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Quartic Trigonometric Bézier Curves and Surfaces with Shape Parameter

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ABSTRACT: In this paper a new kind of quartic trigonometric Bézier curve with a shape parameter λ is presented and the corresponding trigonometric Bézier surfaces are defined. These curves not only inherit most properties of the usual quartic Bézier curves with the Bernstein basis in the polynomial space, but also enjoy some other advantageous properties for shape modelling. The shape parameter provides freedom in terms of design and shape control of the curve. Thus we can construct smooth curves of almost any shape. The shape of the curve can be adjusted by altering the values of shape parameter while the control polygon is kept unchanged. These curves can be used as an efficient model for geometric design in the fields of CAGD.

KEYWORDS: Trigonometric Bézier basis function, Trigonometric Bézier curve, Trigonometric Bézier surface, Shape parameter.

I. INTRODUCTION

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Bézier curves and surfaces are the basic tools for modelling in Computer Aided Geometric Designing (CAGD) and Computer Graphics (CG). Bézier polynomial has several applications in the fields of engineering, science and technology such as highway or railway rout designing, networks, Computer aided design system, animation, environment design, robotics, communications and many other disciplines because it is easy to compute and is also very stable. The classical Bézier curves have some limitations that their shape and position are fixed relative to their control polygon. Thus people attempt to find a solution of the problem in the non-polynomial function space. During the last few years, a major research focus has been the use of trigonometric functions or the blending of polynomial and trigonometric functions.

Trigonometric B-splines were first presented in [1] and the recurrence relation for the trigonometric B-splines of arbitrary order was established in [2]. In recent years, several new trigonometric splines have been studied in the literature; see [3], [4] and [5]. In [6] cubic trigonometric Bézier curve with two shape parameters were presented. In [7], a novel generalization of Bézier curve and surface with n shape parameters are presented. In [8], the cubic trigonometric polynomial spline curve of G^3 continuity is constructed, which can be G^5 continuity under special condition. In [9], uniform T-B-spline basis function of $(n+1)^{th}$ order and its solution is presented. In [10], quartic splines with C^2 continuity are presented for a non-uniform knot vectors which are C^2 and G^3 continuous under special case. Algebraic-Trigonometric blended spline curves are presented in [11] which can represent some transcendental curves. Cubic trigonometric Bézier curve with two shape parameters is presented in [12]. Recently in [13], a quadratic trigonometric Bézier curve with shape parameter is constructed which is G^1 continuous. In [14], the generalized basis functions of degree n+1 with two shape parameters are presented. The cubic trigonometric polynomial spline curve of G^1 continuity is constructed in [15], which can be G^3 continuity under special condition. In [16], the cubic trigonometric polynomial curve similar to the cubic Bézier curves is constructed. In [17], the shape features of the cubic trigonometric polynomial curves with a shape parameter are analysed. An extension of the Bézier model is studied in [18]. In [19], [20], [21] and [22] quartic and cubic trigonometric Bézier curve respectively with shape parameter is presented and the effect of shape parameter is studied. A new rational cubic trigonometric Bézier curve with four shape parameters is defined in [23]. In this paper quartic trigonometric Bézier Curve with a shape parameter is presented

In this paper a new kind of quartic trigonometric Bézier Curve with a shape parameter is presented. The paper is organized as follows. In section II, quartic trigonometric Bézier basis functions with a shape parameter are established

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and the properties of the basis functions are shown. In section III, quartic trigonometric Bézier curves are given and some properties are discussed. By using shape parameter, shape control of the quartic trigonometric Bézier curves is studied. In section IV, the representation of quartic trigonometric Bézier surface has been shown. In section V, the approximability of the quartic trigonometric Bézier curves and the quartic Bézier curves with Bernstein basis corresponding to their control polygon are studied. Conclusion is presented in Section VI.

II. QUARTIC TRIGONOMETRIC BÉZIER BASIS FUNCTIONS

Firstly, the definition of quartic trigonometric Bézier basis functions is given as follows.

The construction of the basis functions

Definition 1. For an arbitrarily selected real values of λ where $\lambda \in$ [-20, 0], the following five functions of t ($t \in \left[0, \frac{\pi}{2}\right]$) are defined as Quartic Trigonometric Bézier basis functions with a shape parameter λ:

$$\begin{cases} b_0(t) = (1-\sin t)^2\cos^2 t - \lambda(1-\sin t)^2(1-\cos t)^2 \\ b_1(t) = \sin t(1-\sin t) + \lambda(1-\sin t)^2(1-\cos t)^2 \\ b_2(t) = 1 - 2\sin^2 t\cos^2 t + (\sin t + \cos t)(2\sin t\cos t - 1) \\ b_3(t) = \cos t(1-\cos t) + \lambda(1-\sin t)^2(1-\cos t)^2 \\ b_4(t) = (1-\cos t)^2\sin^2 t - \lambda(1-\sin t)^2(1-\cos t)^2 \end{cases} \quad (1)$$

The properties of the basis functions

Theorem 1 The basis functions (1) have the following properties:

(a) ***Non-negativity***: $b_i(t) \geq 0$ for $i = 0, 1, 2, 3, 4$.

(b) ***Partition of unity***: $\sum_{i=0}^{4} b_i(t) = 1$

(c) ***Symmetry***: $b_i(t; \lambda) = b_{4-i}(\frac{\pi}{2} - t; \lambda)$, for $i = 0, 1, 2, 3, 4$.

(d) ***Monotonicity***: For a given parameter t, as the shape parameter λ increases (or decreases),$b_0(t)$, $b_4(t)$ decreases (or increases) and $b_1(t), b_3(t)$ increases (or decreases). $b_2(t)$ remains unchanged for any increase or decrease in the shape parameter λ.

Proof (a) For $t \in [0, \frac{\pi}{2}]$ and $\lambda \in [-20, 0]$, then
$0 \leq (1-\sin t) \leq 1, 0 \leq (1-\cos t) \leq 1, 0 \leq \sin t \leq 1, 0 \leq \cos t \leq 1$
It is obvious that $b_i(t) \geq 0$ for $i = 0, 1, 2, 3, 4$.
(b) $\sum_{i=0}^{4} b_i(t) = (1-\sin t)^2\cos^2 t - \lambda(1-\sin t)^2(1-\cos t)^2 + \sin t(1-\sin t) + \lambda(1-\sin t)^2(1-\cos t)^2 + 1 - 2\sin^2 t\cos^2 t + (\sin t + \cos t)(2\sin t\cos t - 1) + \cos t(1-\cos t) + \lambda(1-\sin t)^2(1-\cos t)^2 + (1-\cos t)^2\sin^2 t - \lambda(1-\sin t)^2(1-\cos t)^2 = 1$

The remaining cases follow obviously. The curves of the Quartic Trigonometric Bézier basis functions are shown in Figure 1 for $\lambda = -20$ (red lines), $\lambda = -10$ (blue lines) and $\lambda = 0$ (green lines). Note that $b_2(t)$ (black line) remains unchanged for any increase or decrease in the shape parameter λ.

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III. QUARTIC TRIGONOMETRIC BÉZIER CURVE

The construction of the Quartic Trigonometric Bézier curve

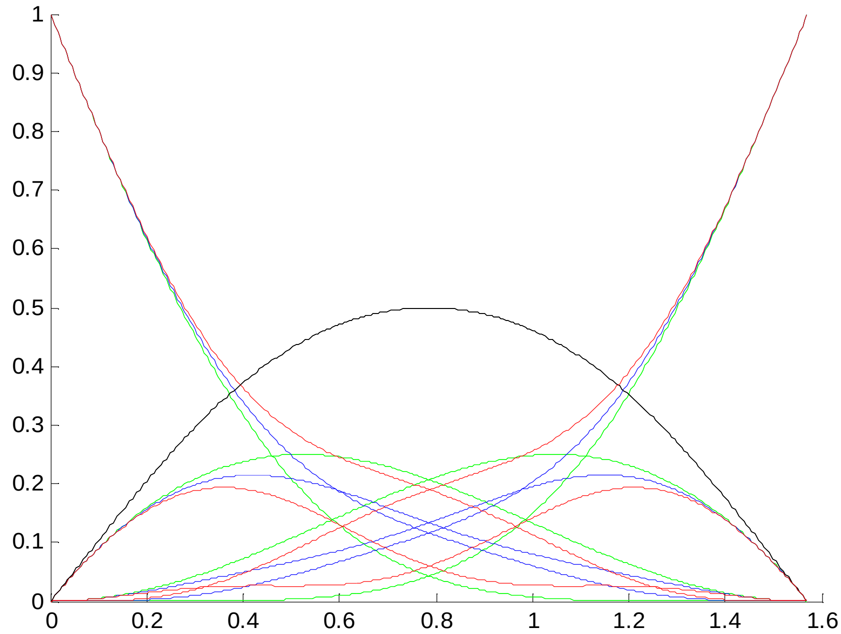

Fig. 1. Quartic Trigonometric Bézier basis functions for different values of λ

Definition 2. Given control points $P_i = 0.1, 2, 3, 4$ in R^2 or R^3, then

$$r(t) = \sum_{i=0}^{4} b_i(t) P_i, \qquad t \in [0,1], \ \lambda \in [-20, 0] \tag{2}$$

is called a Quartic Trigonometric Bézier curve with a shape parameter λ.

From the definition of the basis function, some properties of the Quartic Trigonometric Bézier curve can be obtained as follows:

Theorem 2 The Quartic Trigonometric Bézier curves (2) have the following properties:

a) *Terminal Properties*: $r(0) = P_0, \quad r(\frac{\pi}{2}) = P_4,$ (3)

$r'(0) = -2P_0 + P_1 + P_2, \quad r'(\frac{\pi}{2}) = 2P_4 - P_2 - P_3$ (4)

b) *Symmetry*: P_0, P_1, P_2, P_3, P_4 and P_4, P_3, P_2, P_1, P_0 define the same Quartic Trigonometric Bézier curve in different parametrizations, i.e.,

$$r(t; \lambda; P_0, P_1, P_2, P_3, P_4) = r(\tfrac{\pi}{2} - t; \lambda; P_4, P_3, P_2, P_1, P_0); \quad t \in [0, \tfrac{\pi}{2}], \ \lambda \in [0,1] \tag{5}$$

c) *Geometric invariance*: The shape of a Quartic Trigonometric Bézier curve is independent of the choice of coordinates, i.e. (2) satisfies the following two equations:

$$r(t; \lambda; P_0 + q, P_1 + q, P_2 + q, P_3 + q, P_4 + q) = r(t; \lambda; P_0, P_1, P_2, P_3, P_4) + q$$
$$r(t; \lambda; P_0 * T, P_1 * T, P_2 * T, P_3 * T, P_4 * T) = r(t; \lambda; P_0, P_1, P_2, P_3, P_4) * T \tag{6}$$

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$t \in \left[0, \frac{\pi}{2}\right]$, $\lambda \in [0,1]$

where q is arbitrary vector in R^2 or R^3, and T is an arbitrary $d \times d$ matrix, d=2 or 3.

d) *Convex hull property*: The entire Quartic Trigonometric Bézier curve segment lies inside its control polygon spanned by P_0, P_1, P_2, P_3, P_4.

Shape Control of the Quartic Trigonometric Bézier Curve

For $t \in [0, \frac{\pi}{2}]$, we rewrite (2) as follows:

$$r(t) = \sum_{i=0}^{4} P_i c_i(t) + \lambda(1-\sin t)^2(1-\cos t)^2[(P_1 - P_0) + (P_3 - P_4)] \qquad (7)$$

where $c_0(t) = (1-\sin t)^2 \cos^2 t$,

$c_1(t) = \sin t(1-\sin t)$,

$c_2(t) = 1 - 2\sin^2 t \cos^2 t + (\sin t + \cos t)(2 \sin t \cos t - 1)$,

$c_3(t) = \cos t(1-\cos t)$,

$c_4(t) = (1-\cos t)^2 \sin^2 t$.

Obviously, shape parameter λ affects the curve on the control edges $(P_1 - P_0)$ and $(P_3 - P_4)$. The shape parameter λ serves to effect local control in the curve: as λ increases, the curve moves in the direction of edges $(P_1 - P_0)$ and $(P_3 - P_4)$ and as λ decreases, the curve moves in the opposite direction to the edges $(P_1 - P_0)$ and $(P_3 - P_4)$.

Figure 3 shows a computed example of quartic trigonometric Bézier Curves with different values of shape parameter λ. These curves are generated by setting $\lambda = -20$ (green lines), $\lambda = -10$ (red lines) and $\lambda = 0$ (blue lines). In Figure 4, another example of shape modelling of quartic trigonometric Bézier Curves is presented for $\lambda = 0$.

IV. THE QUARTIC TRIGONOMETRIC BÉZIER SURFACES

Given control points P_{rs} $(r = i, \ldots, i+3;\ s = j, \ldots, j+3)$, $(i = 0,1, \ldots, n-1;\ j = 0,1, \ldots, m-1)$ using the tensor product method, we can construct the Quartic Trigonometric Bézier surface

$$T(u,v) = \sum_{r=i}^{i+3} \sum_{s=j}^{j+3} b_{r,4}(\lambda_1, u) b_{s,4}(\lambda_2, v) P_{rs}; \qquad (u,v) \in [0, \frac{\pi}{2}] \times [0, \frac{\pi}{2}]$$

where $b_{r,4}(\lambda_1, u)$ and $b_{s,4}(\lambda_2, v)$ are quartic trigonometric polynomial base functions. Obviously these surfaces have properties similar to the corresponding quartic trigonometric Bézier curves. An example of quartic trigonometric Bézier surface is presented in Figure 5.

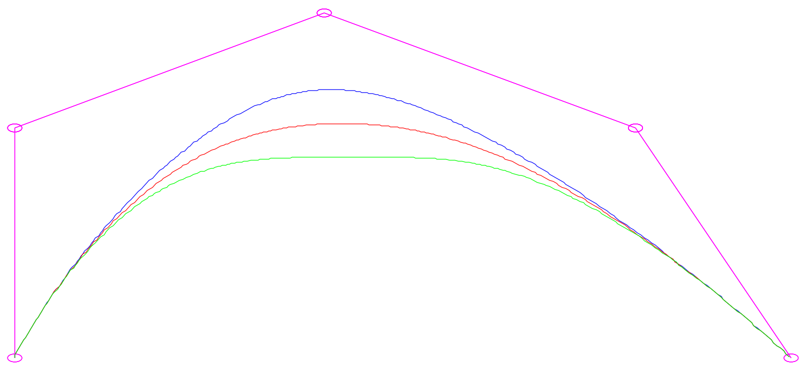

Fig. 3. Quartic Trigonometric Bézier Curves with different values of shape parameter λ

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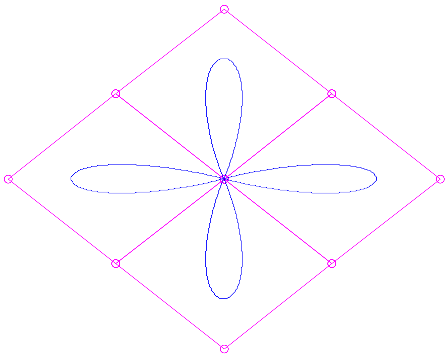

Fig. 4. Quartic Trigonometric Bézier Curves

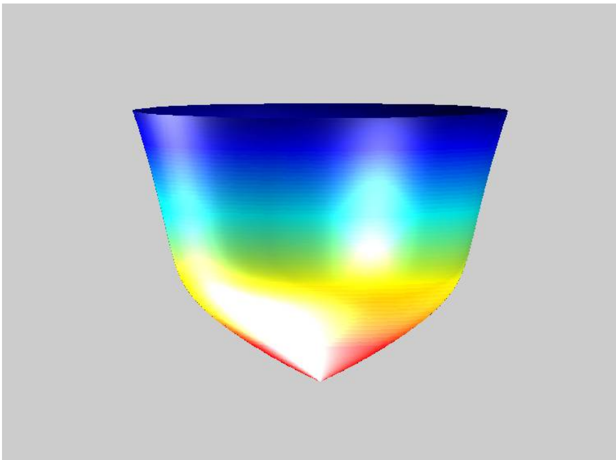

Fig. 5. Quartic Trigonometric Bézier Surface

V. APPROXIMABILITY

Suppose P_0, P_1, P_2, P_3 and P_4 are not collinear; the relationship between Quartic Trigonometric Bézier curve $r(t)$ and the quartic Bézier curve $B(t) = \sum_{i=0}^{4} P_i \binom{4}{i}(1-t)^{4-i}t^i$ with the same control points P_i is given by

$$r\left(\frac{\pi}{4}\right) - P_2 = \frac{(\sqrt{2}-1)^2}{4}\{P_0 - 2P_2 + P_4\} + \frac{(\sqrt{2}-1)}{2}\{P_1 - 2P_2 + P_3\} + \lambda\frac{(\sqrt{2}-1)^4}{4}\{-P_0 + P_1 + P_3 - P_4\}$$

and

$$B\left(\tfrac{1}{2}\right) - P_2 = \tfrac{1}{16}[P_0 + 4P_1 - 10P_2 + 4P_3 + P4] \qquad (8)$$

These equations shows that quartic trigonometric Bézier curve can be made closer to the control polygon by altering the values of shape parameter λ.

VI. CONCLUSION

As mentioned above Quartic Trigonometric Bézier curve have all the geometric properties that classical quartic Bézier curves have. The shape of the curve can be flexibly controlled by the shape parameter without altering the control points. Since there is nearly no difference in structure between a Quartic Trigonometric Bézier curve and a classical

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quartic Bézier curve, it is not difficult to adapt a Quartic Trigonometric Bézier curve to a CAD/CAM system that already uses the classical quartic Bézier curves.

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BIOGRAPHY

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