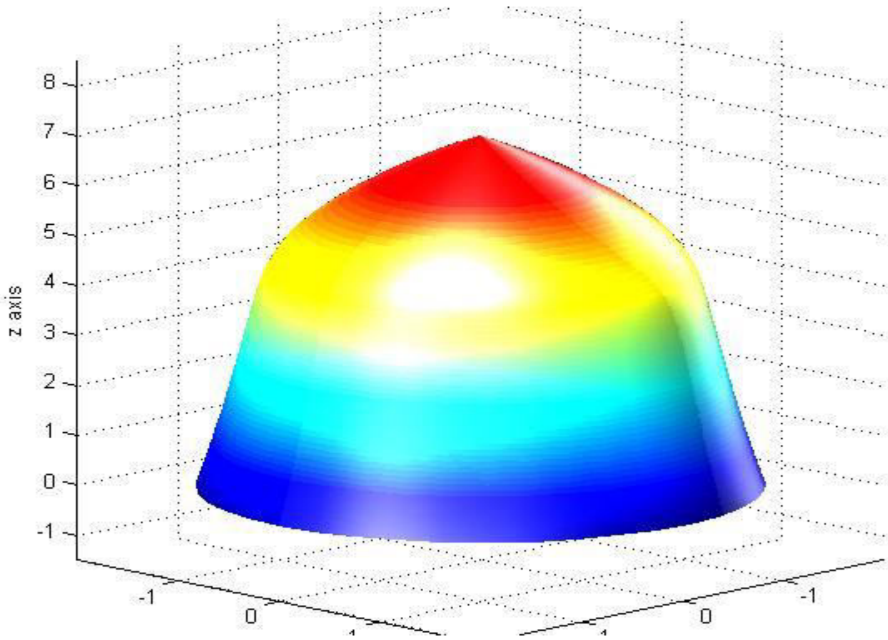

Fig. 5: QT Bézier surface corresponding to the curve in Fig. 4.

VII. CONCLUSION

As mentioned above QT Bézier curve have all the geometric properties that classical quartic Bézier curves have. The shape of the curve can be flexibly controlled by the shape parameters without altering the control points. Since there is nearly no difference in structure between a QT Bézier curve and a classical quartic Bézier curve, it is not difficult to adapt a QT Bézier curve to a CAD/CAM system that already uses the classical quartic Bézier curves.

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BIOGRAPHY

Dr. Reenu Sharma is working as Assistant Professor in Department of Mathematics, Rani Durgawati Govt. P.G. College, Mandla, M.P.She received Ph. D. in 2014 and MSc degree in 1997 from Rani Durgawati University, Jabalpur, M.P. Her research interests are Computer Aided Geometric Designing, Spline Theory etc.

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