

Srinivasa Ramanujan's Contribution To Mathematics

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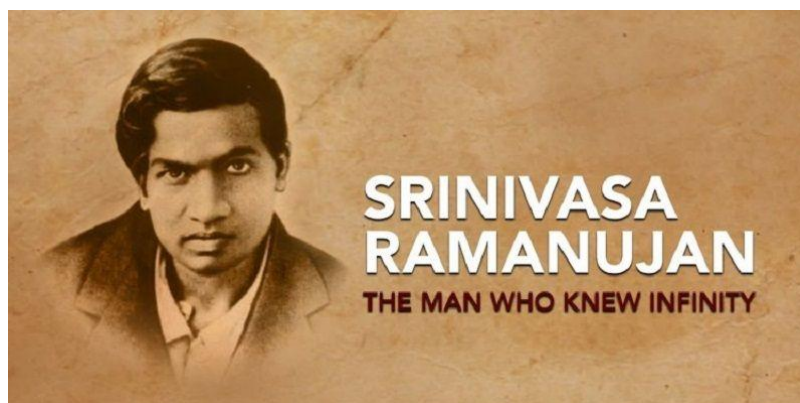
ABSTRACT: Srinivasa Ramanujan (1887-1920), the man who reshaped twentieth-century mathematics with his various contributions in several mathematical domains, including mathematical analysis, infinite series, continued fractions, number theory, and game theory is recognized as one of history's greatest mathematicians. Leaving this world at the youthful age of 32, Ramanujan made significant contributions to mathematics that only a few others could match in their lifetime. Surprisingly, he never received any formal mathematics training. Most of his mathematical discoveries were based only on intuition and were ultimately proven correct. With its humble and sometimes difficult start, his life story is just as fascinating as his incredible work. Every year, Ramanujan's birth anniversary on December 22 is observed as National Mathematics Day.

Born in Erode, Tamil Nadu, India, Ramanujan demonstrated an exceptional intuitive grasp of mathematics at a young age. Despite being a mathematical prodigy, Ramanujan's career did not begin well. He received a college scholarship in 1904, but he quickly lost it by failing in nonmathematical subjects. Another attempt at college in Madras (now Chennai) ended in failure when he failed his First Arts exam. It was around this time that he began his famous notebooks. He drifted through poverty until 1910 when he was interviewed by R. Ramachandra Rao, secretary of the Indian Mathematical Society. Rao was initially sceptical of Ramanujan, but he eventually recognised his abilities and supported him financially.

Srinivasa Ramanujan began developing his theories in mathematics and published his first paper in 1911. He was mentored at Cambridge by GH Hardy, a well-known British mathematician who encouraged him to publish his findings in a number of papers. In 1918, Ramanujan became the second Indian to be included as a Fellow of the Royal Society.

KEYWORDS: Srinivasa Ramanujan, mathematics, Tamil Nadu, theories, paper, Cambridge, Royal Society

I. INTRODUCTION



Ramanujan's major contributions to mathematics:

Ramanujan's contribution extends to mathematical fields such as complex analysis, number theory, infinite series, and continued fractions.¹

Infinite series for pi: In 1914, Ramanujan found a formula for infinite series for pi, which forms the basis of many algorithms used today. Finding an accurate approximation of π (pi) has been one of the most important challenges in the history of mathematics.

Game theory: Ramanujan discovered a long list of new ideas for solving many challenging mathematical problems that have given great impetus to the development of game theory. His contribution to game theory is purely based on intuition and natural talent and is unmatched to this day.²

Mock theta function: He elaborated on the mock theta function, a concept in the field of modular forms of mathematics.

Ramanujan number: 1729 is known as the Ramanujan number which is the sum of the cubes of two numbers 10 and 9.

Circle Method: Ramanujan, along with GH Hardy, invented the circle method which gave the first approximations of the partition of numbers beyond 200. This method contributed significantly to solving the notorious complex problems of the 20th century, such as Waring's conjecture and other additional questions.³

Theta Function: Theta function is a special function of several complex variables. German mathematician Carl Gustav Jacob Jacobi invented several closely related theta functions known as Jacobi theta functions. Theta function was studied by extensively Ramanujan who came up with the Ramanujan theta function, that generalizes the form of Jacobi theta functions and also captures general properties. Ramanujan theta function is used to determine the critical dimensions in Bosonic string theory, superstring theory, and M-theory.⁴

Other notable contributions by Ramanujan include hypergeometric series, the Riemann series, the elliptic integrals, the theory of divergent series, and the functional equations of the zeta function.⁵

Ramanujan's achievements were all about elegance, depth, and surprise beautifully intertwined. Unfortunately, Ramanujan contracted a fatal illness in England in 1918. He convalesced there for more than a year and returned to India in 1919. His condition then worsened, and he died on 26 April 1920. One might expect that a dying man would stop working and await his fate. However, Ramanujan spent his last year producing some of his most profound mathematics.

It has been more than a century, however, his mathematical discoveries are still alive and flourishing. "Ramanujan is important not just as a mathematician but because of what he tells us that the human mind can do".⁵ "Someone with his ability is so rare and so precious that we can't afford to lose them. A genius can arise anywhere in the world. It is our good fortune that he was one of us. It is unfortunate that too little of Ramanujan's life and work, esoteric though the latter is, seems to be known to most of us".⁶Ramanujan compiled around 3,900 results consisting of equations and identities. One of his most treasured findings was his infinite series for pi. This series forms the basis of many algorithms we use today. He gave several fascinating formulas to calculate the digits of pi in many unconventional ways. He discovered a long list of new ideas to solve many challenging mathematical problems,⁷ which gave a significant impetus to the development of game theory. His contribution to game theory is purely based on intuition and natural talent and remains unrivalled to this day.⁸He elaborately described the mock theta function, which is a concept in the realm of modular form in mathematics. Considered an enigma till sometime back, it is now recognized as holomorphic parts of mass forms. One of Ramanujan's notebooks was discovered by George Andrews in 1976⁹ in the library at Trinity College. Later the contents of this notebook were published as a book. 1729 is known as the Ramanujan number. It is the sum of the cubes of two numbers 10 and 9. For instance, 1729 results from adding 1000 (the cube of 10) and 729 (the cube of 9). This is the smallest number that can be expressed in two different ways as it is the sum of these two cubes. Interestingly, 1729 is a natural number following 1728 and preceding 1730.¹⁰Ramanujan's contributions stretch across mathematics fields, including complex analysis, number theory, infinite series, and continued fractions. Ramanujan's other notable contributions include hypergeometric series, the Riemann series, the elliptic integrals, the theory of divergent series, and the functional equations of the zeta function.¹¹

At the age of 32, he died of Tuberculosis. In his short life span, he independently found 3900 results. He worked on real analysis, number theory, infinite series, and continued fractions. Some of his other works such as Ramanujan number, Ramanujan prime, Ramanujan theta function, partition formulae, mock theta function,¹² and many more opened new areas for research in the field of mathematics. He worked out the Riemann series, the elliptic integrals, hypergeometric series, the functional equations of the zeta function, and his theory of divergent series, in which he found a value for the sum of such series, using a technique he invented, that came to be called Ramanujan summation¹³. In England, Ramanujan made further researches, especially in the partition of numbers, i.e., the number of ways in which a positive integer can be expressed as the sum of positive integers.¹⁴ Some of his results are still under research. His journal, Ramanujan Journal, was established to keep a record of all his notebooks and results, both published and unpublished, in the field of mathematics.¹⁵ As late as 2012, researchers studied even the small comments in his book, as they do not want to miss any results or identities given by him, that remained unsuspected until a century after his death¹⁶. From his last letters in 1920 that he wrote to Hardy, it was evident that he was still working on new ideas and theorems of mathematics. In 1976, mathematicians found the 'lost notebook', that contained the works of Ramanujan from the last year of his life.¹⁷ Ramanujan devoted all his mathematical intelligence to his family goddess Namagir Thayar. He once said, "An equation for me has no meaning unless it expresses a thought of God."¹⁸

II. DISCUSSION

1. Infinite series of π

William Shanks, a 19th-century British mathematician tried calculating the value of infinite series of π . In 1873, he calculated the value of π to 707 decimal places. Ramanujan, in 1914, published ‘Modular equations and approximations to π ’,¹⁹ which contained not only one, but 17 different series, that will converge very fastly to π , after calculating just fewer terms of the series.

For estimating π

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)! (1103 + 26390k)}{(k!)^4 396^{4k}}$$

2. Ramanujan number

The number 1729 is known as the Ramanujan number or Hardy-Ramanujan number. It is the smallest natural number that can be expressed as the sum of two cubes, in two different ways, i.e., $1729 = 1^3 + 12^3 = 9^3 + 10^3$. There is a small story behind the discovery of this number. When Ramanujan was under treatment, G.H. Hardy once visited him in the hospital and had a conversation in which he mentioned,²⁰

I remember once going to see him when he was ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavourable omen. “No,” he replied, “it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways.”²¹ This is how the Ramanujan number came into existence. Later on, more properties of this number were discovered.²³

3. Ramanujan Prime

Ramanujan published a two-page paper on the proof of Bertrand’s postulate. At the end of the last page, he mentioned a result, $\pi(x) - \pi(x/2) \geq 1, 2, 3, 4, 5, \dots$, for all $x \geq 2, 11, 17, 29, 41, \dots$ respectively, where $\pi(x)$ is the prime counting function, equals to the number of primes equal or less than x . The n th Ramanujan prime number is the least integer R_n , for which there are at least n primes between x and $x/2$, for all $x \geq R_n$. The first five Ramanujan primes are 2, 11, 17, 29, 41

4. Ramanujan Theta Function²⁴

Ramanujan theta function is the generalised form of the Jacob theta function. In particular, Jacobi triple product can be beautifully represented by the Ramanujan theta function. The Ramanujan theta function is given below.²⁵

$$f(a, b) = \sum_{n=-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}}$$

The Ramanujan theta function.

for $|ab| < 1$. With the help of Ramanujan theta function, Jacobi triple product can be represented as,

$$f(a, b) = (-a; ab)_{\infty} (-b; ab)_{\infty} (ab; ab)_{\infty}.$$

5. Mock Theta Function

Ramanujan in his last letter to G.H. Hardy and in his ‘lost notebook’, gave the first example of mock theta function. A mock theta function is a mock modular form(the holomorphic part of a harmonic weak Maass form), of weight 1/2. His last letter to Hardy contained 17 examples of mock theta functions, and some more examples were mentioned in his ‘lost notebook.’²⁶ Ramanujan gave an order to his mock theta function. Before the attempts of Zwegers, the order of mock theta function was 3, 5, 6, 7, 8, 10.²⁷

6. Partition

Partition or integer partition of an integer ‘n’ is a way of writing ‘n’ as a sum of positive integers. Partitions that differ only in the order of summands are considered as the same partitions. Each summand in the partition is called a part. The number of partitions of an integer ‘n’ is denoted by p(n). For example, integer 4 has 5 partitions as given below.²⁸

4

1+1+1+1

1+2+1

1+3

2+2

Here partition 1+3 is the same as 3+1 and 1+2+1 is the same as 1+1+2 and p(4)=5. Partitions can also be visualised with the help of the Young diagram and Ferrers diagram.²⁹

7. Ramanujan Magic Squares

In his school days, he used to enjoy solving magic squares. Magic squares are the cells in 3 rows and 3 columns, filled with numbers starting from 1 to 9. The numbers in the cells are arranged in such a way that the sum of numbers in each row is equal to the sum of numbers in each column is equal to the sum of numbers in each diagonal. Ramanujan gave a general formula for solving the magic square of dimension 3×3,³⁰

C+Q	A+P	B+R
A+R	B+Q	C+P
B+P	C+R	A+Q

where A, B, C and P, Q, R are in arithmetic progression. The following formula was also given by him.³¹

$2Q+R$	$2P+2R$	$P+Q$
$2P$	$P+Q+R$	$2Q+2R$
$P+Q+2R$	$2Q$	$2P+R$

8. Ramanujan Congruences

Ramanujan obtained three congruences when m is a whole number, $p(5m+4) \equiv 0 \pmod{5}$, $p(7m+5) \equiv 0 \pmod{7}$, $p(11m+6) \equiv 0 \pmod{11}$. Hardy and E.M. Wright wrote, he was first to led the conjecture and then to prove, three striking arithmetic properties associated with the moduli 5, 7 and 11.³²

Values of $p(n)$ for $n \equiv 4 \pmod{5}$, $n \equiv 5 \pmod{7}$, $n \equiv 6 \pmod{11}$ are given in the below tables repectively.³³

n	4	9	14	19	24	29	34	39
$p(n)$	5	30	135	490	1575	4565	12310	31185

n	5	12	19	26	33	40
$p(n)$	7	77	490	2436	10143	37338

n	6	17	28	39
$p(n)$	11	297	3718	31185

9. Highly composite numbers

Composite numbers are the numbers that have factors other than 1 and the number itself. Ramanujan raised an interesting question that if 'n' is a composite number then what properties make a number highly composite. Ramanujan's definition of Highly composite numbers,³⁴

A natural number is a highly composite number if $d(m) < d(n)$ for all $m < n$.

One of the highly composite numbers calculated by Ramanujan is 6746328388800. This number has 13 digits and its prime factorization is $2^6 \times 3^4 \times 5^2 \times 7^2 \times 11^1 \times 13^1 \times 17^1 \times 19^1 \times 23^1$

He also published a paper on highly composite numbers in 1915. According to him, there were infinitely many highly composite numbers.³⁵

10. Symmetric Equation by Ramanujan

Ramanujan noticed symmetry in Diophantine's equation, $xy = yx$. He proved that there exists only one integer solution to this equation, i.e., $x=4$, $y=2$, and an infinite number of rational solutions, for example, $(27/8)(9/4)(27/8)(9/4) = (9/4)(27/8)(9/4)(27/8)$.³⁶

11. Ramanujan-Nagell Equation

Ramanujan-Nagell Equation is the equation of type $22n - 7 = 2x^2$. It is an example of Diophantine equation. In 1913, Ramanujan claimed that this equation had only $1^2+7 = 2^3$, $3^2+7 = 2424$, $5^2+7 = 2525$, $11^2+7 = 2727$, $181^2+7 = 215215$ integral solutions. This conjecture was later on proved by Trygve Navell and is widely used in coding theory.³⁷

12. On Certain Arithmetical Functions

Ramanujan published a paper “On certain arithmetic functions” in 1916, in which he discussed the properties of Fourier coefficients of modular forms. Though the concept of modular forms was not even developed then, he gave three fundamental conjectures. In 1936, after 20 years of his published paper, a German mathematician Erich Hecke developed the Hecke theory with the help of his first two conjectures. His last conjecture played a vital role in the Langlands program (a program that relates representation theory and algebraic number theory). “On certain arithmetical functions” by Ramanujan was very effective in creating a sensation in 20th century mathematics.³⁸

13. On Fermat’s Last Theorem

In 2012, mathematicians found some evidence that revealed Ramanujan was working on Fermat’s last theorem. Pierre de Fermat mentioned that,

if n is a whole number greater than 2, then there are no positive whole number triples x, y and z , such that $x^n + y^n = z^n$.³⁹

Ramanujan claimed that he had found an infinite family of whole numbers that will satisfy (approximately, not exactly) Fermat’s equation for $n=3$. He gave the example of the number 1729, which do not fits into the equation just by the mark of 1, for $x=9, y=10, z=12$. Ramanujan also worked on the equations of the form, $y^2 = x^3 + ax + b$. An elliptic curve is obtained, when the points (x,y) of this equation are plotted. These elliptic curves were of great significance and were used by Sir Andrew Wiles while he was proving Fermat’s last theorem in 1994.⁴⁰

14. Roger-Ramanujan Identities

In 1894, these identities were discovered and proved by Leonard James Rogers. Nearby 1913, Ramanujan rediscovered these identities. He had no proof but found Roger’s paper in 1917. Then they both united and gave a joint new proof.⁴¹

ROGERS-RAMANUJAN IDENTITIES

$$\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q; q)_n} = \prod_{n=1}^{\infty} (1 - q^{5n-1})^{-1} (1 - q^{5n-4})^{-1}$$

$$\sum_{n=0}^{\infty} \frac{q^{n^2+n}}{(q; q)_n} = \prod_{n=1}^{\infty} (1 - q^{5n-2})^{-1} (1 - q^{5n-3})^{-1}$$

15. Roger-Ramanujan Continued Fractions

Roger discovered continued fractions in 1894, which were later rediscovered by Ramanujan in 1912.⁴²

$$R(q) = \frac{q^{\frac{11}{60}} H(q)}{q^{-\frac{1}{60}} G(q)} = q^{\frac{1}{5}} \prod_{n=1}^{\infty} \frac{(1 - q^{5n-1})(1 - q^{5n-4})}{(1 - q^{5n-2})(1 - q^{5n-3})} = q^{1/5} \prod_{n=1}^{\infty} (1 - q^n)^{(n|5)}$$

$$= \frac{q^{1/5}}{1 + \frac{q}{1 + \frac{q^2}{1 + \frac{q^3}{1 + \dots}}}}$$

$(n|m)$ denotes the Jacobi symbol.

Ramanujan found various results concerning $R(q)$, for example, $R(-2e^{-2\pi})$ is given below in the picture and he also calculated $R(-2\sqrt{e^{-2\pi/n}})$ for $n=4, 9, 16, 64$

$$R(e^{-2\pi}) = \sqrt{\frac{5 + \sqrt{5}}{2}} - \frac{\sqrt{5} + 1}{2},$$

$$S(e^{-\pi}) = \sqrt{\frac{5 - \sqrt{5}}{2}} - \frac{\sqrt{5} - 1}{2},$$

$$R(e^{-2\pi\sqrt{5}}) = \frac{\sqrt{5}}{1 + \left(5^{3/4} \left(\frac{\sqrt{5}-1}{2}\right)^{5/2} - 1\right)^{1/5}} - \frac{\sqrt{5} + 1}{2}.$$

16. Ramanujan's Master Theorem

Ramanujan's Master Theorem provides an analytic expression for the Mellin transform of an analytical function. This theorem is used by Ramanujan to calculate definite integrals and infinite series. ⁴³

According to [mathworld](#), Ramanujan's master theorem is the statement that if

$$f(z) = \sum_{k=0}^{\infty} \frac{\phi(k)(-z)^k}{k!}$$

for some function (analytic or integrable) ϕ , then

$$\int_0^{\infty} x^{n-1} f(x) dx = \Gamma(n)\phi(-n).$$

As written it is obviously false as the values of an (analytic or integrable) function ϕ at natural numbers do not determine its values anywhere else. However it turns out that

$$\int_0^{\infty} x^{s-1} f(x) dx = \Gamma(s)\phi(-s)$$

for arbitrary s under growth conditions on ϕ .

17. Properties of Bernoulli Numbers

In 1904, Ramanujan independently studied and rediscovered Bernoulli numbers. In 1911, he wrote his first article on this topic. Bernoulli numbers B_n are the sequence of rational numbers, that appear in the Taylor series expansion of tangent and hyperbolic tangent functions. One of the properties that he discussed states that, the denominator of all Bernoulli numbers are divisible by six. Based on previous Bernoulli numbers, he also suggested a method to calculate Bernoulli numbers. According to the method proposed by him, if n is even but not equal to zero,

1. B_n is a fraction and the numerator of B_n/n in its lowest terms is a prime number.
2. The denominator of B_n contains each of the factors 2 and 3 once and only once.
3. $2^n(2^n - 1)B_n/n$ is an integer and $2(2^n - 1)B_n$ consequently is an odd integer.⁴³

$$\binom{m+3}{m} B_m = \begin{cases} \frac{m+3}{3} - \sum_{j=1}^{\frac{m}{6}} \binom{m+3}{m-6j} B_{m-6j}, & \text{if } m \equiv 0 \pmod{6}; \\ \frac{m+3}{3} - \sum_{j=1}^{\frac{m-2}{6}} \binom{m+3}{m-6j} B_{m-6j}, & \text{if } m \equiv 2 \pmod{6}; \\ -\frac{m+3}{6} - \sum_{j=1}^{\frac{m-4}{6}} \binom{m+3}{m-6j} B_{m-6j}, & \text{if } m \equiv 4 \pmod{6}. \end{cases}$$

18. Euler Mascheroni Constant

Ramanujan calculated the Euler Mascheroni constant also known as the Euler constant, up to 15 decimal places. It is the limiting difference between the harmonic series and the natural logarithm. Later on, a value up to 50 decimal places was calculated and is equal to,⁴⁷ 0.57721566490153286060651209008240243104215933593992.....

$$\gamma = \lim_{n \rightarrow \infty} \left(-\log n + \sum_{k=1}^n \frac{1}{k} \right)$$

$$= \int_1^{\infty} \left(-\frac{1}{x} + \frac{1}{[x]} \right) dx.$$

Here, $[x]$ represents the floor function.

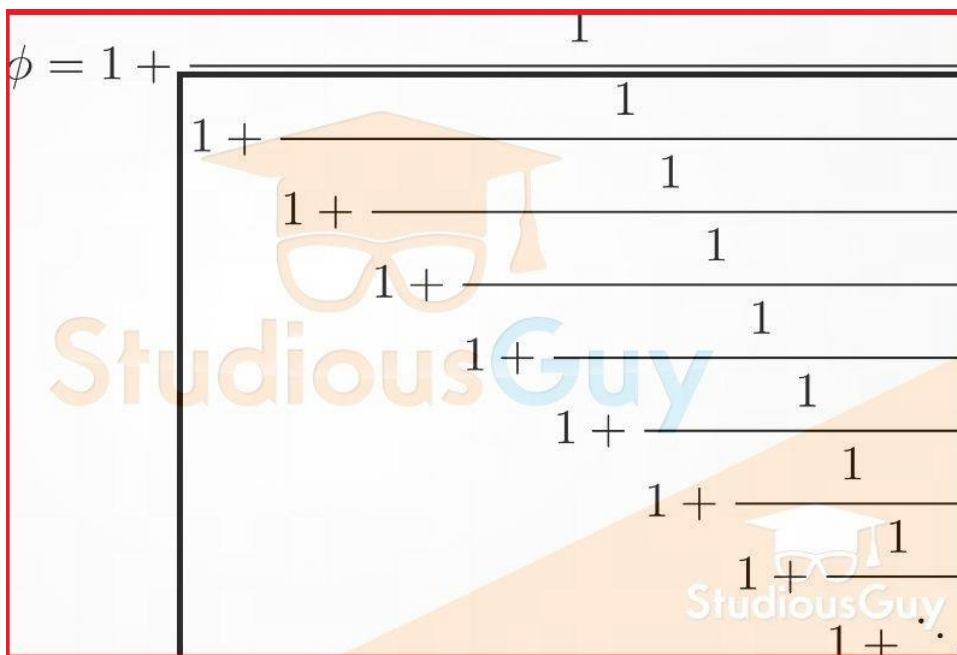
γ denotes the Euler constant

19. Ramanujan Summation

Ramanujan, in one of his books, stated that, if we add up all natural numbers starting from 1 up to infinity, then the sum will be a finite number, i.e., $1+2+3+\dots+\infty = -1/12$

20. Ramanujan Puzzles

- The first puzzle was to prove the equation of infinite nested radical. In 1911, Ramanujan sent the RHS of this equation to a mathematical journal as a puzzle. ⁴⁸
- The next puzzle is to find the value of the Golden ratio(Φ), which is equal to the infinite continued fraction given in the picture below.



The continued fraction in the black box is the same as that in the outer red box. Setting this equal to x , we get $\Phi = 1 + 1/x$, which yields $x^2 - x - 1 = 0$. The solutions of this quadratic equation are $(1+\sqrt{5})/2$ and $(1-\sqrt{5})/2$. Neglecting the negative solution, the value of Φ is $(1+\sqrt{5})/2$

III. RESULTS

Ramanujan (literally, "younger brother of Rama", a Hindu deity)^[12] was born on 22 December 1887 into a Tamil Brahmin Iyengar family in Erode, in present-day Tamil Nadu.^[13] His father, Kuppaswamy Srinivasa Iyengar, originally from Thanjavur district, worked as a clerk in a sari shop.^{[14][2]} His mother, Komalatammal,⁴⁹ was a housewife and sang at a local temple.^[15] They lived in a small traditional home on Sarangapani Sannidhi Street in the town of Kumbakonam.^[16] The family home is now a museum. When Ramanujan was a year and a half old, his mother gave birth to a son, Sadagopan, who died less than three months later. In December 1889, Ramanujan contracted smallpox⁵⁰, but recovered, unlike the 4,000 others who died in a bad year in the Thanjavur district around this time. He moved with his mother to her parents' house in Kanchipuram, near Madras (now Chennai). His mother gave birth to two more children, in 1891 and 1894, both of whom died before their first birthdays.^[12]

On 1 October 1892, Ramanujan was enrolled at the local school.^[17] After his maternal grandfather lost his job as a court official in Kanchipuram,^[18] Ramanujan and his mother moved back to Kumbakonam, and he was enrolled in Kangayan Primary School.^[19] When his paternal grandfather died, he was sent back to his maternal grandparents, then living in Madras. He did not like school in Madras, and tried to avoid attending. His family enlisted a local constable to make sure he attended school.⁵¹ Within six months, Ramanujan was back in Kumbakonam.^[19]

Since Ramanujan's father was at work most of the day, his mother took care of the boy, and they had a close relationship. From her, he learned about tradition and puranas, to sing religious songs, to attend pujas at the temple, and to maintain particular eating habits—all part of Brahmin culture.^[20] At Kangayan Primary School, Ramanujan performed well.⁵² Just before turning 10, in November 1897, he passed his primary examinations in English, Tamil, geography, and arithmetic with the best scores in the district.^[21] That year, Ramanujan entered Town Higher Secondary School, where he encountered formal mathematics for the first time.^[21]

A child prodigy by age 11, he had exhausted the mathematical knowledge of two college students who were lodgers at his home. He was later lent a book written by S. L. Loney on advanced trigonometry.^{[22][23]} He mastered this by the age of 13 while discovering sophisticated theorems on his own. By 14, he received merit certificates and academic awards that continued throughout his school career, and he assisted the school in the logistics of assigning its 1,200 students (each with differing needs) to its approximately 35 teachers.^[24] He completed mathematical exams in half the allotted time, and showed a familiarity with geometry and infinite series.⁵³ Ramanujan was shown how to solve cubic equations in 1902. He would later develop his own method to solve the quartic. In 1903, he tried to solve the quintic, not knowing that it was impossible to solve with radicals.^[25]

In 1903, when he was 16, Ramanujan obtained from a friend a library copy of A Synopsis of Elementary Results in Pure and Applied Mathematics, G. S. Carr's collection of 5,000 theorems.^{[26][27]} Ramanujan reportedly studied the contents of the book in detail.^[28] The next year, Ramanujan independently developed and investigated the Bernoulli numbers and calculated the Euler–Mascheroni constant up to 15 decimal places.^[29] His peers at the time said they "rarely understood him" and "stood in respectful awe" of him.^[24]

When he graduated from Town Higher Secondary School in 1904, Ramanujan was awarded the K. Ranganatha Rao prize for mathematics by the school's headmaster, Krishnaswami Iyer. Iyer introduced Ramanujan as an outstanding student who deserved scores higher than the maximum.^[30] He received a scholarship to study at Government Arts College, Kumbakonam,^{[31][32]} but was so intent on mathematics that he could not focus on any other subjects and failed most of them, losing his scholarship in the process.^[33] In August 1905, Ramanujan ran away from home, heading towards Visakhapatnam, and stayed in Rajahmundry^[34] for about a month.^[33] He later enrolled at Pachaiyappa's College in Madras. There, he passed in mathematics, choosing only to attempt questions that appealed to him and leaving the rest unanswered, but performed poorly in other subjects, such as English, physiology, and Sanskrit.^[35] Ramanujan failed his Fellow of Arts exam in December 1906 and again a year later. Without an FA degree, he left college and continued to pursue independent research in mathematics, living in extreme poverty and often on the brink of starvation.^[36]

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In 1910, after a meeting between the 23-year-old Ramanujan and the founder of the Indian Mathematical Society, V. Ramaswamy Aiyer, Ramanujan began to get recognition in Madras's mathematical circles, leading to his inclusion as a researcher at the University of Madras.^[37]

In 1910, Ramanujan met deputy collector V. Ramaswamy Aiyer, who founded the Indian Mathematical Society.^[52] Wishing for a job at the revenue department where Aiyer worked, Ramanujan showed him his mathematics notebooks.⁵⁵ As Aiyer later recalled:

I was struck by the extraordinary mathematical results contained in [the notebooks]. I had no mind to smother his genius by an appointment in the lowest rungs of the revenue department.^[53]

Aiyer sent Ramanujan, with letters of introduction, to his mathematician friends in Madras.^[52] Some of them looked at his work and gave him letters of introduction to R. Ramachandra Rao, the district collector for Nellore and the secretary of the Indian Mathematical Society.^{[54][55][56]} Rao was impressed by Ramanujan's research but doubted that it was his own work. Ramanujan mentioned a correspondence he had with Professor Saldhana, a notable Bombay mathematician⁸⁸, in which Saldhana expressed a lack of understanding of his work but concluded that he was not a fraud.^[57] Ramanujan's friend C. V. Rajagopalachari tried to quell Rao's doubts about Ramanujan's academic integrity. Rao agreed to give him another chance, and listened as Ramanujan discussed elliptic integrals,⁸⁷ hypergeometric series, and his theory of divergent series, which Rao said ultimately convinced him of Ramanujan's brilliance.^[57] When Rao asked him what he wanted, Ramanujan replied that he needed work and financial support. Rao consented and sent him to Madras.⁸⁶ He continued his research with Rao's financial aid. With Aiyer's help, Ramanujan had his work published in the Journal of the Indian Mathematical Society⁵⁶

Ramanujan departed from Madras aboard the S.S. Nevasa on 17 March 1914.^[88] When he disembarked in London on 14 April,⁵⁷ Neville was waiting for him with a car. Four days later, Neville took him to his house on Chesterton Road in Cambridge.⁵⁸ Ramanujan immediately began his work with Littlewood and Hardy.⁸⁵ After six weeks, Ramanujan moved out of Neville's house and took up residence on Whewell's Court, a five-minute walk from Hardy's room.^[89]

Hardy and Littlewood began to look at Ramanujan's notebooks.⁵⁹ Hardy had already received 120 theorems from Ramanujan in the first two letters, but there were many more results and theorems in the notebooks.⁸⁴ Hardy saw that some were wrong, others had already been discovered, and the rest were new breakthroughs.^[90] Ramanujan left a deep impression on Hardy and Littlewood. Littlewood commented, "I can believe that he's at least a Jacobi",^[91] while Hardy said he "can compare him only with Euler or Jacobi."^[92]

Ramanujan spent nearly five years in Cambridge collaborating with Hardy and Littlewood, and published part of his findings there. Hardy and Ramanujan had highly contrasting personalities.⁸³ Their collaboration was a clash of different cultures⁶⁰, beliefs, and working styles. In the previous few decades, the foundations of mathematics had come into question and the need for mathematically rigorous proofs recognised.⁸² Hardy was an atheist and an apostle of proof and mathematical rigour, whereas Ramanujan was a deeply religious man who relied very strongly on his intuition and insights⁶¹. Hardy tried his best to fill the gaps in Ramanujan's education and to mentor him in the need for formal proofs to support his results, without hindering his inspiration—a conflict that neither found easy.⁶²

Ramanujan was awarded a Bachelor of Arts by Research degree^{[93][94]} (the predecessor of the PhD degree) in March 1916 for his work on highly composite numbers, sections of the first part of which had been published the preceding year⁸¹ in the Proceedings of the London Mathematical Society.⁶³ The paper was more than 50 pages long and proved various properties of such numbers. Hardy disliked this topic area but remarked that though it engaged⁸⁰ with what he called the 'backwater of mathematics', in it Ramanujan displayed 'extraordinary mastery over the algebra of inequalities'.^[95]

On 6 December 1917, Ramanujan was elected to the London Mathematical Society. On 2 May 1918, he was elected a Fellow of the Royal Society,^[96] the second Indian admitted, after Ardaseer Cursetjee in 1841.⁶⁴ At age 31, Ramanujan was one of the youngest Fellows in the Royal Society's history. He was elected "for his investigation in elliptic functions⁷⁹ and the Theory of Numbers." On 13 October 1918, he was the first Indian to be elected a Fellow of Trinity College, Cambridge.^[97]

Ramanujan had numerous health problems throughout his life⁷⁸. His health worsened in England; possibly he was also less resilient due to the difficulty of keeping to the strict dietary requirements of his religion there and because of wartime rationing in 1914–18.⁶⁵ He was diagnosed with tuberculosis and a severe vitamin deficiency, and confined to a sanatorium. In 1919, he returned to Kumbakonam,⁷⁷ Madras Presidency, and in 1920 he died at the age of 32. After

his death, ⁶⁶his brother Tirunarayanan compiled Ramanujan's remaining handwritten notes, consisting of formulae on singular moduli, hypergeometric series and continued fractions.^[43]

Ramanujan's widow, Smt. Janaki Ammal, moved to Bombay. In 1931, she returned to Madras and settled in Triplicane, where she supported herself on a pension from Madras University and income from tailoring. In 1950, she adopted a son,⁶⁷ W. Narayanan, who eventually became an officer of the State Bank of India⁷⁶ and raised a family. In her later years, she was granted a lifetime pension from Ramanujan's former employer, the Madras Port Trust, and pensions from, among others, the Indian National Science Academy and the state governments of Tamil Nadu, Andhra Pradesh and West Bengal. ⁶⁸She continued to cherish Ramanujan's memory, and was active in efforts to increase his public recognition; prominent mathematicians, including George Andrews, Bruce C. Berndt and Béla Bollobás made it a point to visit her while in India. She died at her Triplicane residence in 1994.^{[42][43]}

A 1994 analysis of Ramanujan's medical records and symptoms by Dr. D. A. B. Young^[98] concluded that his medical symptoms—including his past relapses, fevers, and hepatic conditions—were much closer to those resulting from hepatic amoebiasis,⁶⁹ an illness then widespread in Madras, than tuberculosis.⁷⁶ He had two episodes of dysentery before he left India. When not properly treated, amoebic dysentery can lie dormant for years and lead to hepatic amoebiasis⁷⁰, whose diagnosis was not then well established.^[99] At the time, if properly diagnosed, amoebiasis was a treatable and often curable disease;^{[99][100]} British soldiers who contracted it during the First World War were being successfully cured of amoebiasis around the time Ramanujan left England.^[101]

IV. CONCLUSIONS

Ramanujan has been described as a person of a somewhat shy and quiet disposition, a dignified man with pleasant manners.^[103] He lived a simple life at Cambridge.^[104] Ramanujan's first Indian biographers describe him as a rigorously orthodox Hindu. He credited his acumen to his family goddess, Namagiri Thayar (Goddess Mahalakshmi) of Namakkal.⁷¹ He looked to her for inspiration in his work^[105] and said he dreamed of blood drops that symbolised her consort, Narasimha. Later he had visions of scrolls of complex mathematical content unfolding before his eyes.^[106] He often said,⁷² "An equation for me has no meaning unless it expresses a thought of God."^[107]

Hardy cites Ramanujan as remarking that all religions seemed equally true to him.^[108] Hardy further argued that Ramanujan's religious belief had been romanticised by Westerners⁷³ and overstated—in reference to his belief, not practice—by Indian biographers. At the same time, he remarked on Ramanujan's strict vegetarianism.^[109]

Similarly, in an interview with Frontline, Berndt said, "Many people falsely promulgate mystical powers to Ramanujan's mathematical thinking⁷⁴. It is not true. He has meticulously recorded every result in his three notebooks," further speculating that Ramanujan worked out intermediate results on slate that he could not afford⁷⁵ the paper to record more permanently.^[8]

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