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Implementation of New Algorithm for Low Complexity Programmable Fir Filters Based On Extended Double Base Number System Using VHDL

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ABSTRACT: Coefficient multipliers are the stumbling blocks in programmable finite impulse response (FIR) digital filters. As the filters coefficient change either dynamically or periodically, the search for common sub expressions for multipliers less implementation needs to be performed over the entire gamut of integers of the desired precision and the amount of shifts associated with each identified common sub expression needs to be memorized, The complexity of a quality search is thus beyond the existing design algorithms based on conventional binary and signed digit representations. In this paper we present a new design paradigm for the programmable FIR filters by exploiting the extended double base number system (EDBNS). Due to its widely spaced intervals and innate abstraction of the sum of the binary shifted partial products, the sharing of adders in the time-multiplexed multiple constant multiplication blocks of the programmable FIR filters can be maximized by a direct mapping from the quasi-minimum EDBNS. The multiplexing cost can be further reduced by merging double base terms. Logic synthesis results on more than one hundred programmable filters with filter taps ranging from ten to hundred and coefficient word lengths of eight have been verified.

KEYWORDS: Digital Signal Processing, Distributed Arithmetic, Double Based Number, FIR filter, Programmable filter.

I. INTRODUCTION

Finite Impulse Response (FIR) filters offer many advantages, such as computational efficiency in multi-rate applications, desirable numerical property for finite precision, fractional arithmetic and easily attainable linear phase response[2]-[5]. Adaptive filters, in particular, are inevitable in many important applications in communications, image processing, computer vision, data acquisition and control [6]-[10]. With any adaptive filter, there is are requirement for a programmable filter, which is a primary reason behind the increasing dominance of digital instead of analog system implementations. In applications such as multi-rate decimation [7], discrete cosine transform [8], [9], channelization [10], [11], High efficiency Video Coding (HEVC) [12], wide bandwidth photonic filter[13] and high-rate communication [14], the filter coefficients need to be run-time reconfigurable by the error feedback signal or adaptable to varying filtering specifications in real time.

Owing to the huge search space over the complete gamut of integer values of a given precision, the intricacy of common sub expression sharing within and across multiple sets of coefficients in a TM-MCM block beyond the capability of existing CSE algorithms and fixed-coefficient filter design algorithms and fixed –coefficient filter design methodologies even for moderate coefficient word length and filter taps. As the shifters are no longer fixed, more succinct number representation than CSD and binary are explored. DBNS and Multidimensional logarithmic number system (MDLNS) are considered for the design of DSP operators such as constant multipliers [31]-[33]. In [30], logarithmic and double base number representations are used to synthesize inexact fractional filter coefficients for time-invariant filters. To minimize the additions of each coefficient multiplier, multiple-radix DBNS representation is used



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by limiting the maximum power of the second base number in [32]. In this paper, we uniquely exploited the double base number representation to maximize the sub expression sharings for all filter coefficients of a given word length in a more general time-varying filter implementation problem that has no leverage of the fixed quantized coefficients at design time. This piece of work is an extension of [34], which is the first ever attempted to harness the sparseness of the canonic DBNS representation for the complexity reduction of TM-MCM block. In this paper, we extended the double base number system (DBNS) [35] to encapsulate the binary shifts in tandem with several most frequently encountered common sub expressions. A new formulation of the common sub expression search problem as a quasi-minimized extended DBNS(EDBNS) generation problem is proposed, which has led to considerable reduction in the number of distinct partial products for a given word length of programmable coefficients. With this number system, an efficient architecture for the implementation of TM-MCM is derived as shown in figure-1. It consists of a Power –of –b Generator (POBG), N blocks of Power-of b-Selector (POBS) and N blocks of double base coefficient generator(DBCG), where b is the second base number in EDBNS and N blocks of taps. In this design, the unique partial product terms can be generated incrementally with one adder each. The sizes of the multiplexers in the programmable units and the lookup tables for the selector logic are further minimized by exploiting the unique features of EDBNS in the exponential equation.



Fig.1. Sequential realization of a distributed arithmetic FIR filter.

II. DISTRIBUTED ARTHEMETIC

Finite impulse response (FIR) filters are one of the most fundamental components in digital signal processing. Many simplifications in their hardware implementation can be made when the coefficients are constants. However, reconfigurable FIR filters for which the coefficients can be changed in run time are required in many applications like software defined radios. This encourages the researchers to work towards extended optimization techniques that were developed in the context of Multiple Constant Multiplication (MCM) to reconfigurable multiplier blocks [1]-[8], such multiplier blocks are usually realized using additions, subtractions and shifts only. In a reconfigurable multiplier block, additional multiplexes are inserted in the data path to configure the multiplication with a lot of resources of a finite set of coefficients.

The fundamental operation of a digital filter with N taps is the product of two vectors which can be represented as sum-of products of its components.

$$y = c.x = \sum_{n=0}^{N-1} c_n x_n$$
 (1)

Where c_n are usually constants and x_n are the time shifted input samples. If each x_n is represented as a binary B_x bit

 2^{th} complement number, where $x_{n:b}$ denotes the b^{th} bit of x_n .

Therefore equation (1) can be equivalently expressed as



(4)

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where $x \sim_{b}^{N} = (x_{0;b}; :::; x_{N1;b})^{T}$ (3) is a bit vector of length N containing the b_{th} bit of each element of x. The function

$$f(\tilde{x}_b^N) = \sum_{n=0}^{N-1} c_n x_{n,b}$$

can be pre-computed and stored in a single LUT with N inputs. The storage requirement of the LUT is $B_f^N 2^N$ bit, where B_f^N denotes the output word size of the N-input LUT $f(-x_b^N)$. The inner product can now be obtained by accumulating the shifted outputs of the LUT according to (3). A sequential realization of (3) which computes a valid output every N samples is shown in Fig. 1. For higher throughput, a parallel implementation using B_x LUTs can be obtained by unfolding .So far,this N-input LUT cannot be directly mapped to the reconfigurable 4/5-input CFGLUTs described above. Therefore, a method to reduce the LUT input size [11] was used to break the N-input LUT into several 4/5-input LUTs which is described in the following.

A. Dividing LUTs into Smaller Partial LUTs

The input size of the LUT can be reduced by splitting the sum in (4) into several smaller sums

$$f(\tilde{x}_{b}^{N}) = \sum_{l=0}^{\lfloor N/L \rfloor - 1} \underbrace{\sum_{n=lL}^{(l+1)L-1} c_{n} x_{n,b}}_{f_{l}(\tilde{x}_{b}^{L})} + \underbrace{\sum_{n=N-L'}^{N-1} c_{n} x_{n,b}}_{f_{\lfloor N/L \rfloor}(\tilde{x}_{b}^{L'})}$$
(5)

with L < N where

$$f_l(\tilde{x}_b^L) = \sum_{n=lL}^{(l+1)L-1} c_n x_{n,b}$$
(6)

can be realized by partial L-input LUTs. If N is not dividable by L, one additional partial LUT of size $L^0 = N \mod L$ is necessary, which is represented with the last term in (5). By setting L = 4 or L = 5, the LUT $f(\sim x_b^N)$ can be directly mapped to CFGLUTs by using the decomposition of (5). Furthermore, this method reduces the LUT storage requirements for the N-input LUT $f(\sim x_b^N)$ from $B_f^N 2^N$ bits to $bN=Lc B_f^L 2^L + B_f^{L0} 2^{L0}$ bits. Note that for parallel DA, the N-input LUT is used B_x times. For a fixed L, this realization style grows linear with the number of filter taps N in contrast to (4) which grows exponentially. This memory reduction is paid by bN=Lc additional adders.



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A. RESOURCE OPTIMIZED ARCHITECTURE

Modern DSP systems are well suited for VLSI implementation .Indeed, they are often technically feasible or economically viable only if implemented using VLSI technologies. Many DSP systems are produced in very large numbers and require high performance circuits with respect to throughput and power consumption. The combined advances in system design capability and VLSI technology have made it possible to economically design unique integrated circuits or use in dedicated applications, so called Application Specific Integrated Circuits (ASIC). The possibility of incorporating a whole signal processing system into a chip has multitude of effects. It will dramatically increase the processing capacity and simultaneously reduce the size of the system, the power consumption.

Finite-impulse response (FIR) Filter is extensively used in wireless sensor networks as a signal preprocessing step. Because sensor nodes require a long working periods and ultra-low cost, traditional FIR structures are inapplicable as multipliers occupy too much die size for such node's chips. This paper proposes novel FIR filter structures used in the design of application specific integrated circuits (ASICs) for sensor nodes, which can reduce the hardware cost to a minimum. The experiments show that the proposed FIR structure can lead to significant hardware savings from the traditional FIR filter. It's a better choice for sensor node ASICs.

Memory Based structures

The phrase we use "*memory-based structures*" or "*memory-based systems*" for those systems where memory elements like RAM or ROM is used either as a part or whole of an arithmetic unit. Memory-based structures are more regular compared with the multiply-accumulate structures; and have many other advantages, e.g., greater potential for high-throughput and reduced-latency implementation, (since the memory-access-time is much shorter than the usual multiplication-time) and are expected to have less dynamic power consumption due to less switching activities for memory-read operations compared to the conventional multipliers. Memory-based structures are well- suited for many digital signal processing (DSP) algorithms, which involve multiplication with a fixed set of coefficients.

There are two basic variants of memory-based techniques. One of them is based on distributed arithmetic (DA) for inner product computation and the other is based on the computation of multiplication by look-up-table (LUT). In the LUT-multiplier-based approach, multiplications of input values with a fixed-coefficient are performed by an LUT consisting of all possible pre-computed product values corresponding to all possible values of input multiplicand, while in the DA-based approach, an LUT is used to store all possible values of a fixed –N-point vector with any possible N-point bit-vector. If the inner-products are implemented in a straight-forward way, the memory- size of LUT-multiplier based implementation increases exponentially with the word length of input values, while that of the DA based approach increases exponentially with the inner product- length. Attempts have been made to reduce the memory-space in DA- based architectures using offset binary coding (OBC) and group distributed technique. A decomposition scheme is suggested in a recent paper for reducing the memory-size of DA-based implementation of FIR filter. But, it is observed that the reduction of memory size achieved by such decompositions is accompanied by increase in latency as well as the number of adders and latches.

III. PROPOSED FIR FILTER

It consists of 3 blocks those are 1.POBG (power-of- generator) block 2.POBS (power-of- selector) block 3.DBCG (double base coefficient generator) Block





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Fig.2. 8-bit programmable FIR filter

The shifters in DBCG block shift the T outputs from POBS to generate the double base terms 2α tbt β t for t=1,2,.....T, and the subsequent adder tree sums up the T terms to produce the coefficient multiplier output h[i]x[n] of each filter tap. Given T and F min, the EDBNS representations of all -bit coefficients that result in the minimum number of different α t for each of the POBS outputs are selected. Thus, each shifter needs only to realize different amounts of shift, where is determined by the minimum number of distinct exponents α t that can appear in the -th double base term. Each POBS output is hardwire-shifted by different numbers of bits and fed into the data inputs of a multiplexer. One of these shifted versions of the POBS output will be selected from each multiplexer to compose the EDBNS representation of the programmed coefficient. The control signals to the multiplexers and the programmable shifters are generated by an external look up table (LUT). Because the EDBNS representation of any even integers can be obtained by a double base scalar multiplication of a factor and an odd integer, i.e. $c=2\alpha \times c$, where is an even number and c is known as a fundamental, the control information of the even integers are stored together with their fundamentals in the LUT, 2α plus a factor stored for each even number. This will reduce the LUT size by almost half. The complete design procedure is summarized as follows:

Step 1) Compute T min and F min for all -bit coefficients. Generate the EDBNS array for all the -bit coefficients using the search algorithm presented.

Step 2) Implement the POBG block by producing all power-of- integers in F min using the method described.

Step 3) Design the POBS block with T multiplexers. Each power-of- integer from the POBG block is first connected to an input of K different multiplexers, where k is the maximum number of times that power-of- value can appear in the EDBNS representation of any coefficient. Then, minimize the number of input lines to the multiplexers of POBS block by the algorithm presented in Fig 4.

Step 4) Design T programmable shifters for the DBCG block. Extract the amount of shifts α for each power-of- integer and store it in the LUT addressable by the fundamentals.

Step 5) Sum the T double base terms in DBCG by a carry save adder (CSA) tree to reduce the delay.



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Fig.3. POBG block implementation

(a) F min= $\{3, 9, 27, 81\}$ without logic depth optimization

(b) F min = $\{5, 25, 125, 625\}$ without logic depth optimization (c) F min = $\{3, 9, 27, 81\}$ with

logic depth optimization.

The immediate structure is by and large favoured in light of its higher execution and force proficiency. The issue of planning Poly phase decimator has gotten an extraordinary consideration because of expansive number of duplications. This execution must fulfil the authorized examining rate limitations of the constant DSP applications and must require less space and power utilization. Present works have concentrated on outline of Multi rate Poly phase decimator by channels, information generator locks and viper. As the coefficients of an application particular channel are steady, the decay is more proficient than utilizing multipliers. The multifaceted nature of FIR channels for this situation is ruled by the quantity of increases and duplications. The multiplier square of the advanced FIR channel in its immediate structure is actualized in the configuration so that critical effect on the many-sided quality and execution of the outline will be made strides. Additionally, Poly phase channel is outlined utilizing MCM and digit serial adders which overcome issue of multifaceted nature, plan execution and creating low region.

IV. CONCLUSION AND FUTURESCOPE

The method of sub expression has the capability of affecting noteworthy reserve funds in the quantities of augmentations utilized as a part of the usage of FIR channels. It has been watched that different procedures using normal sub expressions in coefficient representations other than CSD might on events lead to less adders, and as a rule can accomplish comparative investment funds. Preparatory studies have demonstrated that just a predetermined number of SPT terms are required to meet a respectable arrangement of particulars if a decent enhancement method exists. Henceforth, to speak to the coefficients of a channel thusly, the coefficient multipliers can be supplanted by a little number of include/subtract-shift operations. The equipment many-sided quality is in this manner to a great extent lessened. By utilizing MILP snake use is further minimized.

There are various techniques generated for designing of FIR filters. Every method has its own merits and demerit. For example design linear-phase FIR filters by a novel weighted BP neural networks algorithm has Some limitation it is not involved in operation of inverse matrix and the window method is also have some limitations like they are not very suitable for designing of filters with any given frequency response. The future work is develop a technique suitable for designing of filters with a given magnitude response and reduce the noise of signal.



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V. SIMULATION RESULTS



Fig 4: POBGS output

entre	Tanac		
📲 m1[]	000000	0000000000101	
🃲 m2[)	000000	000000000000	
📲 m3‡	000000	0000000000000	
📲 sela	00	00	
📲 sela	00	00	
📲 sela	00	00	
Num 📲	000000	000000000000000000000000000000000000000	
Num sum	000000	000000000000000000000000000000000000000	

Fig 5: POBSS output





REFERENCES

[1] Jiajia Chen, Chip-Hong Chang, Feng Feng, Weiao Ding, and Jiatao Ding, Novel Design Algorithm for Low Complexity Programmable FIR Filters Based on Extended Double Base Number System IEEE Transactions on Circuits and Systems—I: Regular papers, Vol. 62, No. 1, January 2015.

[2] Dempster, A. G and MacleodM. D., Use of minimum-adder multiplier blocks in FIR digital filters, *IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process.*, Vol. 42, No. 9, pp. 569–577, Sep. 1995.

[3] Wang Y and Roy.K, "CSDC: A new complexity reduction technique for multiplier less implementation of digital FIR filters, *IEEE Trans Circuits Syst. I, Reg Papers*, Vol. 52, No. 9, pp. 1845–1853, Sep. 2005.

[4] Yao, C. Y., Hsia, W. C. and. Ho, Y. H., Designing hardware-efficient fixed-point FIR filters in an expanding sub expression space, *IEEE Trans. Circuits Syst. I Reg Papers*, Vol. 61, No.1, pp. 202–212, Jan. 2014.

[5] Pan . Yand Meher, P. K., Bit-level optimization of adder-trees for multiple constant multiplications for efficient FIR filter implementation, *IEEE Trans. Circuits Syst. I, Reg Papers*, Vol. 61, No. 2, pp. 455–462, Feb. 2014.

[6] Park,J, Jeong, W, Meimand,H. M., Wang,Y., Choo,H and Roy,K "Computation sharing programmable FIR filter for low-power and highperformance applications," *IEEE J. Solid-State Circuits*, Vol. 39, No. 2, pp. 348–357, Feb. 2004.

[7] Grayver, E andDaneshrad, B., Low power, area efficient programmable filter and variable rate decimator, *Proc. IEEE Int. Symp. Circuits Syst.*, Geneva, Switzerland, May 2000, pp. 341–344.

[8] Demirsoy, S. S., Beck, R., Dempster, A. G and Kale, I. Reconfigurable implementation of recursive DCT kernels for reduced quantization noise , *Proc. IEEE Int. Symp. Circuits Syst.*, Bangkok, Thailand, May 2003, pp. 289–292.

[9] Chen, J and. Chang, C. H., High-level synthesis algorithm for the design of reconfigurable constant multiplier, *IEEE Trans.Comput.-Aided Design Integr. Circuits Syst.*, Vol. 28, No. 12, pp.1844–1856, Dec. 2009.

[10] Mahesh, R and Vinod, A. P., Reconfigurable low area complexity filter bank architecture based on frequency response masking for no uniform channelization in software radio receivers, *IEEE Trans. Aerosp.Electron. Syst.*, Vol. 47, No. 2, pp. 1241–1255, Apr. 2011.



(An ISO 3297: 2007 Certified Organization)

Website: www.ijircce.com

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[11] Chen, T., Zakharow, Y. V and Liu, C., Low-complexity channel-estimate based adaptive linear equalizer, IEEE Signal Process. Lett., Vol.18, No. 7, pp. 427–430, Jul. 2011.

[12] Hautala, I, Boutellier, J and Hannuksela, J., Programmable lowpower implementation of the HEVC adaptive loop filter, *IEEE Int. Conf.Acous., Speech, Signal Process.*, Vancouver, BC, Canada, May 2013, pp. 2664–2668.

[13] Norberg, E. J., Guzzon, R. S., Parker, J. S., Johansson, L. A and Coldren, L. A., Programmable photonic microwave filtersmonolithically integerated in InP-InGaAsP, *J. Light wave Technol.*, Vol. 29, No. 11, pp.1611–1619, Jun. 2011.

[14] Zahabi, M. R., Meghdadi, V., Cances, J. P and Saemi, A, Mixed signal matched filter for high-rate communication systems, *IET Signal Process.*, Vol. 2, No. 4, pp. 354–360, Dec. 2008.

[15] Prakash, M. S and Shaik, R. A., Low-area and high-throughput architecture for an adaptive filter using distributed arithmetic, *IEEE Trans.Circuits Syst. II, Exp. Briefs*, Vol. 60, No. 11, pp. 781–785, Nov. 2013.

[16] Chilo, J and Lindblad, T., Hardware implementation of 1D wavelet transform on an FPGA for infrasound signal classification, *IEEETrans. Nuclear Sci.*, Vol. 55, No. 1, Feb. 2008.

[17] Kamp, W and Brainbridge-Smith, A, Multiply accumulate unit optimized for fast dot-product evaluation, *Proc. Int. Conf. Field-Programmable Tech.*, Kitakyushu, Japan, Dec. 2007, pp. 349–352.

[18] Cieplucha, M, High performance FPGA-based implementation of a parallel multiplier-accumulator, Proc. 20th Int. Conf. Mixed Design Integr. Circuits Syst., Gdynla, Poland, Jun. 2013, pp. 485–489.

[19] Tummeltshammer, P., Hoe, J. C.and Püschel, M., Time-multiplexed multiple-constant multiplication, *IEEE Trans. Comput., -Aided Design Integr. Circuits Syst.*, Vol. 26, No. 9, pp. 1551–1563, Sep. 2007.

[20] Hartley, R. I., Subexpression sharing in filters using canonic signed digit multipliers, *IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process.* Vol. 43, No. 10, pp. 677–688, Oct. 1996.

[21] Paško, R., Schaumont, P., Derudder, V., Vernalde, S and Ďuračková, D, A new algorithm for elimination of common sub expressions, *IEEE Trans. Comput.-Aided Design Integr. Circuits Syst.*, Vol. 18, No. 1, pp. 58–68, Jan. 1999.

[22] Chang, C. H., Chen, J. and Vinod, A. P., Information theoretic approach to complexity reduction of FIR filter design, *IEEE Trans. Circuits Syst. I, Reg. Papers*, Vol. 55, No. 8, pp. 2310–2321, Sept. 2008.

[23] Peiro, M. M., Boemo, E. I.and Wanhammar, L., Design of high-speed multiplierless filters using a non-recursive signed common sub expression algorithm, *IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process.* Vol. 49, No. 3, pp. 196–203, Mar. 2002.

[24] Xu, F., Chang, C.H and Jong, C. C., Design of low-complexity FIR filters based on signed-powers-of-two coefficients with reusable common sub expressions, *IEEE Trans. Comput.-Aided Design Integr. Circuits Syst.*, Vol. 26, No. 10, pp. 1898–1907, Oct. 2007.

[25] Mahesh, R and Vinod, A. P., New reconfigurable architectures for implementing FIR filters with low complexity, IEEE *Trans. Comput.- Aided Design Integr. Circuits Syst.*, Vol. 29, No. 2, pp. 275–288, Feb.2010.

[26] Khoo, K., Kwentus, A and Willson, A. N., Aprogrammable FIR digital filter using CSD coefficients, *IEEE J. Solid-State Circuits*, Vol. 31, No.6, pp. 869–874, June. 1996.

[27] El-Kheir, N. T. A., El-Kharashi, M. S and EL-Moursy, M. A., A low power programmable FIR filter using sharing multiplication technique, *Proc. IEEE Int. Conf. IC Design Tech.*, Austin, TX, USA, May 2012, pp. 1–4.

[28] Tang, Z., Zhang, J. and Min, H., A high-speed programmable, CSD Coefficient FIR filters, *IEEE Trans. Consum. Electron.*, Vol. 48, No. 4, pp. 834–837, Nov. 2002.

[29] Reitwiesner, R. W., Binary arithmetic, Advances in Computers. New York: Academic, 1960, Vol. 1, pp. 231-308.

[30]. De Micheli, G., Synthesisand Optimization of Digital Circuits. New York: McGraw-Hill, Inc., 1994.

[31] Dimitrov, V. S., Eskritt, J., Imbert, L., Jullien, G. Aand Miller, W. C., The use of the multi-dimensional logarithmic number system in DSP applications, Proc. 15th IEEE Symp. Comput. Arith. Vail, CO, USA, Jun. 2001, pp. 247–254.

[32] Dimitrov, V. S., Imbert, L and Zakaluzny, A., Multiplication by a constant is sub linear, *Proc. 18th IEEE Symp. Comput. Arith.*, Montpellier, France, Jun. 2007, pp. 261–268.

[33] Adikari, J., Dimitrov, V. S and Imbert, I., Hybrid binary-ternary number system for elliptic curve cryptosystems, *IEEE Trans. Comput.*, Vol. 60, No. 2, pp. 254–265, Feb. 2011.

[34] Chen, J and Chang, C. H., Design of programmable FIR filters using canonical double based number representation, *Proc. IEEE Int. Symp. Circuits Syst.*, Melbourne, Australia, Jun. 2014, pp. 1183–1186.

[35] Dimitrov, V. S., Jullien, G. A and Muscedere, R., *Multiple-Base Number System: Theory and Applications*. Boca Raton, FL, USA: CRC, 2012.
[36] Dimitrov, V. S., Jullien, G. A and Liller, W. C., Theory and applications of the double-base number system, *IEEE Trans. Comput.*, Vol. 48, No. 10, pp. 1098–1106, Oct. 1999.

[37] Dimitrov, V. S., Jullien, G. A and Liller, W. C., Theory and applications of the double-base number system, *IEEE Trans. Comput.*, Vol. 48, No. 10, pp. 1098–1106, Oct. 1999.

[38] Chen, X., Harris, F. J., Venosa, E and Rao, B.D., Non-maximally decimated analysis/synthesis filter banks: Applications in wideband digital filtering," *IEEE Trans. Signal Process.*, Vol. 62, No. 4, pp. 852–867.