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Using the Rational Valued Characters Table for Cyclic Groups C_2 *and* $G = \bigoplus_{i=1}^{n} C_2$ in Transformation

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ABSTRACT: In this paper, we introduce a new method to transform image by using the rational character table for point group C_2 , $G = \bigoplus_{i=1}^{n} C_2$ by considering character table as square matrix of size $m \times m$ and designing an algorithm for it, which includes the transformation matrix of the image to the sets of matrices square of size $\mathbf{m} \times \mathbf{m}$.

KEYWORDS:point group, character table, cipher and anti – cipher.

I. Introduction

Image processing refers to the various operations; Purpose of transformation is to convert the data into a form where compression is easier. This transformation will transform the pixels which are correlated into a representation where they are decorrelated . The new values are usually smaller on average than the original values. The net effect is to reduce the redundancy of representation. For lossy compression, the transform coefficients can now be quantized according to their statistical properties, producing a much compressed representation of the original image data , in this paper we presented a new method to cipher and anti-cipher see[5],[6]and[9].

II. Material and Methods

Point group(1.1): [7]

Particularly we will consider the following point groups which molecules can belong to the group Cn. A molecule belongs to the group Cn if it has a n-fold axis. C_2 group as it has the elements E and C_2. The group $G = \bigoplus_{i=1}^{n} C_2$ A molecule belongs to the group $G = \bigoplus_{i=1}^{n} C_2$ if in addition to the identity E and a Cn axis, it has n vertical mirror planes

Character table of finite abelian group(1.2): [3]

If G =C_n the cyclic group of order n generated by r. Then the general formula of the Character table of C_n is given in the table (1.1).

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$\equiv(\boldsymbol{C_n}) =$

CL_α	I	r	r^2	$\cdots$	r^{n-1}
$\lvert CL_\alpha\rvert$	1	1	1	$\cdots$	1
$\lvert C_G(CL_\alpha)\rvert$	n	n	n	$\cdots$	n
χ_1	1	1	1	$\cdots$	1
χ_2	1	ω	ω^2	$\cdots$	ω^{n-1}
χ_3	1	ω^2	ω^4	$\cdots$	ω^{n-2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
χ_n	1	ω^{n-1}	ω^{n-2}	$\cdots$	ω

Table (1.1)

Where $\boldsymbol{\omega} = \boldsymbol{e}^{2\pi i/n}$

$\equiv(C_2) =$

CL_α	1	r
$\lvert CL_\alpha\rvert$	1	1
$\lvert C_G(CL_\alpha)\rvert$	2	2
χ_1	1	1
χ_2	1	-1

Table (1.2)

Theorem(1.3) : [4]

Let $T_1: G_1 \to GL(n,F)$ and $T_2: G_2 \to GL(m,F)$ are two matrix representations of the groups G_1 and G_2, χ_1 and χ_2 be two characters of T_1 and T_2 respectively ,then the character of $T_1 \otimes T_2$ is $\chi_1 \cdot \chi_2$.

Theorem(1.4) : [8]

The rational valued Character table of the cyclic group C_{p^s} of the rank $s+1$ where p is a prime number which is denoted by ($\equiv^* (C_{p^s})$) is given as follows :

Γ				$\cdot\cdot$	
	1	1	1	$\cdots$	1
	p-1	-1	p-1	$\cdots$	p-1
	P(p-1)	0	-p	$\cdots$	P(p-1)
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	p	0	0	$\cdots$	
	p	0	0	$\cdots$	
	p	0	0	$\cdots$	

Table (1.3)

Where its rank s+1 represents the number of all distinct Γ-classes.

Example(1.5):

To find the rational valued Character table of the cyclic group C_2 by using theorem (1.4) .

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$\equiv^* (C_2) =$

$\Gamma - classes$	$[I]$	$[r]$
θ_1	1	1
θ_2	1	-1

Table (1.4)

Definition(1.6):

. $\bigoplus_{i=1}^{n} C_2$ is the direct sum group The group G

Theorem (1.7):

If all irreducible characters of $G = \bigoplus_{i=1}^{n} C_2$ have the values in Z ,then the rational valued Characters of G has the block form :

$$\equiv^* (\boldsymbol{G}) = \begin{bmatrix} \boldsymbol{A_{n-1}} & \boldsymbol{A_{n-1}} \\ \boldsymbol{A_{n-1}} & -\boldsymbol{A_{n-1}} \end{bmatrix} \text{, Where } \boldsymbol{A_{n-1}} = \equiv^* \left(\bigoplus_{\boldsymbol{i=1}}^{\boldsymbol{n-1}} \boldsymbol{C_2} \right).$$

Example(1.8):

To find the rational valued Character table of the $G = \bigoplus_{i=1}^{3} C_2$ by using theorem(1.7).

$$\equiv^* \left(\boldsymbol{G} = \bigoplus_{\boldsymbol{i=1}}^{\boldsymbol{3}} \boldsymbol{C_2} \right) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

III. ALGORITHM

In this section will divide the matrix into the blocks matrix of the same order mxm.

Definition(2.1) : [6],[5]

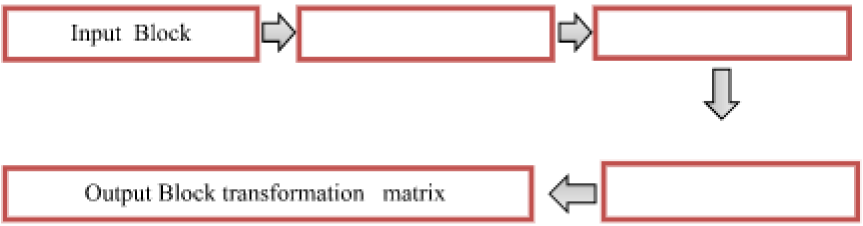
Let an matrix f be represented as an $n \times n$ matrix of integer numbers $f = \begin{bmatrix} f_1 & \cdots & f_m \\ \vdots & \ddots & \vdots \\ f_n & \cdots & f_k \end{bmatrix}$, where f_i are blocks matrix of order ixi .

General transform $F = P\,f\,Q$, If P and Q are non-singular (non-zero determinants), inverse matrices exist and

$$f = P^{-1}\,F\,Q^{-1}$$

Rule (2.2):

- Let $P = Q$
- $P = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \equiv^* C_2$, when P of size 2x2
- $P = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = \equiv^* \left(\boldsymbol{G} = \bigoplus_{\boldsymbol{i=1}}^{\boldsymbol{2}} \boldsymbol{C_2}\right)$, when P of size 4x4
- Transformation $f_i$matrix when P be of size $m \times m$:



So we get the final matrix encoded F_nAnd for the purpose of obtaining the original matrix and without the use of inverse matrix, so the answer will be in the following main theorems

IV. Result and Discussion

In this section we present important theorems on open cipher , where its cipher an n loop

Theorem (3.1):

If $P = \begin{bmatrix} A_{n-1} & A_{n-1} \\ A_{n-1} & -A_{n-1} \end{bmatrix}$ where P is matrix of size $m \times m$, Where $A_{n-1} = \equiv^{*} \begin{pmatrix} n-1 \\ \bigoplus \; C_2 \\ i=1 \end{pmatrix}$.

Then $P^{-1} = \frac{1}{m}\begin{bmatrix} A_{n-1} & A_{n-1} \\ A_{n-1} & -A_{n-1} \end{bmatrix}$

Example(3.2):

To find$P^{-1}$by theorem (3.1)

$$\text{If } P = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \text{ then} P^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

and

$$\text{If } P = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \text{ then } P^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

Theorem (3.3):

Let $P_1, P_2, \cdots, P_{n-1}, P_n$ are matrices n-time of size $m \times m$ and n even then$P_1 \times P_2 \times \ldots \times P_{n-1} \times P_n = m^{\frac{n}{2}} I_m$, where I_m identity matrix .

Proof:

$$P_1 \times P_2 \times \ldots \times P_{n-1} \times P_n = \underbrace{\begin{bmatrix} A_{n-1} & A_{n-1} \\ A_{n-1} & -A_{n-1} \end{bmatrix} \times \cdots \times \begin{bmatrix} A_{n-1} & A_{n-1} \\ A_{n-1} & -A_{n-1} \end{bmatrix}}_{n-time} \quad , \quad where \quad A_{n-1} = \equiv^{*} \begin{pmatrix} n-1 \\ \bigoplus \; C_2 \\ i=1 \end{pmatrix}.$$

$$= mI_m \times mI_m \times \cdots \times mI_m$$

$$= m^{\frac{n}{2}} I_m$$

Example(3.4):

Let P_1 and P_2 are matrices of size 4×4 , then $P_1 \times P_2 = 4I_4$, where I_4 identity matrix of size 4×4.

Proof: by theorem (3.3)

Example (3.5):

Let P_1 , P_2 , … P_{n-1} , P_n are matrices n-time of size 4×4 and n even to find $P_1 \times P_2 \times \ldots \times P_{n-1} \times P_n$ by theorem (3.3).

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$P_1 \times$

$= 4^{\frac{n}{2}} I_4$ where I_4 identity matrix .

Theorem(3.6) :
Let P be matrix of size $m \times m$, and F_i is transformation matrix , for level n then**:**

- Keys when n is even number:

- Keys when n is odd number :

Proof:
If n is even level:
Let F_n be be transformation for n-time , then we need n-time inverse matric for P . i.e:

$f = [P^{-1}.P^{-1}$

$f = \{\frac{1}{m}P.\frac{1}{m}P$

$f = \{\frac{1}{m^n}(P.P$

$f = \{\frac{1}{m^n}\left(m^{\frac{n}{2}}\right.$

$f = \{\frac{1}{m^n}\left(m^{\frac{n}{2}}\right.$

$f = \{\frac{m^n}{m^{2n}}(I_m$

$f = \{\frac{1}{m^n}\ (F_n$

If n is odd level:
Let F_n be be transformation for n-time , then we need n-time inverse matric for P . i.e:

$f = [P^{-1}.P^{-1}$

$f = \{\frac{1}{m}P.\frac{1}{m}P$

$f = \{\frac{1}{m^n}(P.P$

$f = \{\frac{1}{m^n}\left(m^{\frac{n}{2}}\right.$

$f = \{\frac{m^n}{m^{2n}}\ (I_m$

$f = \{\frac{1}{m^n}\ (P^{-1}$

Example(3.7) :
Let P be matrix of size 8×8, and F_i is transformation matrix, for level n then:

- Keys when n is even number:

$f = \frac{1}{8^n}([F_n])$

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- Keys when n is odd number :

Proof: by theorem (3.6)

If n is even level:

Let F_n be be transformation for n-time, then we need n-time inverse matric for . i.e:

$$f = [P^{-1}. P^{-1}$$

$$f = \{\frac{1}{8}P.\frac{1}{8}P$$

$$f = \{\frac{1}{8^n}(P.P$$

$$f = \{\frac{1}{8^n}\left(8^{\frac{n}{2}}I_8\right.$$

$$f = \{\frac{1}{8^n}\left(8^{\frac{n}{2}}I_8\right.$$

$$f = \{\frac{1}{8^n}\ (F_n$$

If n is odd level:

Let F_n be be transformation for n-time, then we need n-time inverse matric for . i.e:

$$f = [P^{-1}. P^{-1}$$

$$f = \{\frac{1}{8}P.\frac{1}{8}P$$

$$f = \{\frac{1}{8^n}(P.P$$

$$f = \{\frac{1}{8^n}\left(8^{\frac{n}{2}}I_8\right.$$

$$f = \{\frac{8^n}{8^{2n}}\ (I_8$$

$$f = \{\frac{1}{8^n}\ (P^{-1}$$

V. APPLICATION

Now we can applied the transformation

$$f = \begin{bmatrix} f_1 & \cdots \\ \vdots & \ddots \\ f_m & \cdots \end{bmatrix}$$

Then the block matrix f_1 of matrix f be

$$f_1 = \begin{bmatrix} 12 & 34 & 68 & 19 \\ 20 & 78 & 25 & 100 \\ 33 & 65 & 87 & 90 \\ 200 & 44 & 268 & 11 \end{bmatrix}.$$

Now the transformation be : $F = P\ f\ Q$, where $P = Q = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$ Then

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Where n=4 and degree of partition 4x4 then:

VI. CONCLUSION

Transferring messages secretly between participants has interested people for long time, and it's been really important and needed in our modern world, especially with the advent of electronic messaging and the internet. This paper is a suggestion of a transformation that transform the pixels which are correlated into a representation where they are decor related. The new values are usually smaller on an average than the original values. The net effect is to reduce the redundancy of representation. For lossy compression, the transform coefficients can now be quantized according to their statistical properties, producing a much compressed representation of the original the image data. This idea of this transformation can be applied in encrypting and decrypting important data, and that would be my future modification of this paper.

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