Analysis of Switching Function Based Three Phase Voltage Source Inverter for Induction Motors

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ABSTRACT: It is essential to Model an Inverter driven Induction Motor in present industrial applications. This paper proposes very fundamental Switching Model as well as Dynamic Model for the Three Phase Inverter and Induction Motor respectively. With this developed functional model, the simplification of Static Power Circuits can be obtained so that quick convergence can be achieved during the simulation of larger systems. The Switching Function Model and Induction Machine Model are simulated in MATLAB/ SIMULINK and the corresponding results are also presented in this paper.

I.INTRODUCTION

A. Three Phase VSI

The behaviour of Power Electronic circuits like converters and Inverters are analysed by many technical computing programs or models. In the case of model based design, the power electronic circuits can be schematically expressed by using power semiconductor device models. Also, the control circuits can be expressed in terms of analog components. This kind of design is suitable only at the circuit level. But if the design is involved with non – linear systems, it will consume more time and sometimes it may face convergence problems. [4].

The main disadvantage of Matlab is, it is an interpreted language which can execute the programs very slowly than compiled languages. Moreover the exact state equations has to be designed to realize the Power Electronic circuit. But designing the proper state equation based on the circuit configuration is very difficult and time-consuming. Because for small change in the circuit also, the state equations has to be modified. So, a straight forward method to describe the Power converter and Inverter circuits is required, which should not be involved with state equations.

The Switching Function model is a prevailing tool to find behaviour of the Power converters and Inverters [1]–[5]. By using the Switching Function model, the Power Electronic circuits can be designed according to their tasks, instead of their circuit topologies [2]. The Switching Function model has the following advantages.
1) The Power Converter circuits can be generalised.
2) It is easy to design and investigate the performance.
3) The State equations are not required in order to realize the Power Converter circuit.
4) The Simulation time is reduced and also because of the absence of state equations, convergence problem will not occur.

B.Induction Machine

Whenever an electrical machine is supplying a constant load with respect to time, the performance of the machine can be analysed with its steady state model. But most of the loads used in industries are not of constant load. Under these conditions, the transient performance of the drives is very important in addition with the steady state

performance. The Induction machine models are available in MATLAB/SIMULINK[1-3], but it will not have any specific internal details. By using software codes the the control over the internal parameters is achieved. Again this process will be time consuming and the programmer should have the knowledge of writing the software codes. The models using software codes will have lesser simulation time than a simulink blocks. But Simulink models can also be made to run faster using "accelerator" functions. Both of these need additional expense and can be avoided if the simulation speed is not that significant. Another approach is using the Simulink Power System Blockset [4] that can be purchased with Simulink.

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With the proposed system, all the internal machine parameters are controllable unlike the simulink block of a Machine. This can be first achieved by first forming the mathematical equations which will best describe the Induction machine.
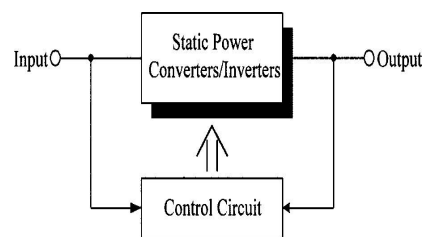
II. BASICS OF SWITCHING FUNCTION MODEL

Fig. 1. Block diagram of the static power conversion system.

As shown in Fig. 1, the Power converters and Inverters can be modeled as a Transfer function with the input and output ports. The type of supply i.e, the direct or alternating current is decided by the operation [4]. The transfer function is obtained to describe the task to be performed by the circuits. In general, the transfer function is used to compute a dependent variable from the independent variable [3]. In Pulse Width Modulation technique, the waveform to be modulated is considered the independent variable and the resulting modulated waveform is the dependent variable. For example, in case of a Voltage Source Inverter, the output voltage is dependent variable and the Input Voltage is an Independent variable. Therefore, the transfer function can be defined as [3]

$$\text{Transfer Function} = \frac{\text{Dependent Variable}}{\text{Independent Variable}}$$

$$= \frac{\text{Unmodulated Waveform}}{\text{Modulated Waveform}} \qquad (1)$$

With the proposed paper, each transfer function consists of particular switching functions. Therefore, achieving proper switching function is very important in order to realize the Power converters and Inverters. The detailed theoretical explanation of the switching function is described in the references [1]–[3].

III. PROPOSED FUNCTIONAL MODEL FOR THREE-PHASE VOLTAGE SOURCE INVERTER

A. Switching Function for VSI

Fig. 2(a) shows the circuit configuration of the VSI and Fig. 2(b) shows the input and output variables to be considered while analyzing and designing the circuit.
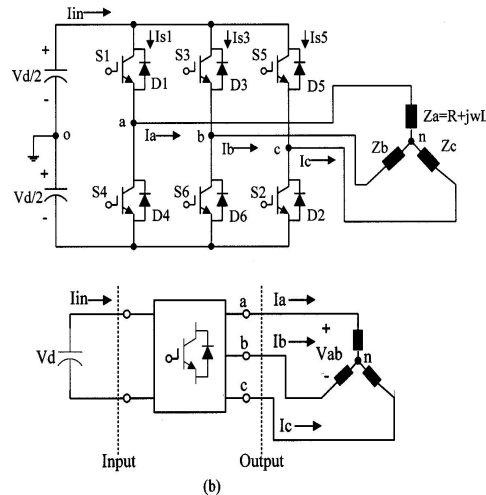
Fig. 2. (a) Circuit configuration of VSI. (b) Input and output variables of VSI.

Based on the transfer function theory, in the VSI, input current (I_{in}) and output voltage (V_{ab}, V_{bc}, V_{ca}) are the dependent variables and input voltage (V_d) and output current (I_a, I_b, I_c) are the independent variables. Therefore, the relationship between the input and output variables can be expressed as [3].

$$[V_{ab}, V_{bc}, V_{ca]} = TF . V_d \qquad (2)$$
$$I_{in} = TF . [I_a, I_b, I_c] \qquad (3)$$

where TF is the transfer function of VSI. Generally, the transfer function consists of the various switching functions as follows.

$$TF = [SF_1, SF_2, SF_3,....] \qquad (4)$$

In order to obtain the switching functions, a type of control strategy to be used has to be choosen. In this paper, the sinusoidal PWM (SPWM) technique as shown in Fig. 3(a) is considered as a control strategy. Based on the SPWM, Fig. 3(b) and (c) shows the two switching functions .
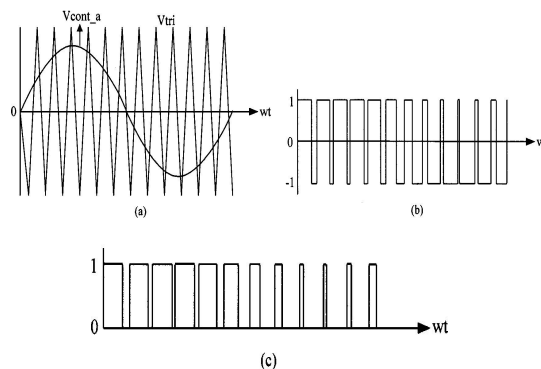

Fig. 3. SPWM control strategy and switching functions. (a) Carrier (V_{tri}) and control (V_{cont_n}) signals. (b) Switching function SF_1 (c) Switching function SF_2 .

The switching function SF_1 expresses the V_{ao}, V_{b0}, and V_{co}, and it is used to compute the inverter line-to-line voltages (V_{ab}, V_{bc}, V_{ca}) and phase voltages (V_{an}, V_{bn}, V_{cn}). And, the switching function SF_2 expresses the voltage

across the switch. The load currents (I_a, I_b, I_c) are derived as ratios of voltages and corresponding impedances using the switching function SF_2. The Switching Functions SF_1 SF_2 are given by [2]

$$SF_1 = \sum_{n=1}^{\infty} A_n \sin(n\omega t) \qquad (5)$$

$$SF_2 = B_0 + \sum_{n=1}^{\infty} B_n \sin(n\omega t). \qquad (6)$$

B. Realization of Switching Function Model

Based on the switching functions SF_1 and SF_2, a model for the VSI is built by using MATLAB Simulink [6]. Fig. 4 shows the proposed overall functional model for computing the design parameters of the VSI. As shown in Fig. 4, it consists of five functional blocks: SPWM generator, switching function block, inverter block, load current block, and pure switch and diode current generating block.
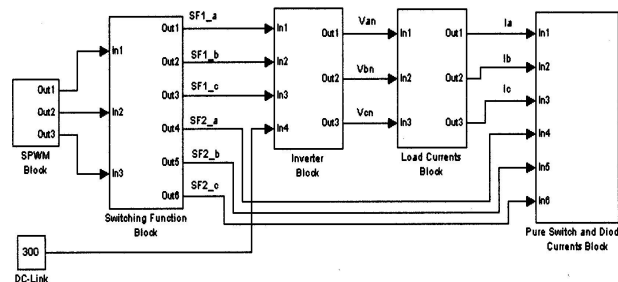

Fig. 4. Overall block diagram of the proposed simulation model
for VSI using switching function concept.

In the SPWM block, the carrier signal (V_{tri}) is compared with three sinusoidal signals (V_{cont_a}, V_{cont_b}, V_{cont_c}). The output of SPWM block is feeding the input to the switching function block to produce the two switching functions. (SF_1 and SF_2).

1) Voltage Variables

Each phase has two Switching Functions such as SF_{1_a}, SF_{2_a} for phase'a', SF_{1_b}, SF_{2_b} for phase'b', SF_{1_c}, SF_{2_c} for phase'c'. Using the Switching Function $SF_{1\text{-a,b,c}}$, the V_{ao}, V_{bo}, V_{co} can be obtained as,

$$\bar{V}_{ao} = \frac{V_d}{2} \cdot SF_{1\_a} = \frac{V_d}{2} \cdot \sum_{n=1}^{\infty} A_n \sin(n\omega t) \qquad (7)$$

$$V_{bo} = \frac{V_d}{2} \cdot SF_{1\_b} = \frac{V_d}{2} \cdot \sum_{n=1}^{\infty} A_n \sin n(\omega t - 120°) \qquad (8)$$

$$V_{co} = \frac{V_d}{2} \cdot SF_{1\_c} = \frac{V_d}{2} \cdot \sum_{n=1}^{\infty} A_n \sin n(\omega t + 120°). \qquad (9)$$

Once the Switching Function is produced, the phase and line voltages in the Voltage Source Inverter can be obtained and is obtained in the inverter block in Fig. 4.

2) Current Variables (Load, Switch, and Inverter Input Currents):

Next, the load current block is used to obtain the load currents (I_a, I_b, I_c). Assuming the load currents consists of an balanced *R-L* load, the load currents are found by taking the ratios of the phase voltages and respective impedance.

The switch currents (I_{s1}, I_{s3}, I_{s5}) are calculated by the product of the load currents with the corresponding switching function.

For obtaining the current rating of the Power Semiconductor Switch, the pure switch current and the pure diode current are needed. Actually, the switch current (I_{S1}) can be divided into two namely pre switch current I_{S1_s} and pure diode current I_{S1_d}.

From the switch currents, the inverter input current (I_{in}) can be obtained.

3) Average and RMS Ratings:
Finally, the average and rms values of current parameters can be expressed by the vector form such as

$$I_{S1\_S,\,\text{avg}} = \frac{1}{T} \int_0^T I_{S1\_S}(t)\, dt \qquad (10)$$

$$= \frac{1}{2n} \sum_{k=1}^{n} [I_{S1\_S}(k) + I_{S1\_S}(k-1)]$$

$$I_{S1\_S,\,\text{rms}} = \sqrt{\frac{1}{T} \int_0^T I_{S1\_S}^2(t)\, dt}$$

$$= \sqrt{\frac{1}{2n} \sum_{k=1}^{n} [I_{S1\_S}^2(k) + I_{S1\_S}^2(k-1)]} \qquad (11)$$

In a similar way, the gate signals are produced for current control and SVPWM control strategies. This will be given to the Power Semiconductor Switches to drive the inverter or converter, to achieve the desired outputs. These gate signals can be converted to the switching function signals.

IV. INDUCTION MOTOR MODEL

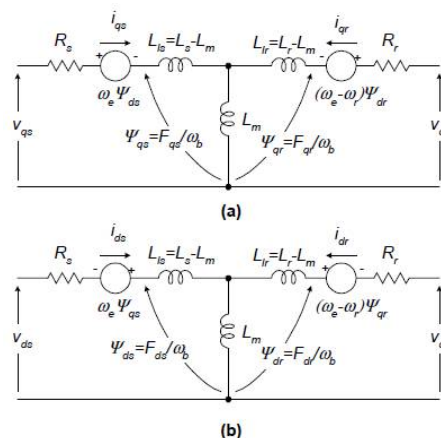The induction machine d-q or dynamic equivalent circuit is shown in Fig. 5.

Fig. 5. Dynamic or d-q equivalent circuit of an induction machine.

One of the most popular induction motor models derived from this equivalent circuit is Krause's model detailed in [13].

According to his model, the modeling equations in flux linkage form are as follows:

$$\frac{dF_{qs}}{dt} = \omega_b\left[v_{qs} - \frac{\omega_e}{\omega_b}F_{ds} + \frac{R_s}{x_{ls}}\left(F_{mq} + F_{qs}\right)\right] \quad (12)$$

$$\frac{dF_{ds}}{dt} = \omega_b\left[v_{ds} + \frac{\omega_e}{\omega_b}F_{qs} + \frac{R_s}{x_{ls}}\left(F_{md} + F_{ds}\right)\right] \quad (13)$$

$$\frac{dF_{qr}}{dt} = \omega_b\left[v_{qr} - \frac{\left(\omega_e - \omega_r\right)}{\omega_b}F_{dr} + \frac{R_r}{x_{lr}}\left(F_{mq} - F_{qr}\right)\right] \quad (14)$$

$$\frac{dF_{dr}}{dt} = \omega_b\left[v_{dr} + \frac{\left(\omega_e - \omega_r\right)}{\omega_b}F_{qr} + \frac{R_r}{x_{lr}}\left(F_{md} - F_{dr}\right)\right] \quad (15)$$

$$F_{mq} = x_{ml}^*\left[\frac{F_{qs}}{x_{ls}} + \frac{F_{qr}}{x_{lr}}\right] \quad (16)$$

$$F_{md} = x_{ml}^*\left[\frac{F_{ds}}{x_{ls}} + \frac{F_{dr}}{x_{lr}}\right] \quad (17)$$

$$T_e - T_L = J\left(\frac{2}{p}\right)\frac{d\omega_r}{dt} \quad (18)$$

$$\frac{dF_{qr}}{dt} = \omega_b\left[-\frac{\left(\omega_e - \omega_r\right)}{\omega_b}F_{dr} + \frac{R_r}{x_{lr}}\left(\frac{x_{ml}^*}{x_{ls}}F_{qs} + \left(\frac{x_{ml}^*}{x_{lr}} - 1\right)F_{qr}\right)\right]$$

$$\frac{dF_{dr}}{dt} = \omega_b\left[\frac{\left(\omega_e - \omega_r\right)}{\omega_b}F_{qr} + \frac{R_r}{x_{lr}}\left(\frac{x_{ml}^*}{x_{ls}}F_{ds} + \left(\frac{x_{ml}^*}{x_{lr}} - 1\right)F_{dr}\right)\right]$$

$$\frac{d\omega_r}{dt} = \left(\frac{p}{2J}\right)\left(T_e - T_L\right)$$

where d : direct axis, q : quadrature axis,
s : stator variable, r : rotor variable

Fij is the flux linkage ($i=q$ or d and $j=s$ or r),
vqs, vds : q and d–axis stator voltages,
vqr, vdr : q and d–axis rotor voltages,
Fmq, Fmd : q and d axis magnetizing flux linkages,
Rr : rotor resistance,
Rs : stator resistance,
Xls : stator leakage reactance (ω $eLls$),
Xlr : rotor leakage reactance (ω $eLlr$),
iqs, ids : q and d–axis stator currents,
iqr, idr : q and d–axis rotor currents,
J : moment of inertia,
Te : electrical output torque,
TL(or Tl) : load torque,
ω e : stator angular electrical frequency,
ω b : motor angular electrical base frequency, and
ω r : rotor angular electrical speed.

For a squirrel cage induction machine as in the case of this paper, vqr and vdr in (14) and (15) are set to zero.

An induction machine model can be represented with differential equations as seen above. To solve these equations, they have to be rearranged in the state-space form, $dx/dt = Ax + b$, where $x = [Fqs\ Fds\ Fqr\ Fdr\ \omega r]^T$ is the state vector. Note that $Fij = \psi ij \cdot \omega b$, where Fij is the flux linkage ($i=q$ or d and $j=s$ or r) and ψij is the flux.

In this case, state-space form can be achieved by substituting (16) and (17) in (12 – 15).

Combining the similar terms together so that each state derivative is a function of only other state variables and model inputs.

Then, the modeling equations of a squirrel cage induction motor in state-space become

$$\frac{dF_{qs}}{dt} = \omega_b\left[v_{qs} - \frac{\omega_e}{\omega_b}F_{ds} + \frac{R_s}{x_{ls}}\left(\frac{x_{ml}^*}{x_{lr}}F_{qr} + \left(\frac{x_{ml}^*}{x_{ls}} - 1\right)F_{qs}\right)\right]$$

$$\frac{dF_{ds}}{dt} = \omega_b\left[v_{ds} + \frac{\omega_e}{\omega_b}F_{qs} + \frac{R_s}{x_{ls}}\left(\frac{x_{ml}^*}{x_{lr}}F_{dr} + \left(\frac{x_{ml}^*}{x_{ls}} - 1\right)F_{ds}\right)\right]$$

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V. SIMULATION IMPLEMENTATION

A. Voltage Source Inverter

The proposed model for the VSI is implemented using MATLAB Simulink and the simulation voltage and current waveforms are presented in Figs. 6 and 7.

The simulation parameters are as follows: dc-link input voltage V_d = 300V, R = 5Ω, L= 20 mH, carrier signal frequency 1 kHz, control signal frequency(f_c) = 50 Hz, and modulation index, M_a = 0.8 .

Fig. 7 shows the switching function signals and for the three phases, obtained from the SPWM control strategy.

Based on the switching functions , the inverter voltage parameters, such as , line-to-line voltages and phase voltages can be obtained as shown in Fig. 7.

Also, the balanced load currents I_a, I_b, I_c are obtained as shown in Fig. 7(d). By multiplying the load currents with the switching function , the switch currents , and can be calculated.
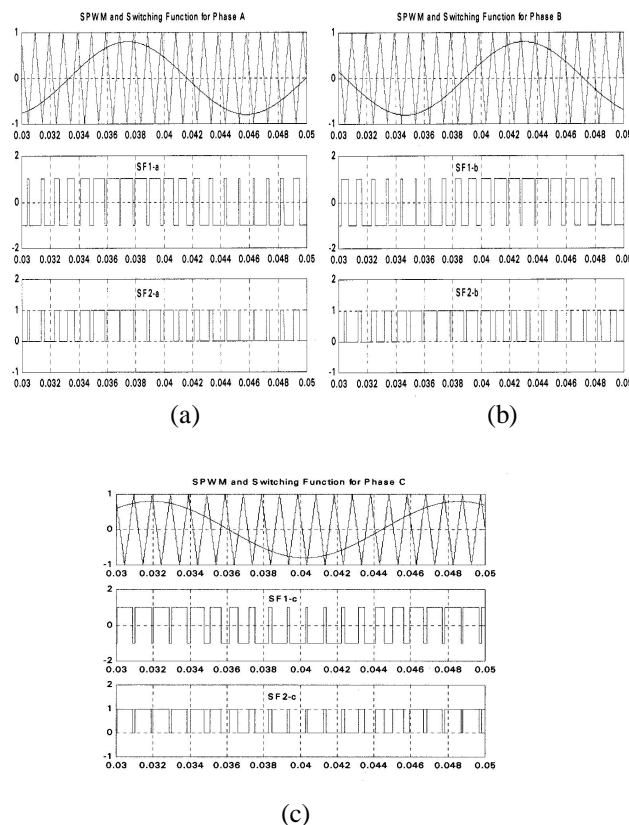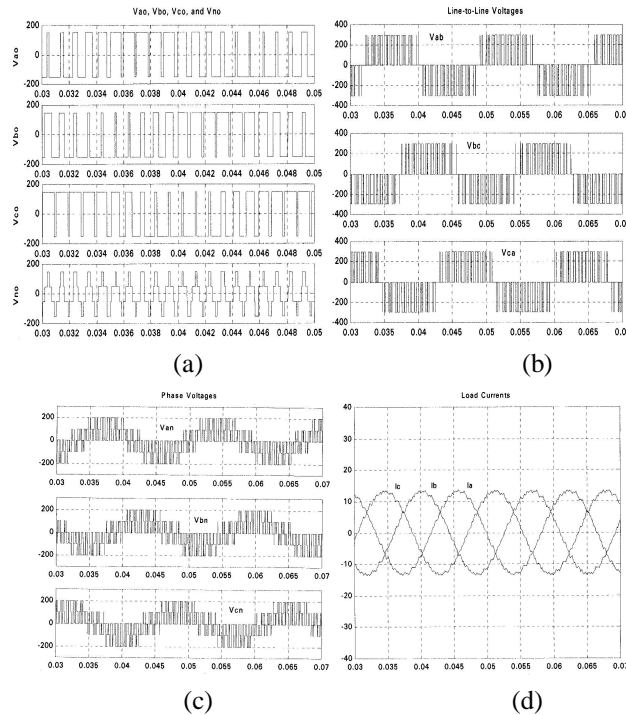

(a) (b)

(c)

Fig. 6. Switching functions SF_1 and SF_2 with the SPWM control
(a) Phase A. (b) Phase B. (c) Phase C.

Fig. 7. Voltage and current waveforms of VSI with the SPWM control (a) V_{ao} ; V_{bo} ; V_{co} (b) Line-to-line voltages (c) Phase voltages (d) Load currents

B. Three phase Induction Machine

Generally change of variables are used in analysis of ac machines to eliminate time varying inductances, this is also employed in the analysis of various static, constant parameter power and control system components.

Here Synchronously rotating reference frame is considered. So, the induction machine model will have three phase to to phase conversion (abc- dq block) and two phase to three phase conversion (dq – abc block) to convert back.

The propsed overall induction machine model is shown in Fig. 8. It consists of five major blocks: the o-n conversion, abc-syn conversion, syn-abc conversion, unit vector calculation, and the induction machine d-q model blocks.
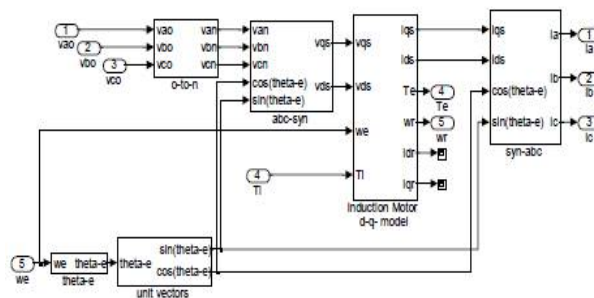

Fig. 8. The complete induction machine Simulink model

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o-n conversion block - This block is required for an isolated neutral system, otherwise it can be bypassed.

Unit vector calculation block - Unit vectors *cos* θe and *sin* θe are used in vector rotation blocks, "abc-syn conversion block" and "syn-abc conversion block". The angle, θe is calculated directly by integrating the frequency of the input three-phase voltages, ωe.

abc-syn conversion block -To convert three-phase voltages to voltages in the two phase synchronously rotating frame, they are first converted to two-phase stationary frame and then from the stationary frame to the synchronously rotating frame.

syn-abc conversion block -This block does the opposite of the abc-syn conversion block for the current variables the same implementation techniques as before.

Induction machine d-q model block - First consider the flux linkage state equations because flux linkages are required to calculate all the other variables.
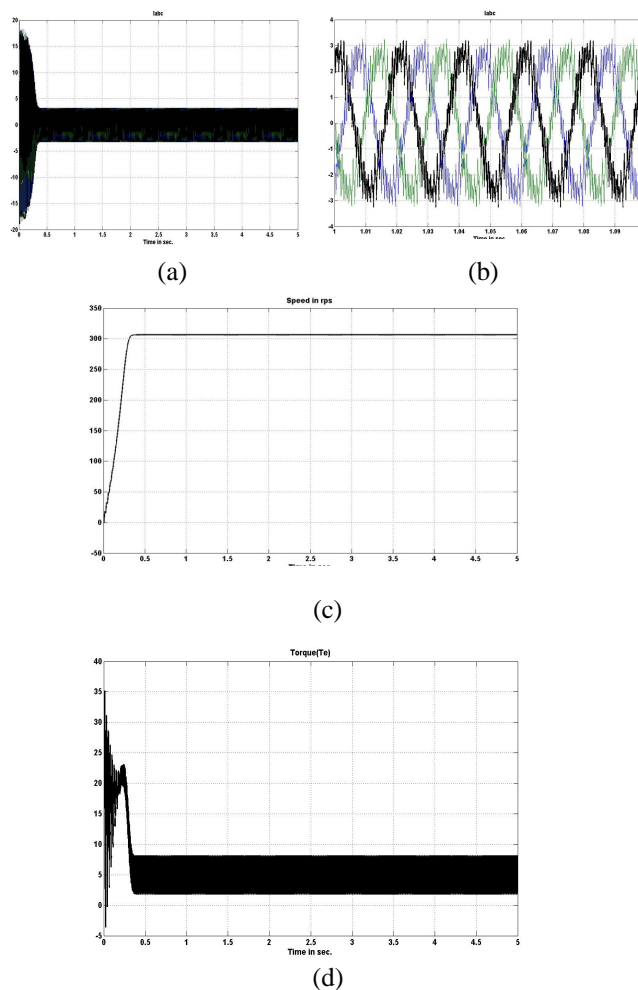

(a)

(b)

(c)

(d)

Fig.9 (a) Three Phase Currents (b) Steady state Three Phase Currents (c) Speed (d) Torque

These equations could be implemented using Simulink "State-space" block, but to have access to each point of the model, implementation using discrete blocks is preferred. Fig. 9 shows the three-phase currents, torque, and speed of the machine.

VI. CONCLUSIONS

The Switching function model three-phase Voltage Source Inverter has been studied. It is simulated in MATLAB / SIMULINK and the results are verified. The Modelling of Induction Machine is done and also implemented in SIMULINK. In this all the variables of Induction machine can be easily accessed to analyse the transient behavior. By using this combined model, the convergence problem is greatly reduced.

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