



Performance Analysis of Digital Finite Impulse Response Filter Design using Windowing Techniques

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ABSTRACT: A filter is a frequency selective circuit that allows a certain frequency to pass while attenuating the others. So by using it one can improve quality of the inputs and other is to process or extract information from the inputs. Filters could be analog or digital. Analog filters use electronic components such as resistor, capacitor, transistor etc. to perform the filtering operations. These are mostly used in communication for noise reduction, video/audio signal enhancement etc. Filters can be used to shape the signal spectrum in a desired way or to perform mathematical operations such as differentiation and integration. Digital filters are a very important part of DSP. In fact, their extraordinary performance is one of the key reasons that DSP has become so popular. Digital filters are categorized as per their frequency response in two categories: finite impulse response (FIR) and infinite impulse response (IIR) filters. This paper is on design of FIR filters.

KEYWORDS: DSP, Filter FIR, IIR.

I. INTRODUCTION

A filter is a frequency selective circuit that allows a certain frequency to pass while attenuating the others. So by using it one can improve quality of the inputs and other is to process or extract information from the inputs. Filters could be analog or digital. Analog filters use electronic components such as resistor, capacitor, transistor etc. to perform the filtering operations. These are mostly used in communication for noise reduction, video/audio signal enhancement etc. Filters can be used to shape the signal spectrum in a desired way or to perform mathematical operations such as differentiation and integration. Digital filters are a very important part of DSP. In fact, their extraordinary performance is one of the key reasons that DSP has become so popular. Digital filters are used for two general purposes: (1) separation of signals that have been combined, and (2) restoration of signals that have been distorted in some way. Signal separation is needed when a signal has been contaminated with interference, noise, or other signals. For example, imagine a device for measuring the electrical activity of a baby's heart (EKG) while still in the womb. The raw signal will likely be corrupted by the breathing and heartbeat of the mother. A filter might be used to separate these signals so that they can be individually analyzed. Signal restoration is used when a signal has been distorted in some way. For example, an audio recording made with poor equipment may be filtered to better represent the sound as it



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actually occurred. Another example is the deblurring of an image acquired with an improperly focused lens, or a shaky camera

II. ADVANTAGE OF DIGITAL FILTER

Digital filters are becoming increasingly popular because:

- They can be implemented with software running on a general purpose computer. Therefore, they are relatively easy to build and test.
- They are based on solely on the arithmetic operations of addition and multiplication. Therefore, they are extremely stable – they do not change with time or temperature.
- They are easier to modify than their analog counterparts.
- They are easier to understand.

All digital filters can be either of two forms: non recursive or recursive. A non recursive filter generates its output by simply weighting the inputs by constants and then summing the weighted inputs. The constant are called coefficient and these constant determine the filter.

Digital filters are categorized as per their frequency response in two categories: finite impulse response (FIR) and infinite impulse response (IIR) filters. FIR filter is non recursive type and output depends on past and present samples of the input. The FIR digital filter is presented as:

$$y(n)=\sum_{i=0}^N a_i x(n - i) \dots\dots\dots(i)$$

IIR filter is recursive type and output depends on past and present samples of the input and also on past output samples. The IIR digital filter is presented as:

$$y(n) = \sum_{i=0}^N a_i x(n - i) - \sum_{k=1}^M b_k y(nk) \dots\dots\dots(ii)$$

In FIR there is a lot of uncertainty. And at every step you are not sure whether you are proceeding in the correct manner and whether the specification are satisfied or not; so it has a blind start. In FIR, you only have empirical formulas which may or may not work. If empirical formulas give the order of 12 you might have to use an order of 16 or 17. Empirical formulas always have this kind of tolerance and uncertainty. But despite these drawbacks, what is the real need to use FIR? It is because FIR is unconditionally stable and has a linear phase. Linear phase is a strict requirement in many situations. So Although FIR filters are more complex, they have certain advantages over IIR filters due to which they are more widely used in filtering applications. IIR filters do not provide stability at higher orders whereas the FIR counterparts are always stable and are particularly useful for applications where exact linear phase response is required. For example, in data processing, if a rectangular pulse becomes smeared because of delay distortion, then it does not convey what you wish to convey. In speech processing, linear phase is a strict requirement so you have to use it. There are two advantages: one is, it is linear phase and the other is that it is unconditionally stable. There is a third advantage. If you have a non-causal FIR then you can make it causal by simply shifting the impulse response to the right by multiplying the transfer function by the required number of delays. So reliability of FIR is not a problem.

III.METHODS TO DESIGN FIR FILTER

There are three methods for designing linear phase FIR filter:

- Window method
- Frequency sampling
- Optimum equiripple Method



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A. Window design method

The window design method is first designs an ideal IIR filter and then truncates the infinite impulse response by multiplying it with a finite length window function. The result is a finite impulse response filter whose frequency response is modified from that of the IIR filter. Multiplying the infinite impulse by the window function in the time domain results in the frequency response of the IIR being convolved with the frequency response of the window function.

$$h(n)=w(n) h_d(n) \dots\dots\dots(iii)$$

According to the complex convolution formula, in the time domain can be expressed as the product of relationships in the frequency domain of the periodic convolution relations, then the design of FIR digital filter frequency response.

$$H(e^{jw}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\theta)W(w-\theta)d\theta \dots\dots\dots(iv)$$

Wherein, W(w) as a truncated window functions of frequency characteristics. Thus, the actual FIR digital filter frequency response H(e^{jw}) is approximation to the desired filter frequency response H_d(e^{jw}) which depends entirely on the window function of frequency characteristics.

Design Steps of Window Function Method

The design steps Window function method includes H_d(e^{jw})

1. Give the frequency response function of the FIR digital filter
2. Compute the unit impulse response h_d(n) = IDTFT [H_d(e^{jw})] of the FIR digital filter (i.e., inverse Fourier transform).
3. According to the requirements of the transition bandwidth and minimum stopband attenuation, selected window function shapes w(n) by checking the table and calculate the size of filter order N.
4. Computer the unit impulse response h(n)=w(n) h_d(n) of the designed FIR digital filter.
5. Computer the frequency response H(e^{jw}) = DTFT[h(n)] of the designed FIR digital filter (i.e., Fourier transform). Test whether to meet the design requirements, if not, you will need to design.

B. Design Processing of Window Function Method:

Assume the group delay of the linear phase filter is τ, and then the ideal frequency response of the filter is

$$H_d(e^{jw})=e^{-jw\tau}$$

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{jw})e^{jwn} dw$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{jw(n-\tau)} dw$$

It is the symmetric infinite non causal sequence at the central point. Now using the rectangular window R_N(n) to intercept the infinite length sequence and get the finite length sequence, the window function is

$$w(n)=R_N(n) \dots\dots(viii)$$

And the frequency characteristic of window function is

$$W_R(e^{jw})=\sum_{n=0}^{N-1} w(n)e^{-jwn} =\sum_{n=0}^{N-1} e^{-jwn}$$

$$= \frac{\sin(\frac{wN}{2})}{\sin(\frac{w}{2})} e^{-jw\tau} \approx W_R(w) e^{-jw\tau}$$

In which

$$W_R(w) = \frac{\sin(\frac{wN}{2})}{\sin(\frac{w}{2})}$$

$$\tau = \frac{N-1}{2}$$

Finally we can get the frequency response of the designed FIR digital filter by complex convolution formula



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$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\theta) W_R(\omega - \theta) d\theta$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\theta}) e^{j(\omega - \theta)} d\theta.$$

The several effects of windowing the Fourier coefficients of the filter on the result of the Frequency response of the filter is as follows:

- (i) A major effect is that discontinuities in $H(\omega)$ become transition bands between values on either side of the discontinuity.
- (ii) The width of the transition bands depends on the width of the main lobe of the frequency response of the window function, $w(n)$ i.e. $W(\omega)$.
- (iii) Since the filter frequency response is obtained via a convolution relation, it is clear that the resulting filters are never optimal in any sense.
- (iv) As M (the length of the window function) increases, the mainlobe width of $W(\omega)$ is reduced which reduces the width of the transition band, but this also introduces more ripple in the frequency response.
- (v) The window function eliminates the ringing effects at the band edge and does result in lower sidelobes at the expense of an increase in the width of the transition band of the filter

Some of the windows commonly used are as follows:

1. Rectangular window, that is $w(n) = 1$ for $0 \leq n \leq N - 1$ and 0 otherwise. The spectrum is

$$W_R(\omega) = \frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)} e^{-j\omega\tau}$$

The first 0 shall occur at $\omega = 2\pi/N$ and not at $\omega = 0$ because at $\omega = 0$, the value is N . The next 0 shall occur at $4\pi/N$ and so on. So in terms of this spectrum what you want is that the Main Lobe Width (MLW) which is $4\pi/N$ should be as small as possible. Since we want an approximation to the impulse function, the Side Lobe Height (SLH) should be as small as possible.

Unfortunately, these two requirements are contradictory. That is, if you want to decrease the main lobe width, then N should increase; as N increases it shrinks, but at the same time the side lobe height increases. The ratio MLW/SLH is approximately a constant. This is the problem in FIR filter design. Whatever window you choose, it would be a compromise between main lobe width and the side lobe height and there is hardly much of a choice except two windows which we shall not discuss in detail in the class; one is the Kaiser window and the other is Dolph Chebyshev Window, the idea of the latter being taken from Antenna Array Design.

The truncation of the infinite length will introduce ripples in frequency response. The oscillatory behavior near the band edge of the filter is called the Gibbs phenomenon. When the N is increased The transition band of the filter will decrease But the relative amplitude of the peaky values will remain constant.

2. Bartlett window: Since the Gibbs phenomenon results from the fact that the rectangular window has a sudden transition from 0 to 1 (or 1 to 0), Bartlett suggested a more gradual transition in the form of a triangular window. The width of main lobe is $8\pi / N$.

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$$w(n) = \begin{cases} \frac{2n}{N-1}, & 0 \leq n \leq \frac{N-1}{2} \\ 2 - \frac{2n}{N-1}, & \frac{N-1}{2} < n \leq N-1 \end{cases}$$

$$W(e^{j\omega}) \approx \frac{2}{N} \left[\frac{\sin\left(\frac{N\omega}{4}\right)}{\sin\left(\frac{\omega}{2}\right)} \right]^2 e^{-j\left(\frac{N-1}{2}\right)\omega}, \quad (N \gg 1, N-1 \approx N)$$

3. Hann Window, which is a smooth window; there are no abrupt discontinuities. It is $\frac{1}{2} [1 - \cos 2\pi n/(N-1)]$ for $0 \leq n \leq N-1$ and 0 otherwise.

$$w(n) = \frac{1}{2} \left[1 - \cos\left(\frac{2\pi n}{N-1}\right) \right]$$

If you plot it, it looks like a cosine wave and is shown in the slide. For $n = 0$, the value is 0, and for $n = N-1$, again the value is 0. The maximum occurs when the angle $2\pi n/(N-1)$ is π , so that the maximum value is 1. It occurs at $n = (N-1)/2$. Obviously an odd N is to be preferred. There is also another reason as to why N odd should be preferred. It is because the delay is an integer, and a half delay is not very easy to accommodate in a DSP. A point to notice about Hann window is that effectively the length is $N-2$ because two of the samples are 0. You have not been able to utilize the efforts you have put in aiming for the length N ; the effective length becomes $N-2$.

4. Hamming window, Hann window is $(1/2) - (1/2) \cos 2\pi n/(N-1)$. Instead of the first $1/2$, Hamming window uses 0.54 for the first term. Naturally for the second $1/2$, you have to use 0.46; then only the maximum value becomes 1.

$$w(n) = \left[0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) \right]$$

The advantage of this is that you are utilizing the full length window. At $n = 0$, $w(0) = 0.08$ and this is also same as the $w(N-1)$. So instead of rising from the base of zero it is a cosine shaped wave form, but it has been raised by the amount point 0.08. So, Hamming window is also called Raised Cosine Window. The effect of increasing the length of the window is very similar to Hann window except that Hamming allows for a little more reduction in side lobe height. It is always a compromise between main lobe width and side lobe height.

5. Blackman window : This is a 2-order raised cosine window

$$w(n) = \left[0.42 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right) + 0.08 \cos\left(\frac{4\pi n}{N-1}\right) \right]$$

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6. Kaiser window: This is one of the most useful and optimum windows

$$w(n) = \frac{I_0\left(\beta \sqrt{1 - \left(1 - \frac{2n}{N-1}\right)^2}\right)}{I_0(\beta)}$$

Where $I_0(\cdot)$ is the modified zero-order Bessel functions, and β is a parameter that can be chosen to yield various transition widths and stopband attenuation. This window can provide different transition widths for the same N.

$\beta = 0 \rightarrow$ rectangular window

$\beta = 5.44 \rightarrow$ Hamming window

$\beta = 8.5 \rightarrow$ Blackman window

Table 1.1
Summary of window function characteristics

Window name	Window function		Filter	
	Peak value of side lobe	The width of main lobe	Transition width	Min. stopband attenuation
Rectangular	-13 dB	$4\pi/N$	$1.8\pi/N$	-21 dB
Bartlett	-25 dB	$8\pi/N$	$4.2\pi/N$	-25 dB
Hanning	-31 dB	$8\pi/N$	$6.2\pi/N$	-44 dB
Hamming	-41 dB	$8\pi/N$	$6.8\pi/N$	-53 dB
Blackman	-57 dB	$12\pi/N$	$11\pi/N$	-74 dB

Conclusion: The Bartlett window reduces the overshoot in the designed filter but spreads the transition region considerably. The Hanning, Hamming and Blackman windows use progressively more complicated cosine functions to provide a smooth truncation of the ideal impulse response and a frequency response that looks better. The best window results probably come from using the Kaiser window, which has parameter β that allows adjustment of the compromise between the overshoot reduction and transition region width spreading. The major advantages of using window method are their relative simplicity as compared to other methods and ease of use. The fact that well defined equations are often available for calculating the window coefficients has made this method successful.

REFERENCES

- [1] T.W.Parks and C.S.Burrus, Digital Filter Design. New York:Wiley,1987.
- [2] L.R. Rabiner and B.Gold, Theory and Applications of Digital Signal Processing.New Jersey: Prentice-Hall, 1975
- [3] J.G.Proakis and D.G.Manolakis, Digital Signal Processing-Principles,Algorithms and Applications New Delhi: Prentice-Hall, 2000
- [4] Tao Zhang, research on design FIR digital filter using MATLAB and window function method, Journal of Theoretical and Applied Information Technology 10th February 2013.



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