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A Novel Scheme for Estimation of Probability of Inference

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ABSTRACT*:* In the theory of probability one has to idealize a random experiment by making assuming hypothesis, so that relative frequency of an event happening can be directly mapped to probability of occurrence. In many real world scenarios the desired outcome of a random experiment is either known or certain measures have to be taken to meet the desired performances. For example in detection of radar signal the probability of false alarm and probability of detection is specified by the user and one implements a system to realize the specifications. However there are situations where it is not possible to draw inference as to possible outcomes of observations.

In this paper we consider atypical problem of making inference when there is no definite answers nevertheless the experiments lend themselves to logical analysis. A large body of such material in the theory of probability used for analyzing random experiments can be applied to derive an expression for the Probability of Inference.

KEYWORDS: Binary Symmetric Channel, BSC; Radar Signals, Probability False Alarm; Probability of Detection, Channel Capacity &Inference

I. INTRODUCTION

The fundamental idea in any probabilistic model is the concept of making observations, which have certain measures of randomness and can't be predicted with certainty. Flipping a coin s, drawing a card, observing a signal in the presence of noise etc cannot be analyzed without making certain assumptions and assigning probability measures for the event that could occur. The term random experiment or random phenomenon does not lend themselves to be defined by a precise mathematical expression but can be used to predict what could be the result from what appears to be essentially identical process. For example repeated measurements on certain physical objects such as communication, radar, sonar signals generally leads to a set of readings not all of which are exactly alike , regardless of the care we exercise in making the measurements correctly, random variations which sometimes called random errorscreep in wide beyond our precautions and make the observer decide on erroneous outcomes. This is applicable in wide variety of problems which describe experiments involving random variables in areas such as physical, biological and social sciences and other areas of human endeavors.[1][2][3] Sample space for making inference, an expert which evolve if measurements one or more random variables has certain outcomes that other elementary results of the experiment, for example in the flipping of the pin head and tail are the possible outcomes , similarly in the throwing of a dice 1,2,3,4,5,6 are the possible outcomes.

II. MATHEMATICAL MODEL

For discrete sample spaces, any subset of the sample space is an event, that is, a probability can be defined for it. For instance, in throwing a die various events such as “the outcome is even,” “the outcome is greater than 3,” and “the outcome divides 3” can be considered. For a non discrete sample space, not every subset of Ω can be assigned a probability without sacrificing basic intuitive properties of probability. To overcome this difficulty, we define aσ-field βon the sample space Ω as a collection of subsets of Ω such that the following conditions are satisfied:

1. $\Omega \in \beta$.

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2. If the subset (event) $E \in \beta$ then $E^c \in \beta$ where E^cdenotes the complement of E.
3. If $E_i \in \beta$ for all *i*, then$\bigcup_{i=1}^{\infty} E_i \in \beta$.

We define a *probability measure P* on β as a set function assigning nonnegative values to all events E in β such that the following conditions are satisfied:
1. $0 \leq P(E) \leq 1$ for all $E \in \beta$.
2. $P(\Omega) = 1$.
3. For disjoint events, $E_1, E_2, E_3, ..$ (i.e., events for which $E_i \cap E_j = \phi$ for all $i \neq j$, where ϕ is the null set), we have $P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} p(E_i)$.The triple (Ω, β, P) is called a *probability space*.

Some basic properties of the probability measure follow easily from the set theoretical properties of events together with the basic properties of probability measure.
Some of the most important properties are.
1. $P(E^c) = 1 - P(E)$.
2. $P(\phi) = 0$.
3. $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.
4. If $E_1 \subset E_2$ then $P(E_1) \leq P(E_2)$.

2.1. Conditional Probability

Let us assume that the two events E_1 and E_2 are defined on the same probability space with corresponding probabilities $P(E_1)$ and $P(E_2)$.[4][5] Then, if an observer receives the information that the event E_2 has in fact occurred, the observer's probability about event E_1 will not be $P(E_1)$ any more.[6] In fact, the information that the observer received changes the probabilities of various events, and new probabilities, called *conditional probabilities*, is defined. The conditional probability of the event E_1 given the event E_2 is defined by

$$P(E_1|E_2) = \begin{cases} \frac{p(E_1 \cap E_2)}{p(E_2)} & , P(E_2) \neq 0 \\ 0, & otherwise \end{cases} (1)$$

If it happens that $P(E_1|E_2) = P(E_1)$ then knowledge of E_2 does not change the probability of occurrence ofE_1. In this case, the events E_1 and E_2 are said to be *statistically independent*. For statistically independent events,[7]$P(E_1 \cap E_2) = P(E_1)P(E_2)$

2.2. Random Variables

A (real) *random variable* is a mapping from the sample space Ω of the set of real numbers. A schematic diagram representing a random variable is given in Figure 1.
Random variables are denoted by capital letters *X*, *Y*, etc.; individual values of the random variable *X* are $X(\omega)$. A random variable is *discrete* if the range of its values is either finite or countable infinite. This range is usually denoted by $\{x_i\}$.
The *cumulative distribution function (CDF)* of a random variable *X* is defined as

$$F_X(x) = P(\omega \in \Omega: X(\omega) \leq x) \quad (2)$$

This can be simply written as.

$$F_X(x) = P(X \leq x) \quad (3)$$

And has the following properties:
1. $0 \leq F_X(x) \leq 1$.
2. $F_X(x)$is nondecreasing.
3. $\lim_{x \to -\infty} F_X(x) = 0$ *and*
$\lim_{x \to +\infty} F_X(x) = 1$.
4. $F_X(x)$is continuous from the right; i.e.$lim_{\varepsilon \downarrow 0} F(X+\varepsilon) = F(X)$
5. $P(a < X \leq b) = F_X(a) - F_X(b)$
6. $P(X = a) = F_X(a) - F_X((a-)$.

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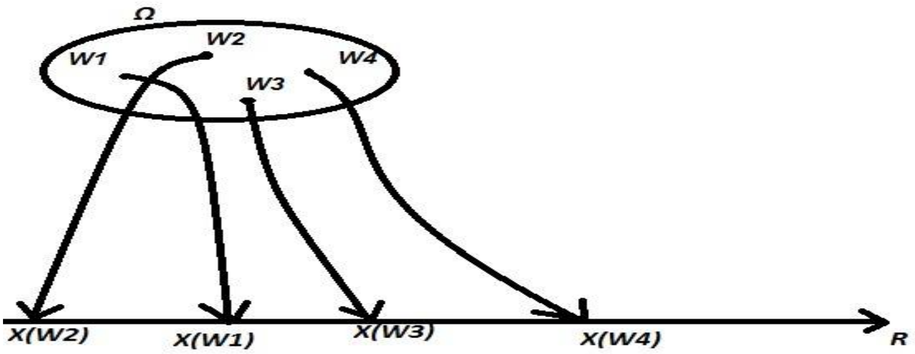

Figure1.Random variable as a mapping from Ω to R.

For discrete random variables$F_X(x)$ is a stair-case function. A random variable is called *continuous* if$F_X(x)$) is a continuous function. A random variable is called *mixed* if it is neither discrete nor continuous. Examples of CDFs for discrete, continuous, and mixed random variables are shown in Figures 2, 3, and 4, respectively.
The *probability density function (PDF)* of a random variable *X* is defined as the derivative of $F_X(x)$);

i.e.,$f_X(x) = \frac{d}{dx} F_X(x)$ (4)

In case of discrete or mixed random variables, the PDF involves impulses. The basic properties of PDF are :

1. $f_X(x) \geq 0$.

2.$\int_{-\infty}^{\infty} f_X(x)dx = 1$

3. $\int_{a^+}^{b^+} f_X(x)dx = P(a < X \leq b)$.

4. in general, $P(X \in A) = \int f_x(x)dx$

5. $F_X(x) = \int_{-\infty}^{x^+} f_X(u)\, du$.

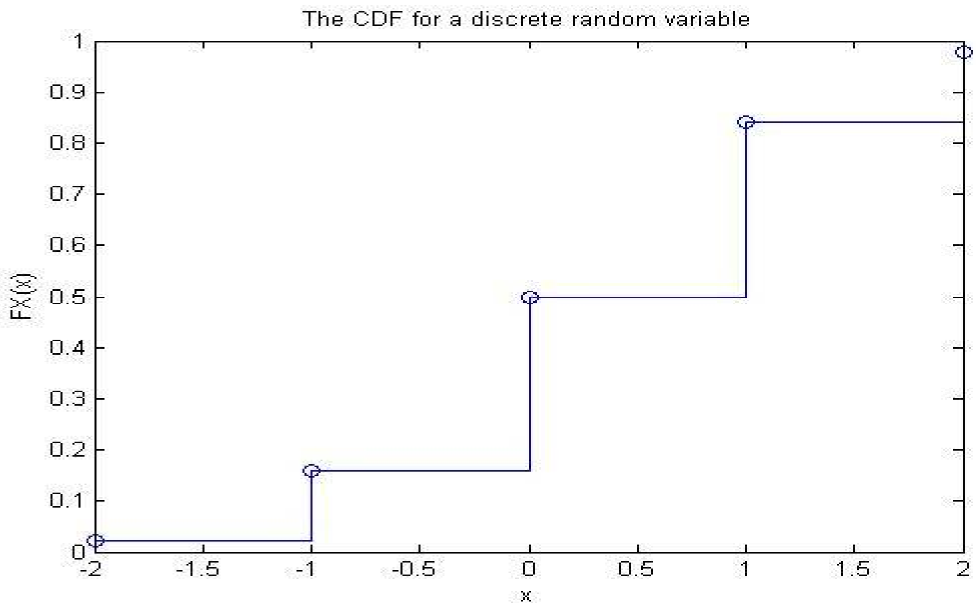

Figure 2 CDF for discrete random variables

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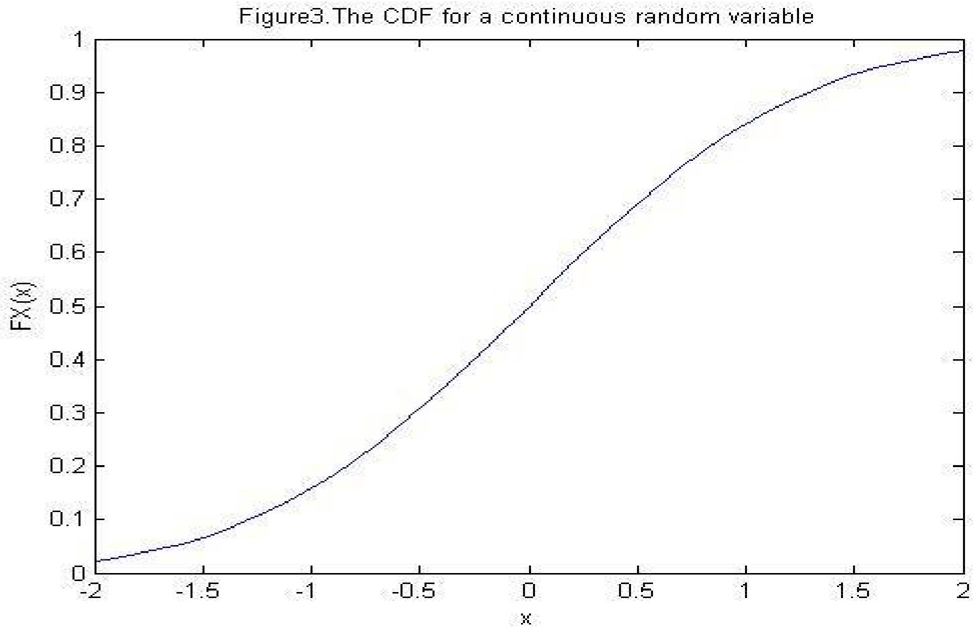

Figure 3 CDF for continuous random variables

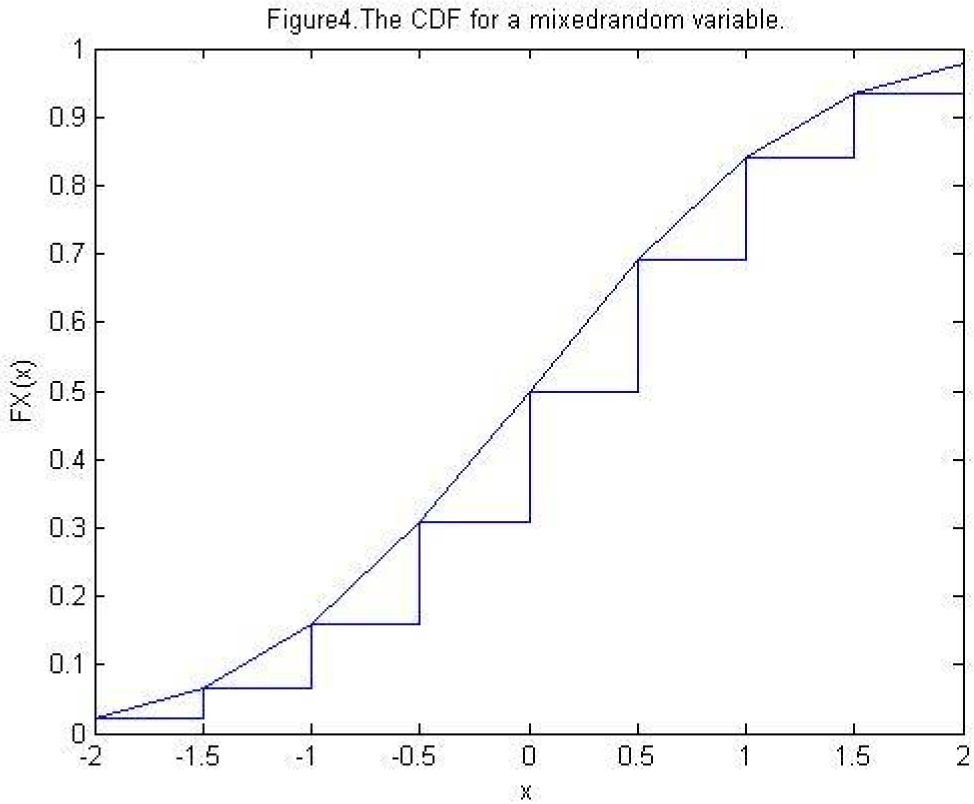

Figure 4 CDF for mixed random variables

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For discrete random variables, it is more common to define the *probability mass function (p.m.f),* which is defined as $\{p_i\}$ where $p_i = P(X = x_i)$. Obviously for all i we have $p_i \geq 0$ .and $\sum_i p_i = 1$.

III. APPLICATIONS

In this paper we choose two applications to apply our inference scheme which are:

3-1 Gaussian Channel Capacity

A discrete-time Gaussian channel with input power constraint is characterized by the input–output relation

$$Y = X + Z \qquad (5)$$

Where Z is a zero-mean Gaussian random variable with variance P_N, and for n large enough, an input power constraint of the form:

$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 \leq P \qquad (6)$$

Applies to any input sequence of length n. For blocks of length n at the input, the output, and the noise, we have

$$\mathbf{y} = \mathbf{x} + \mathbf{z}$$

If n is large, by the law of large numbers, we have

$$\frac{1}{n}\sum_{i=1}^{n} z_i^2 = \frac{1}{n}\sum_{i=1}^{n} (y_i - x_i)^2 \leq P_N \qquad (7)$$

Or

$$\|\boldsymbol{y} - \boldsymbol{x}\|^2 \leq nP_N \qquad (8)$$

This means that, with probability approaching one (as n increases), **y** will be located in an n-dimensional sphere (hyper sphere) of radius $\sqrt{nP_N}$ and centered at **x**. On the other hand, due to the power constraint of P on the input and the independence of the input and noise, the output power is the sum of the input power and the noise power, i.e.

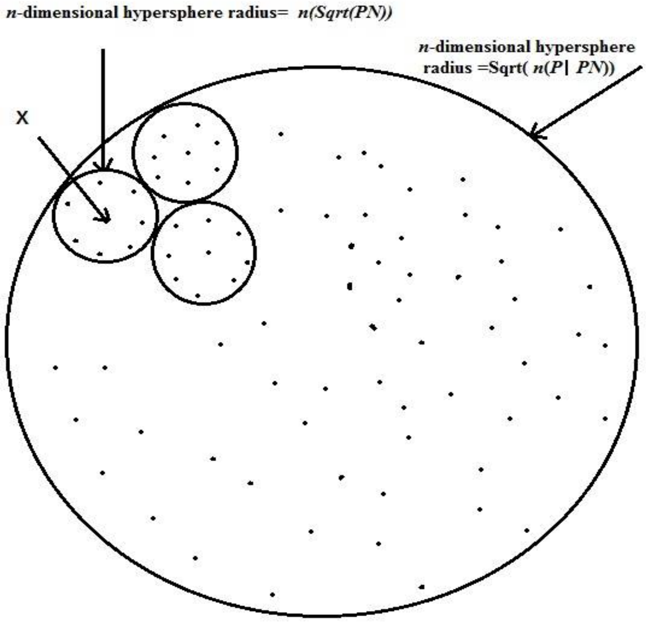

Figure 5: The output sequences of a Gaussian channel with power constraint

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$$\frac{1}{n}\sum_{i=1}^{n} y_i^2 \leq P + P_N \tag{9}$$

Or

$$\|y\|^2 \leq n\langle P + P_N \rangle \tag{10}$$

This implies that the output sequences (again, asymptotically and with high probability) will be inside an n-dimensional hyper sphere of radius $\sqrt{n(P + P_N)}$and centered at the origin. Figure 5 shows the sequences in the output space.

The question now is: How many **x** sequences can we transmit over this channel such that the hyper spheres corresponding to these sequences do not overlap in the output space? Obviously if this condition is satisfied, then the input sequences can be decoded reliably. An equivalent question is: how many hyper spheres of radius $\sqrt{nP_N}$can we pack in a hyper sphere of radius$\sqrt{n(P_N + P)}$? The answer is roughly the ratio of the volumes of the two hyper spheres. If we denote the volume of an n-dimensional hyper sphere by $V_n = K_n R^n$where R denotes the radius and $K_n$is independent of R, we see that the number of messages that can be reliably transmitted over this channel is equal to

$$M = \frac{K_n(n(P_N+P))^{\frac{n}{2}}}{K_n(nP_N)^{\frac{n}{2}}} = \left(\frac{P_N+P}{P_N}\right)^{\frac{n}{2}} = \left(1 + \frac{P}{P_N}\right)^{\frac{n}{2}} \tag{11}$$

Therefore, the capacity of a discrete-time additive white Gaussian noise channel with input power constraint P is given by

$$C = \frac{1}{n}\log M = \frac{1}{n} \cdot \frac{n}{2}\log\left(1 + \frac{P}{P_N}\right) = \frac{1}{2}\log\left(1 + \frac{P}{P_N}\right) \tag{12}$$

When dealing with a continuous-time, band limited additive white Gaussian noise channel with noise power-spectral density$\frac{N_0}{2}$, input power constraint P, and bandwidth W, one can sample at the Nyquist rate and obtain a discrete-time channel. The power/sample will be $P$and the noise power/sample will be

$$P_N = \int_{-W}^{+W} \frac{N_0}{2} df = WN_0 \tag{13}$$

Substituting these results in the above Equation, we obtain

$$C = \frac{1}{2}\log\left(1 + \frac{P}{N_0 W}\right)\text{bits/transmission} \tag{14}$$

If we multiply this result by the number of transmissions/sec, which is $2W$, we obtain the channel capacity in bits/sec

$$C = W\log\left(1 + \frac{P}{N_0 W}\right)\text{bits/sec} \tag{15}$$

This is the celebrated Shannon's formula for the capacity of an additive white Gaussian noise channels shown in figure, 6 and 7.

3.2 Radar Signal Detection

As a simple example of calculation maximum probability of inference,[8]consider a radar system with bandwidth 1 MHz, a required probability of detection 95 percent,[9] [10]and an average interval of 3hr between false alarms. Then the energy will be 10^4 joules, and the probability of false allowance 10^-10. For probability of signal detection 95 percent the figure shows that S/N is 15.5 db[11][12].

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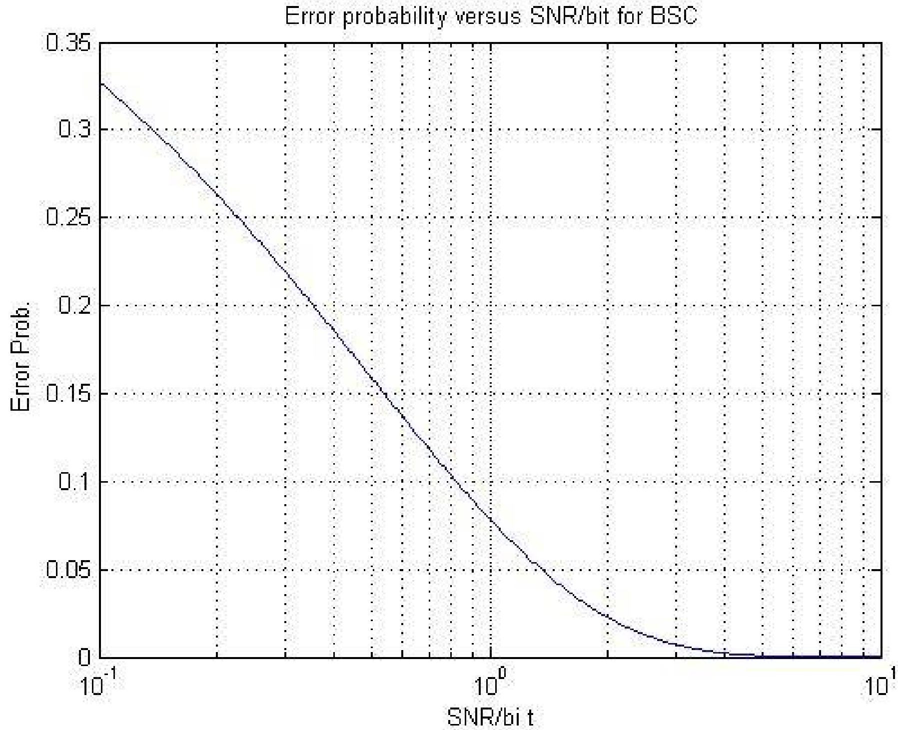

Figure 6 Error Probabilities versus SNR in Binary Symmetric Channel

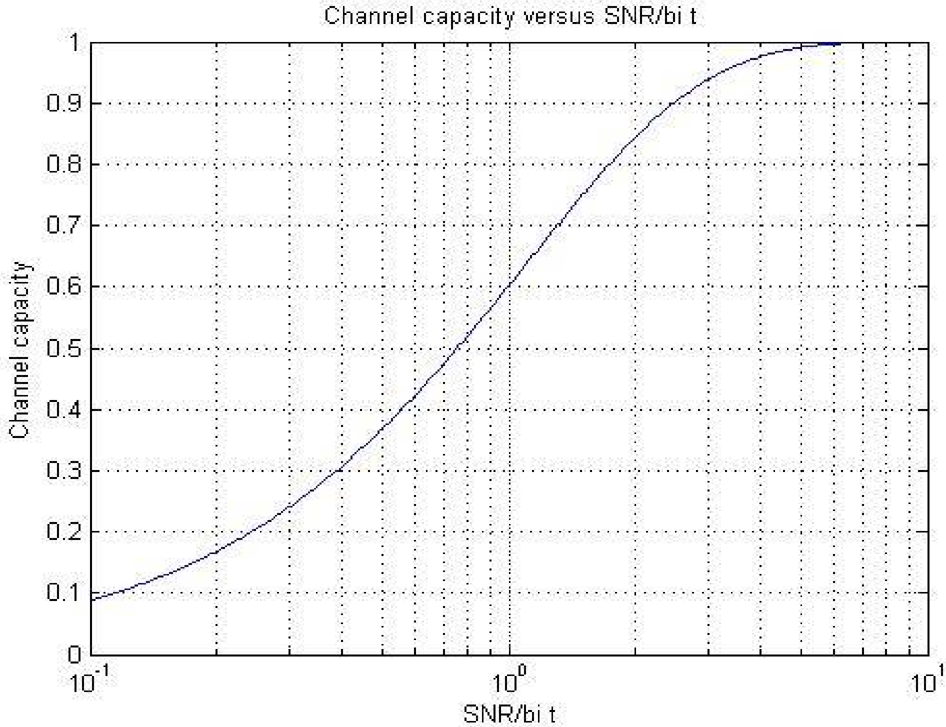

Figure7 Channel Capacity versus SNR in BSC

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SNR=35 at the detector input. From the required value of signal energy we can work backward to find the transmitted power required. A similar calculation with the probability of detection 99 percent requires the signal to noise power to be increased to 50 (17 db)[13].

Figure 8 and 9 shows the probability of detection with different values of false alarm and the improvement factor using the probability of inference method.

P_D	P_{fa}								
	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}	10^{-11}	10^{-12}
0.1	6.19	7.85	8.95	9.94	10.44	11.12	11.62	12.16	12.65
0.3	8.25	9.50	10.44	11.10	11.75	12.37	12.81	13.25	13.65
0.5	9.45	10.62	11.25	11.95	12.60	13.11	13.52	14.00	14.35
0.7	10.50	11.50	12.31	12.75	13.31	13.87	14.20	14.59	14.95
0.9	11.85	12.65	13.31	13.85	14.25	14.62	15.00	15.45	15.75
0.95	12.40	13.12	13.65	14.25	14.64	15.10	15.45	15.75	16.12
0.98	13.00	13.62	14.25	14.62	15.12	15.47	15.85	16.25	16.50
0.99	13.37	14.05	14.50	15.00	15.38	15.75	16.12	16.47	16.75
0.998	14.05	14.62	15.06	15.53	16.05	16.37	16.70	16.89	17.25
0.999	14.25	14.88	15.25	15.85	16.13	16.50	16.85	17.12	17.44
0.9995	14.50	15.06	15.55	15.99	16.35	16.70	16.98	17.35	17.55
0.9999	14.94	15.44	16.12	16.50	16.87	17.12	17.35	17.62	17.87

Table (1) Single Pulse SNR (dB)

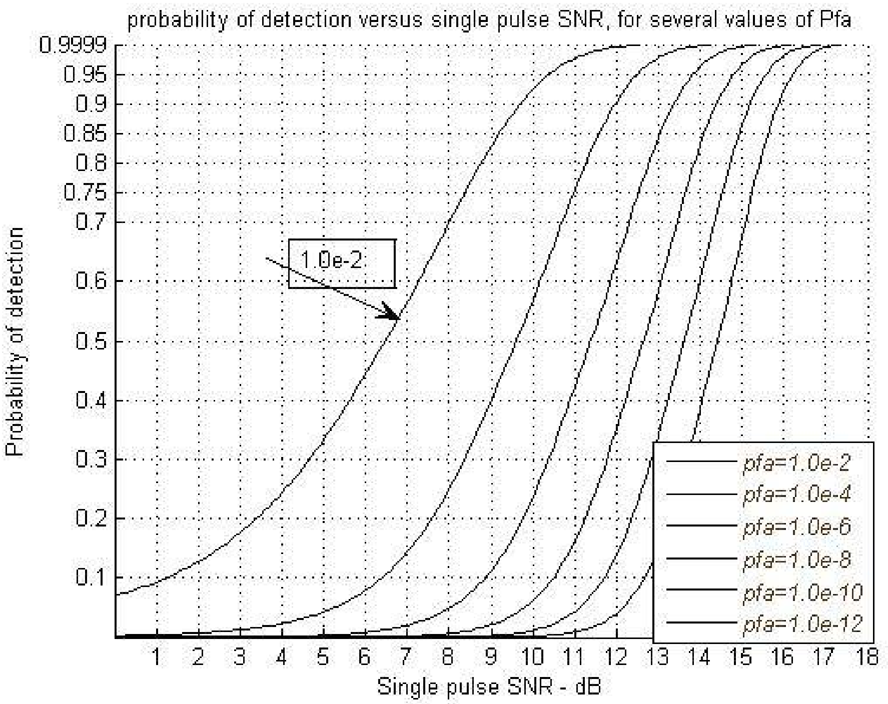

Figure 8.Probability of Detection Versus single Pulse SNR , for several Pfa

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The probability of detection can be written as $p_d = \log_{10}\langle 1 + \frac{p_d}{p_f}\rangle$ (16)

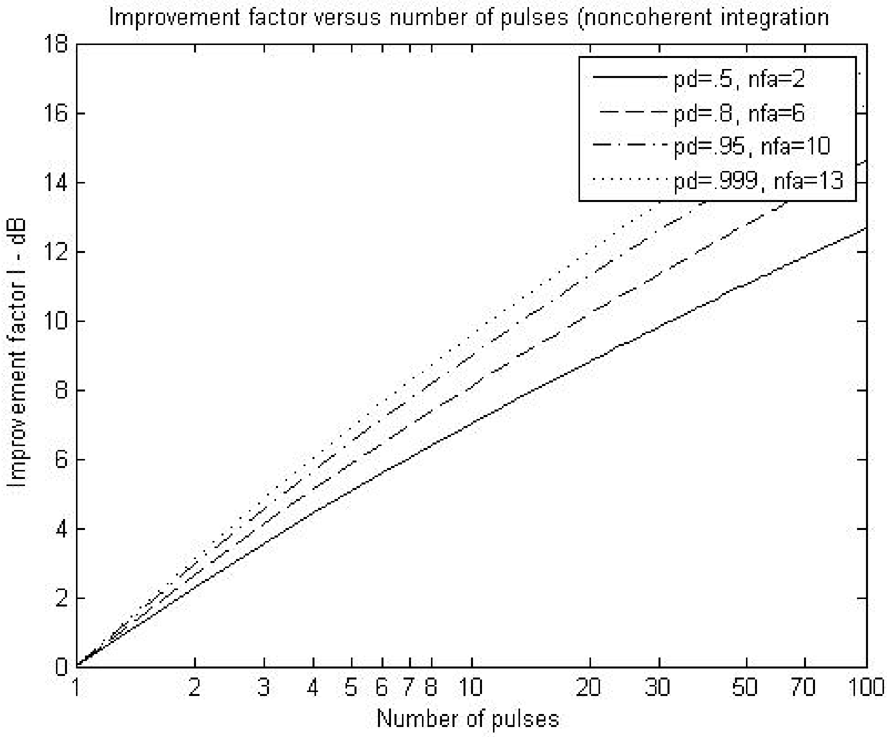

Figure 9 Improved Factors Versus Number of Pulses.

IV. CONCLUSIONS

This paper deals with a scheme for computing probability of inference and applying this scheme in different applications in real life like, Binary symmetric channel and radar signals. This is done by use cumulative distribution function and probability mass function to calculate the inference and channel capacity of BSC and value of probability of false alarm in radar signals.

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REFERENCES

[1] D. J. C. MacKay, *Information theory, inference and learning algorithms*. Cambridge university press, 2003.
[2] J. G. Proakis and M. Salehi, *Communucation Systems Engineering*. 2002.
[3] J. Proakis, M. Salehi, and G. Bauch, *Contemporary communication systems using MATLAB*. Nelson Education, 2012.
[4] A. Leon-Garcia and A. Leon-Garcia, *Probability, statistics, and random processes for electrical engineering*. Pearson/Prentice Hall 3rd ed. Upper Saddle River, NJ, 2008.
[5] A. Papoulis and S. U. Pillai, *Probability, random variables, and stochastic processes*. Tata McGraw-Hill Education, 2002.
[6] G. W. Domhoff, *Fifth Edition*. .
[7] H. D. Wacker and J. Borcsok, “Probability of Undetected Error on a Binary Symmetric Channel without Memory via Bayesian Inference,” in *Systems, 2007. ICONS’07. Second International Conference on*, 2007, p. 38.

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[8] S. Kingsley and S. Quegan, *Understanding radar systems*, vol. 2. SciTech Publishing, 1999.
[9] M. A. Richards, J. A. Scheer, and W. A. Holm, *Principles of modern radar*. Citeseer, 2010.
[10] P. Swerling, "Probability of detection for fluctuating targets," *IRE Trans. Inf. Theory*, vol. 6, no. 2, pp. 269–308, 1960.
[11] M. Schwartz, "Information transmission, modulation and noise," 1970.
[12] "Mark A. Richards-Fundamentals of Radar Signal Processing -McGraw-Hill (2005).pdf." .
[13] Mahafza, *Radar Systems Analysis and Design using Matlab*. 2000.

BIOGRAPHY

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