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A Fuzzy Replenishment Policy for Non-instantaneous Deteriorating Inventory System under Partial Backlogging and Inflation Effect

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ABSTRACT: This paper explores an Fuzzy inventory model for Non-instantaneous deteriorating items with time varying demand under the effect of inflation. In the proposed model, shortages are allowed and or partially backlogged assuming that backlogging rate varies inversely as the waiting time for the next replenishment. Our goal is to minimize the total Fuzzy cost function with respect to optimal order quantity and optimal interval of the total cost function over a finite planning horizon. All the inventory costs involved here are taken as pentagonal fuzzy number. Graded mean representation method is used to defuzzify the model. The model is illustrated with the help of numerical examples. Sensitivity analysis of the optimal solution with respect to various parameters of the system is carried out and the results are discussed in detail.

KEYWORDS AND PHRASES: Linear time dependent demand, Inflation, Partial backlogging, Non-instantaneous deterioration, pentagonal fuzzy numbers.

I. INTRODUCTION

DETERIORATION plays a significant role in many inventory system. Deterioration is defined as decay, damage, spoilage, evaporation ,obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decrease usefulness. Most physical good undergo decay or deterioration over time, examples being medicines, volatile liquids , blood banks and so on. So decay or deterioration of physical goods in stock is very realistic factor and there is big need to consider this in inventory modeling.

Many researcher assume that the deterioration of an item in an inventory starts from the instant of their arrival in stock. Infact most goods would have span of maintaining quality or original condition.(e.g Vegetables,Fruits,Fish,Meat and so on), namely, during that period there is no deterioration is occurring defined as "non-instantaneous deterioration". In the real world, this type of phenomenon exist commonly such as first hand vegetables and fruits have short span of maintaining fresh quality, in which there is almost no spoilage. After words, some of the items will start to decay. For this kind of items, the assumption that the deterioration is starts from the instant of arrival in stock may cause retailer to make in appropriate replenishment policies due to over value the total annual relevant inventory cost. Therefore, in the field of inventory management, it is necessary to consider the inventory problems for non-instantaneous deteriorating items.

In recent years, inventory problems for deteriorating items have been widely studied after Ghare and Schrader [4] .They presented an EOQ model for an exponentially decaying inventory. Later Covert and Philip [3] formulated the model with variable deterioration rate with two-parameter Weibull distribution. Philip [9] then developed the inventory model with a three-parameter Weibull distribution rate without shortages. Shah [11] extended Philip's model and considered that shortage was allowed. Goyal and Giri [14] provided a detailed review of deteriorating inventory literatures. Sana, Goyal and Chaudhuri [10] developed a production inventory model for deteriorating items. In all the above literatures, almost all the inventory models for deteriorating items assume that the deterioration occurs as soon as the retailer receives the commodities. However, in real life, most of the goods would

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have a span of maintaining quality or the original condition, for some period. That is during that period there was no deterioration occurring. We term the phenomenon as "non - instantaneous deterioration". Recently, Wu et.al. [13] developed an inventory model for non-instantaneous deteriorating items with stock–dependent demand.

Furthermore, when the shortages occur, it is assumed that it is either completely backlogged or completely lost. But practically some customers are willing to wait for backorder and others would turn to buy from other sellers. Researchers such as Park [8], Hollier and Mak [6] and Wee [12] developed inventory models with partial backorders. Goyal and Giri [5] developed production inventory model with shortages partially backlogged.

In all the models mentioned above, the inflation and time value of money were disregarded. It has happened most because of the belief that the inflation and the time value of money would not influence the inventory policy to any significant degree. However, in the last several years most of the countries have suffered from large-scale inflation and sharp decline in the purchasing power of money. As a result, while determining the optimal inventory policy, the effects of inflation and time value of money cannot be ignored. Recently Hou [7] developed an inventory model for deteriorating items with stock dependent demand under inflation. He considered that shortages are completely backordered. R.Udhayakumar et.al have developed a model for Non-instantaneous deteriorating inventory system with partial backlogging.

To fit into realistic circumstances we have developed a finite planning horizon fuzzy inventory model for non-instantaneous deteriorating items with time-dependent consumption rate. In which the Deterioration is a Weibull two parameter distribution and shortages are allowed and partially backlogged. In addition, the effects of inflation and time value of money on replenishment policy under instantaneous replenishment with zero lead-time are also considered. All the inventory cost involved here are taken as pentagonal fuzzy numbers. Graded mean representation method is used for defuzzification. An optimization frame work is presented to derive optimal replenishment policy when the present value of total cost is minimized. Numerical examples are provided to illustrate the optimization procedure. In addition, the sensitivity analysis of the optimal solution with respect to parameters of the system is carried out.

II. FUZZY PRELIMINARIES

Definition 1

Let X denotes a universal set. Then the fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}}(x): X \to [0,1]$ which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1], to each element x∈ X where the value of $\mu_{\tilde{A}}(x)$ at x shows the grade of membership of x

Definition 2

A fuzzy set $\tilde{A}$ on R is convex if $\tilde{A}\,(\lambda x_1 + (1-\lambda) x_2) \geq min\left[\tilde{A}(x_1), \tilde{A}(x_2)\right]$ for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$.

Definition 3

A fuzzy set $\tilde{A}$ in the universe of discourse X is called as a fuzzy number in the universe of discourse X.

Definition 4

A pentagonal fuzzy number (PFN)[9] $\tilde{A} = (a, b, c, d, e)$ is represented with membership function $\mu_{\tilde{A}}$

As: $$\mu_{\tilde{A}}(x) = \begin{cases} L_1(x) = \dfrac{x-a}{b-a}, & a \leq x \leq b \\ L_2(x) = \dfrac{x-b}{c-b}, & b \leq x \leq c \\ 1 \quad , & x = c \\ R_1(x) = \dfrac{d-x}{d-c}, & c \leq x \leq d \\ L_2(x) = \dfrac{e-x}{e-d}, & d \leq x \leq e \\ 0 \quad , & otherwise \end{cases}$$

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The a-cut of $\tilde{A}=(a,b,c,d), 0\le a\le 1$ is $A(\alpha)=[A_L(\alpha), A_R(\alpha)]$ where $A_{L_1}(\alpha)=a+(b-a)\alpha=L_1^{-1}(\alpha)$,

$A_{L2}(\alpha)=b+(c-b)\alpha=L_2^{-1}(\alpha)$ and

$A_{R_1}(\alpha)=d-(d-c)\alpha=R_1^{-1}(\alpha)$ $\quad A_{R_2}(\alpha)=e-(e-d)\alpha=R_2^{-1}(\alpha)$

So

$$L^{-1}(\alpha)=\frac{L_1^{-1}(\alpha)+L_2^{-1}(\alpha)}{2}=\frac{a+(b-a)\alpha+b+(c-b)\alpha}{2}=\frac{a+b+(b-a+c-b)\alpha}{2}=\frac{a+b+(c-a)\alpha}{2}$$

$$R^{-1}(\alpha)=\frac{R_1^{-1}(\alpha)+R_2^{-1}(\alpha)}{2}=\frac{d-(d-c)\alpha+e-(e-d)\alpha}{2}=\frac{d+e-(d-c+e-d)\alpha}{2}=\frac{d+e-(e-c)\alpha}{2}$$

Definition 5:

If $\tilde{A}=(a,b,c,d,e)$ is a pentagonal fuzzy number then the graded mean integration representation of $\tilde{A}$

Is defined as $P(\tilde{A})=\dfrac{\int_0^{W_A} h\left(\dfrac{L^{-1}(h)+R^{-1}(h)}{2}\right)dh}{\int_0^{W_A} hdh}$ with $0\le h\le W_A$ and $0\le W_A\le 1$

$$P(\tilde{A})=\frac{1}{2}\frac{\int_0^1 h\left(\dfrac{a+b+(c-a)h}{2}+\dfrac{d+e-(e-c)h}{2}\right)dh}{\int_0^1 hdh}=\frac{a+3b+4c+3d+e}{12}$$

III. ASSUMPTIONS AND NOTATIONS

The following assumptions are made:

1. The Consumption rate D(*t*) at time t is assumed to be $D(t)=\begin{cases}a+bt, & 0\le t\le t_d, t_d\le t\le t_j\\ B, & t_j\le t\le T\end{cases}$ where a is a positive constant, b is the time-dependent consumption rate parameter, $0\le b\le 1$
2. The replenishment rate is infinite and lead time is zero.
3. The system operates for a prescribed period of a planning horizon.
4. Shortages are allowed and the backlogged rate is defined to be $\dfrac{1}{1+\delta(T-t)}$ when inventory is negative. The backlogging parameter δ is a positive constant and
5. It is assumed that during certain period of time the product has no deterioration (i.e., fresh product time). After this period, a fraction, θ (0<θ<1), of the n-hand inventory deteriorates according to Weibull two parameter distribution. i.e. $\theta(t)=\alpha\beta t^{\beta-1}$ is the Weibull tow parameter deterioration where 0<α<1, β>0 are called scale and shape parameter.
6. Product transactions are followed by instantaneous cash flow.

The following notations are used:

r - discount rate, representing the time value of money.

i - inflation rate

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R	- r-i, representing the net discount rate of inflation (which is constant.)
H	- planning horizon.
T	- Replenishiment cycle.
M	- the number of replenishment during the planning horizon, $m=H/T$
T_j	- the total time that elapsed up to and including the j^{th} replenishment cycle ($j=1,2,\ldots,m$). where $T_0=0$, $T_1=T$, ..., $T_m=H$.
t_j	- the time at which the inventory level in the j^{th} replenishment cycle drops to zero ($j=1,2,\ldots,m$).
t_d	- the length of time in which the product has no deterioration (Fresh produc
$T_j - t_j$	- time period when shortage occurs ($j=1,2,\ldots,m$).
Q	- the 2^{nd}, 3^{rd},…, m th replenishment lot size
I_m	- maximum Inventory level.
I_b	- maximum amount of shortage demand to be backlogged.
$\tilde{K}$	- Fuzzy Ordering cost of inventory, $ per order.
$I(t)$	- The inventory level at time t.
θ	- Deterioration rate, a fraction of the on hand inventory, follows weibull two parameter distribution.
$\tilde{p}$	- Fuzzy Purchase cost, $ per unit
$\tilde{h}$	- Fuzzy Holding cost excluding interest charges, $ per unit/ year.
$\tilde{s}$	- Fuzzy Shortage cost, $ per unit/year
$\tilde{\pi}$	- Fuzzy Opportunity cost due to lost sales, $ per unit
$T\tilde{R}C(t_1,T)$	- The Fuzzy average total inventory cost per replenishment.
$TC_{DG}(t_1,T$	- Defuzzified average total inventory cost per unit time.
$T\tilde{C}(t_1,T)$	- The Fuzzy average total inventory cost for planning horizon H

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IV. MODEL FORMULATION

Suppose that the planning horizon H is divided into m equal parts of length $T=H/m$. Hence the reorder times over the planning horizon H are $T_j=jT$ $(j=0,1,2,\ldots,m)$. When the inventory is positive, demand rate is dependent on linear function of time, whereas for negative inventory, the demand is partially backlogged. The period for which there is no-shortage in each interval $[jT,(j+1)T]$ is a fraction of the scheduling period T and is equal to kT $(0<k<1)$. Shortages occur at time $t_j=(k+j-1)T$, $(j=1,2,\ldots,m)$ and are accumulated until time $t=jT$ $(j=1,2,\ldots,m)$ before they are backordered. This model is illustrated in Fig.1. The first replenishment lot size of I_m is replenished at $T_0=0$. During the interval $[0,t_d]$, the inventory level decreases due to time-dependent demand rate. The inventory level drops to zero due to time-dependent demand and deterioration during the time interval $[t_d,t_1]$. During the interval $[t_1, T]$, shortages occur and are accumulated until $t=T$ before they are partially backlogged. $I_1(t)$ denotes the inventory level at time t $(0\le t\le t_d)$ in the product which the product has no deterioration. $I_2(t)$ is the inventory level at time t $(t_d\le t\le t_1)$ in which the product has no shortage.

Therefore the inventory system at any time t can be represented by the following differential equations:

$$\frac{dI_1(t)}{dt}=-(a+bt) \qquad 0\le t\le t_d \tag{1}$$

$$\frac{dI_2(t)}{dt}+\theta I_2(t)=-(a+bt) \qquad t_d\le t\le t_j \tag{2}$$

$$\frac{dI_3(t)}{dt}=-\frac{B}{1+\delta(T-t)} \qquad t_j\le t\le T \tag{3}$$

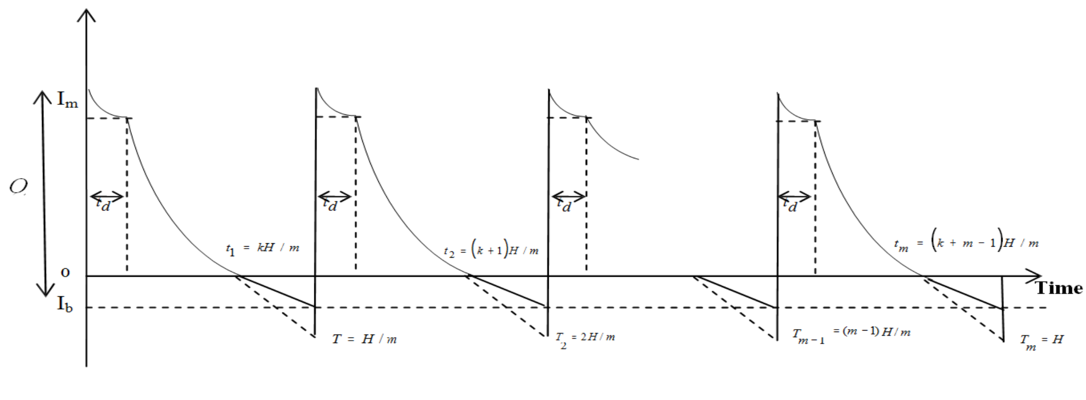

Fig.1 Graphical representation of inventory system

The solutions of the above differential equations after applying the boundary conditions $I_1(0)=I_m$, $I_2(t_1)=0$; $I_3(t_1)=0$, $I_3(T)=I_b$

$$I_1(t)=-at+\frac{bt^2}{2}+I_m \qquad 0\le t\le t_d \tag{4}$$

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$$I_2(t)=e^{-\alpha t^{\beta}}\left\{a(t_1-t)+\frac{a\alpha(t_1^{\beta+1}-t^{\beta+1})}{\beta+1}+\frac{b^2}{2}(t_1^{2}-t^{2})+b\alpha\frac{(t_1^{\beta+2}-t^{\beta+2})}{\beta+2},\quad t_d\le t\le t_1\right\} \qquad (5)$$

$$I_3(t)=-\frac{B}{\delta}\{\log\,[1+\delta(T-t_1)]-\log\,[1+\delta(T-t)]\}\qquad t_1\le t\le T \qquad (6)$$

Considering the continuity of I(t) at $t=t_d$ it follows that $I_1(t_d)=I_2(t_d)$ which implies that

$$I_m=e^{-\alpha t_d^{\beta}}\left\{a(t_1-t_d)+\frac{a\alpha(t_1^{\beta+1}-t_d^{\beta+1})}{\beta+1}+\frac{b^2}{2}(t_1^{2}-t_d^{2})+b\alpha\frac{(t_1^{\beta+2}-t_d^{\beta+2})}{\beta+2}\right\}+at_d-\frac{bt_d^{2}}{2} \qquad (7)$$

At $t=T$ the maximum amount of shortage demand to be backlogged during first replenishment cycle is

$$I_b=-\frac{B}{\delta}\left\{\log\left[1+\delta\frac{H}{m}(1-k)\right]\right\} \qquad (8)$$

And also the maximum inventory level during first replenishment cycle are

$$I_m=e^{-\alpha t_d^{\beta}}\left\{a(\frac{kH}{m}-t_d)+\frac{a\alpha((\frac{kH}{m})^{\beta+1}-t_d^{\beta+1})}{\beta+1}+\frac{b^2}{2}\left(\left(\frac{kH}{m}\right)^2-t_d^{2}\right)+b\alpha\frac{\left(\left(\frac{kH}{m}\right)^{\beta+2}-t_d^{\beta+2}\right)}{\beta+2}\right\}+at_d-\frac{bt_d^{2}}{2} \qquad (9)$$

There are m cycles during the planning horizon. Since, inventory is assumed to start and end at zero, an extra replenishment at $T_m=H$ is required to satisfy the backorders of the last cycle in the planning horizon. Therefore there are $m+1$ replenishments in the entire planning horizon H. The first replenishment lot size is I_m . The 2nd , 3rd ,…mth replenishment lot size is

$$Q=I_m+I_b \qquad (10)$$

And the last or (m+1)th replenishment lot size is I_b Since replenishment in each cycle is done at the start of each cycle, the present value of fuzzy ordering cost/ setup cost during the first cycle is $\tilde{K}$

Inventory occurs during period $0\le t\le t_d\,,t_d\le t\le t_1$ therefore the present value of fuzzy holding cost during the first replenishment cycle is

$$\text{HC}=\tilde{h}\int_0^{t_d}I_1(t)e^{-Rt}dt+\tilde{h}\int_{t_d}^{t_1}I_2(t)e^{-Rt}dt$$

$$=\tilde{h}\left\{-\left(at_d+\frac{bt_d^{2}}{2}\right)\frac{e^{-Rt_d}}{R}+\left(e^{-\alpha t_d^{\beta}}\left\{a(\frac{kH}{m}-t_d)+\frac{a\alpha\left(\left(\frac{kH}{m}\right)^{\beta+1}-t_d^{\beta+1}\right)}{\beta+1}+\frac{b^2}{2}\left(\left(\frac{kH}{m}\right)^2-t_d^{2}\right)+b\alpha\frac{\left(\left(\frac{kH}{m}\right)^{\beta+2}-t_d^{\beta+2}\right)}{\beta+2}\right\}\right.\right.$$

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$$+at_d-\frac{bt_d^{\,2}}{2}\Bigg)\left(\frac{\left(1-e^{-Rt_d}\right)}{R}\right)-\frac{a}{2}\left(\left(\frac{kH}{m}\right)^2-t_d^{\,2}\right)+\frac{\alpha a}{\beta+2}\left(\left(\frac{kH}{m}\right)^{\beta+2}-t_d^{\,\beta+2}\right)+\frac{\alpha b^2}{2(\beta+3)}\left(\left(\frac{kH}{m}\right)^{\beta+3}-t_d^{\,\beta+3}\right)\Bigg\} \tag{11}$$

Fuzzy Deterioration cost in $(0, T_1)$ denoted by DC is given by

$$DC=\tilde{p}\int_{t_d}^{t_1}\theta(t)I_2(t)e^{-Rt}dt=\tilde{p}\int_{t_d}^{t_1}\alpha\beta t^{\beta-1}I_2(t)e^{-Rt}dt$$

$$=\tilde{p}\alpha\beta\left\{-\frac{a}{\beta+1}\left(\left(\frac{kH}{m}\right)^{\beta+1}-t_d^{\,\beta+1}\right)+\frac{\alpha a}{2(\beta+1)}\left(\left(\frac{kH}{m}\right)^{2\beta+1}-t_d^{\,2\beta+1}\right)+\frac{\alpha b^2}{2(2\beta+2)}\left(\left(\frac{kH}{m}\right)^{2\beta+2}-t_d^{\,2\beta+2}\right)\right\} \tag{12}$$

The Fuzzy shortage cost in the interval $[t_1, T)$ denoted by SC is given by

$$SC=\tilde{s}\int_{t_1}^{T}I_3(t)dt$$

$$=\frac{B}{R\delta^2}\tilde{s}\left\{\log[1+\delta(T-\frac{kH}{m})]\left\{\delta(e^{-R\frac{H}{m}}+1+\frac{H}{m})+1\right\}-\delta(T-\frac{kH}{m})\right\} \tag{13}$$

The Fuzzy opportunity cost due to lost sales denoted by OC is given by

$$OC=\tilde{\pi}\int_{t_1}^{T}\left(B-\frac{B}{1+\delta(T-t)}\right)dt$$

$$OC=\frac{\tilde{\pi}B}{\delta}\left\{\delta\frac{H}{m}(1-k)-\log[1+\delta\frac{H}{m}(1-k)]\right\} \tag{14}$$

Fuzzy purchasing cost for the first replenishment cycle is given by

$$PC=\tilde{p}I_m+\tilde{p}e^{-Rt}I_b$$

$$PC=\tilde{p}\left(e^{-\alpha t_d^{\beta}}\left\{a(\frac{kH}{m}-t_d)+\frac{a\alpha((\frac{kH}{m})^{\beta+1}-t_d^{\,\beta+1})}{\beta+1}+\frac{b^2}{2}\left(\left(\frac{kH}{m}\right)^2-t_d^{\,2}\right)+b\alpha\frac{\left(\left(\frac{kH}{m}\right)^{\beta+2}-t_d^{\,\beta+2}\right)}{\beta+2}\right\}+at_d-\frac{bt_d^{\,2}}{2}\right)$$

$$+\tilde{p}e^{-RT}\left(-\frac{B}{\delta}\left\{\log\left[1+\delta\frac{H}{m}(1-k)\right]\right\}\right) \tag{15}$$

Consequently, the present value of fuzzy total cost of system during the first replenishment cycle can be formulated as

$$\tilde{TRC}=\tilde{K}+HC+DC+SC+OC+PC \tag{16}$$

So, the present value of fuzzy total cost of the system over a finite planning horizon

H is $$\tilde{TC}(m,k)=\sum_{j=0}^{m-1}\tilde{TRC}e^{-RT_j}+Ke^{-RH}=\tilde{TRC}(\frac{1-e^{-RH}}{1-e^{-RH/M}})+Ae^{-RH} \tag{17}$$

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Where T=H/m and $T\tilde{R}C$ is got by substituting the equation 11 to 15 in equation 17

On simplification we get

$$T\tilde{C}(m,k) = \Bigg\{\tilde{K} +$$

$$\tilde{h}\left\{-\left(at_d + \frac{bt_d^{\,2}}{2}\right)\frac{e^{-Rt_d}}{R} + \left(e^{-\alpha t_d^{\beta}}\left\{a(\frac{kH}{m}-t_d) + \frac{a\alpha\left(\left(\frac{kH}{m}\right)^{\beta+1} - t_d^{\,\beta+1}\right)}{\beta+1} + \frac{b^2}{2}\left(\left(\frac{kH}{m}\right)^2 - t_d^{\,2}\right) + b\alpha\frac{\left(\left(\frac{kH}{m}\right)^{\beta+2} - t_d^{\,\beta+2}\right)}{\beta+2}\right\} + at_d - \frac{bt_d^{\,2}}{2}\right)\left(\frac{\left(1-e^{-Rt_d}\right)}{R}\right)$$

$$-\frac{a}{2}\left(\left(\frac{kH}{m}\right)^2 - t_d^{\,2}\right) + \frac{\alpha a}{\beta+2}\left(\left(\frac{kH}{m}\right)^{\beta+2} - t_d^{\,\beta+2}\right) + \frac{\alpha b^2}{2(\beta+3)}\left(\left(\frac{kH}{m}\right)^{\beta+3} - t_d^{\,\beta+3}\right)\right\}$$

$$+\tilde{p}\alpha\beta\left\{-\frac{a}{\beta+1}\left(\left(\frac{kH}{m}\right)^{\beta+1} - t_d^{\,2}\right) + \frac{\alpha a}{2(\beta+1)}\left(\left(\frac{kH}{m}\right)^{2\beta+1} - t_d^{\,2\beta+1}\right) + \frac{\alpha b^2}{2(2\beta+2)}\left(\left(\frac{kH}{m}\right)^{2\beta+2} - t_d^{\,2\beta+2}\right)\right\}$$

$$+\frac{B}{R\delta^2}\tilde{s}\left\{\log[1+\delta\frac{H}{m}(1-k)]\left\{\delta(e^{-R\frac{H}{m}}+1+\frac{H}{m})+1\right\} - \delta\frac{H}{m}(1-k)\right\} + \frac{\tilde{\pi}B}{\delta}\left\{\delta\frac{H}{m}(1-k) - \log[1+\delta\frac{H}{m}(1-k)]\right\}$$

$$+\tilde{p}\left(e^{-\alpha t_d^{\beta}}\left\{a(\frac{kH}{m}-t_d) + \frac{a\alpha((\frac{kH}{m})^{\beta+1} - t_d^{\,\beta+1})}{\beta+1} + \frac{b^2}{2}\left(\left(\frac{kH}{m}\right)^2 - t_d^{\,2}\right) + b\alpha\frac{\left(\left(\frac{kH}{m}\right)^{\beta+2} - t_d^{\,\beta+2}\right)}{\beta+2}\right\} + at_d - \frac{bt_d^{\,2}}{2}\right)$$

$$+\tilde{p}e^{-RT}\left(-\frac{B}{\delta}\left\{\log\left[1+\delta\frac{H}{m}(1-k)\right]\right\}\right)\Bigg\}\left(\frac{1-e^{-RH}}{1-e^{-RH/M}}\right) + \tilde{K}e^{-RH}$$

Equation (17) is convex with respect to m, k i.e.

$$\frac{\partial T\tilde{C}(m,k)}{\partial m} = 0 \qquad \text{...........(18)}$$

and $$\frac{\partial T\tilde{C}(m,k)}{\partial k} = 0 \qquad \text{...........(19)}$$

provided they satisfy the sufficient conditions

$$\left[\frac{\partial T\tilde{C}(m,k)}{\partial m}\right]_{at(m^*,k^*)} > 0,$$

$$\left[\frac{\partial T\tilde{C}(m,k)}{\partial k}\right]_{at(m^*,k^*)} > 0$$

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and $$\left[\frac{\left(\frac{\partial \tilde{TC}(m,k)}{\partial m}\right)\left(\frac{\partial \tilde{TC}(m,k)}{\partial k}\right)-}{\left[\frac{\partial^2 \tilde{TC}(m,k)}{\partial m \partial k}\right]^2}\right]_{at\ (m^*,k^*)} > 0$$

V. NUMERICAL EXAMPLES

To illustrate the preceding theory the following examples are presented.

Example-1: Expressing the fuzzy inventory costs (pentagonal fuzzy numbers) in relevant units. Let $\tilde{K}$ =(100,150,200,250,300), $\tilde{p}$ =(10,15,20,25,30), $\tilde{h}$ =(1,3,5,7,9), t_d =0.08, $\tilde{s}$ =(5,10,15,20,25), $\tilde{\pi}$ =(8,10,12,14,16), a =100, b=.09, θ=0.08, α=0.1, β=2, R =0.2 B=30, H=10,δ =.56 From the results presented in Table.1 we see that when the number of replenishment m=12, the total cost $TC_{DG}(m,k)$ (defuzzified value) becomes minimum. Hence the optimal values of m and k are m^* =12, k^*=0.1937 respectively, the minimum defuzzified total cost $TC_{DG}(m^*, k^*)$,=4021. We then have T^*=H/ m^*=10/12=0.8333, t_1^*=k^*H/m^*=0.1614, Q^*=100.78

Example-2: Expressing the fuzzy inventory costs (pentagonal fuzzy numbers) in relevant units. Let $\tilde{K}$ =(200,250,300,350,400), $\tilde{p}$ =(11,13,15,17,19), $\tilde{h}$ =(3,5,7,9,11), t_d =0.08, $\tilde{s}$ =(10,15,20,25,30), $\tilde{\pi}$ =(6,8,10,12,14), a =80, b=.025, θ=0.08, α=0.1, β=2, R =0.2 B=15, δ =.56, H=10.

From the results presented in Table.2 we see that when the number of replenishment m=12, the total cost $TC_{DG}(m,k)$ (defuzzified value) becomes minimum. Hence the optimal values of m and k are m^* =9, k^*=0.2670 respectively, the minimum defuzzified total cost $TC_{DG}(m^*, k^*)$=2415. We then have T^*=H/ m^*=10/9=1.111, t_1^*=k^*H/m^*=0.2966, Q^*=93.60.

We now study the effects of changing the parameters θ, R, t_d and δ on the optimal replenishment policy of the Example 1. The results are summarized in Table 3. Based on Table 3, the observations can be made as follows:

- When the deterioration rate θ is increasing, the optimal cost is increasing and the order quantity is decreasing.
- When the net discount rate of inflation R is increasing, the optimal cost is decreasing
- When the length of fresh product time 't_d ' is increasing, the total cost is decreasing And the order quantity is increasing.
- When the backlogging rate δ is increasing, the total cost and the order quantity is increasing.

VI. SENSITIVITY ANALYSIS

The change in the values of parameters may happen due to uncertainties in any decision-making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in decision-making. Using the numerical examples given in the preceding section, the sensitivity analysis of various parameters has been done. Let the estimated values of order quantity and total cost (defuzzified) be Q' and TC' respectively, while the true value of these are Q and TC. The results of sensitivity analysis are summarized in Tables 4 and 5. The following inferences can be made.

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1. When the consumption rate (a) decreases or increases the ordering quantity (Q) and the present value of total cost (TC) will also decrease or increase. Similarly, the ordering quantity (Q) and the present value of total cost (TC) will also decrease or increase as the ordering cost (A) decrease or increase. That is changes in (a) and (A) will lead to the positive changes in (Q) and (TC).
2. The change in the stock-dependent consumption rate (b) leads to a positive change in the present value of the total cost (TC).
3. The change in net discount rate of inflation (R) leads to a negative change in the present value of total cost (TC).
4. The change in deterioration rate (θ) leads to a negative change on the ordering quantity (Q) and a positive change in the present value of the total cost (TC). That is Q decreases with the increase of (θ).Whereas (TC) increases with the increase of (θ).
5. Increasing the fresh product time (t_d) increases the order quantity (Q) and decreases the total cost (TC).
6. When the backlogging parameter (δ) increases the ordering quantity (Q) and the present value of total cost (TC) increases. That is change in (δ) leads to a positive change in (Q) and (TC).
7. Changes in carrying cost (h) and shortage cost (s) result in a positive change in the present value of total cost (TC) and a negative change in the ordering quantity (Q).
8. When the opportunity cost (π) increases, the ordering quantity (Q) and the present value of total cost (TC) increases.
9. Change in purchase cost (p) leads to positive change in the present value of total cost(TC) and negative change in the ordering quantity (Q).
10. The present value of total cost (TC) is more sensitive to the consumption rate (a), the unit purchase cost (p) and the net discount rate of inflation (R) as compared to other parameters.
11. Tables 4 and 5 imply that the effect of (R) on (TC) is quite significant for 50% over or under estimation of (R). It implies that the effect of inflation and time value of money on present value of total cost is significant.

Table 1
Optimal solution with shortages

M	*k*(*m*)	*T*	*Q*	*TC*(*m*,*k*)
1	0.7180	10.0000	888.20	7557
2	0.6829	5.0000	436.52	6657
3	0.6650	3.3333	229.98	5189
4	0.5750	2.5000	227.48	4723
5	0.5139	2.0000	182.33	4497
6	0.4614	1.6667	156.04	4356
7	0.4130	1.4286	138.65	4259
8	0.3668	1.2500	126.21	4188
9	0.3222	1.1111	116.88	4133
10	0.2786	1.0000	109.59	4089
11	0.2359	0.9091	103.77	4052
12*	0.1927*	0.8333*	100.78*	4021*
13	0.1928	0.7692	100.12	4057

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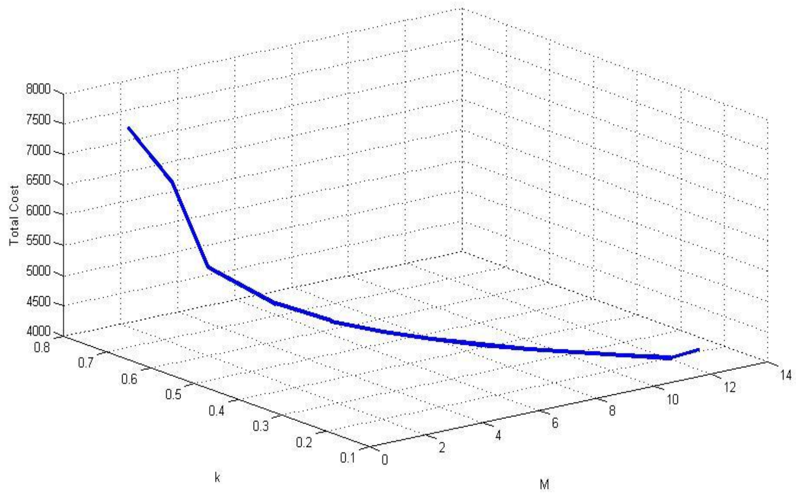
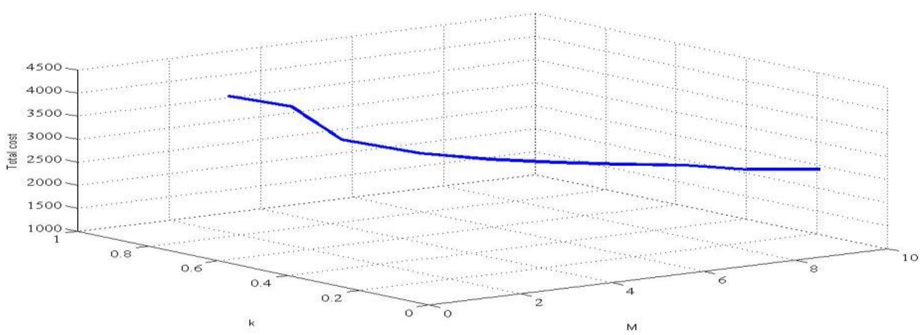


Graph for Table 1

Table 2
Optimal solution with shortages

M	k(m)	T	Q	TC(m,k)
1	0.7008	10.0000	1440.50	4295
2	0.6507	5.0000	331.10	4027
3	0.6373	3.3333	180.16	3202
4	0.5446	2.5000	171.57	2935
5	0.4783	2.0000	165.96	2790
6	0.4208	1.6667	145.05	2690
7	0.3674	1.4286	131.12	2614
8	0.3164	1.2500	101.13	2550
9*	0.2670*	1.1111*	93.60*	2415*
10	0.1913	1.0000	92.71	2418



Graph for Table 2

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Table 3

Effects of changing the parameter θ, R, t_d, δ on the optimal replenishment policy

parameter	parameter value	*m*	*k*	*T*	*Q*	*TC*(*m*,*k*)
θ	0.04	12	0.1901	0.8333	100.14	4009
	0.06	12	0.1930	0.8333	100.52	4015
	0.08	12	0.1937	0.8333	100.78	4021
	0.10	12	0.1934	0.8333	100.80	4023
R	0.10	12	0.2144	0.8333	117.80	8716
	0.15	12	0.2044	0.8333	116.40	6243
	0.20	12	0.1937	0.8333	100.78	4021
	0.25	12	0.1734	0.8333	100.20	3997
t_d	0.0417	12	0.1913	0.8333	100.70	4025
	0.0625	12	0.1926	0.8333	100.74	4023
	0.0833	12	0.1937	0.8333	100.78	4021
	0.1041	12	0.1952	0.8333	100.84	4019
δ	0.28	9	0.1459	1.1111	98.04	4017
	0.42	11	0.0822	0.9091	100.45	4018
	0.56	12	0.1937	0.8333	100.48	4021
	0.70	13	0.2808	0.7692	120.14	4134

Table 4

Sensitivity analysis for Example 1 with respect to various parameters on order quantity and total cost for Time-dependent consumption rate model.

Parameter		Percentage of under estimation and over estimation of parameter				
		-50%	-25%	0%	25%	50%
a	Q'/Q	0.7795	0.8540	1	1.1120	1.1994
	TC'/TC	0.5728	0.7902	1	1.2045	1.4051
b	Q'/Q	1.0096	1.0047	1	0.9955	0.9913
	TC'/TC	0.9981	0.9991	1	1.0008	1.0016
R	Q'/Q	1.0313	1.0154	1	0.9850	0.9703
	TC'/TC	1.4620	1.1985	1	0.8481	0.7300
θ	Q'/Q	1.0005	1.0003	1	0.9997	0.9995
	TC'/TC	0.9999	1.0000	1	1.0001	1.0002
t_d	Q'/Q	0.9999	0.9999	1	1.0001	1.0002
	TC'/TC	1.0002	1.0001	1	1.0000	0.9999

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$\tilde{K}$	Q'/Q	0.6618	0.7996	1	1.1386	1.3179
	TC/TC	0.8988	0.9537	1	1.0409	1.0787
δ	Q'/Q	0.5832	0.8396	1	1.0972	1.2710
	TC/TC	0.8291	0.9235	1	1.0638	1.1180
$\tilde{p}$	Q'/Q	1.1881	1.0876	1	0.8162	0.7512
	TC/TC	0.7503	0.8823	1	1.1051	1.1976
$\tilde{h}$	Q'/Q	1.0519	1.0224	1	0.9823	0.9681
	TC/TC	0.9940	0.9974	1	1.0021	1.0038
$\tilde{s}$	Q'/Q	1.3242	1.0581	1	0.9453	0.8945
	TC/TC	0.8800	0.9494	1	1.0393	1.0712
$\tilde{\pi}$	Q'/Q	0.8200	0.9647	1	1.0354	1.0709
	TC/TC	0.8956	0.9495	1	1.0480	1.0936

Table 5

Sensitivity analysis for Example 2 with respect to various parameters on order quantity and total cost for Time-dependent consumption rate model.

Parameter		Percentage of under estimation and over estimation of parameter				
		-50%	-25%	0%	25%	50%
a	Q'/Q	0.7795	0.8540	1	1.1120	1.1994
	TC/TC	0.5728	0.7902	1	1.2045	1.4051
b	Q'/Q	1.0096	1.0047	1	0.9955	0.9913
	TC/TC	0.9981	0.9991	1	1.0008	1.0016
R	Q'/Q	1.0313	1.0154	1	0.9850	0.9703
	TC/TC	1.4620	1.1985	1	0.8481	0.7300
θ	Q'/Q	1.0005	1.0003	1	0.9997	0.9995
	TC/TC	0.9999	1.0000	1	1.0001	1.0002
t_d	Q'/Q	0.9999	0.9999	1	1.0001	1.0002
	TC/TC	1.0002	1.0001	1	1.0000	0.9999
$\tilde{K}$	Q'/Q	0.6618	0.7996	1	1.1386	1.3179
	TC/TC	0.8988	0.9537	1	1.0409	1.0787

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δ	Q'/Q	0.5832	0.8396	1	1.0972	1.2710
	TC'/TC	0.8291	0.9235	1	1.0638	1.1180
$\tilde{p}$	Q'/Q	1.1881	1.0876	1	0.8162	0.7512
	TC'/TC	0.7503	0.8823	1	1.1051	1.1976
$\tilde{h}$	Q'/Q	1.0519	1.0224	1	0.9823	0.9681
	TC'/TC	0.9940	0.9974	1	1.0021	1.0038
$\tilde{s}$	Q'/Q	1.3242	1.0581	1	0.9453	0.8945
	TC'/TC	0.8800	0.9494	1	1.0393	1.0712
$\tilde{\pi}$	Q'/Q	0.8200	0.9647	1	1.0354	1.0709
	TC'/TC	0.8956	0.9495	1	1.0480	1.0936

VII. CONCLUSION

In this article, a Fuzzy inventory model has been framed for Non-instantaneous deteriorating items with time-dependent consumption rate over a finite planning horizon. Shortages are allowed and partially backlogged. Further we have considered the effects of inflation and the time value of money in formulating the Fuzzy inventory replenishment policy. Weibull deterioration is considered. All the costs involved in this model are taken as pentagonal Fuzzy numbers and defuzzification by graded mean representation method. Sensitivity analysis with respect to various parameters has been carried out. Our research results implies that, the effect of inflation and time value of money on present value of total cost is more significant and increasing the fresh product time increases the order quantity and decreases the total cost.

Thus, this model incorporates some realistic features that are likely to be associated with some kinds of inventory. The model is very useful in their retail business. It can be used for electronic components, fashionable clothes, domestic goods and other products which are more likely with the characteristics above.

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