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Integral solutions of Ternary Cubic Diophantine equation $8\alpha^2 - 5\beta^2 = 3\gamma^3$

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ABSTRACT: The ternary cubic Diophantine equation given by $8\alpha^2 - 5\beta^2 = 3\gamma^3$ is analyzed for its patterns of non-zero distinct integral solutions. A few interesting relations between the solutions and special polygonal numbers are exhibited.

KEYWORDS: Ternary cubic, integral solutions, polygonal numbers.

I. INTRODUCTION

Ternary quadratic equations are rich in variety [1-3]. For an extensive review of sizable literature and various problems, one may refer [4-18]. In this communication, we consider yet another interesting ternary cubic equation $8\alpha^2 - 5\beta^2 = 3\gamma^2$ and obtain infinitely many non-trivial integral solutions. A few interesting relations between the solutions and special polygonal numbers are presented.

II. NOTATIONS USED

- $t_{m,n}$- Polygonal number of rank 'n' with size 'm'
- $CP_{m,n}$- Centered Pyramidal number of rank 'n' with size 'm'
- Pr_n - Pronic number of rank 'n'
- P_n^m- Pyramidal number of rank 'n' with size 'm'
- $F_{m,n}$ – Figurative number of rank 'n' with size 'm'
- Gno_n – Gnomic number of rank 'n'

III. METHOD OF ANALYSIS

The Cubic Diophantine equation with three unknowns to be solved for its non zero distinct integral solutions is

$$8\alpha^2 - 5B^2 = 3\gamma^3 \quad (1)$$

We illustrate methods of obtaining non Zero distinct integer solutions to (1)

On substituting the linear transformations

$$\alpha = X_1 + 5T_1; \quad \beta = X_1 + 8T_1 \quad (2)$$

in (1), leads to

$$X_1^2 - 40T_1^2 = \gamma^3 \quad (3)$$

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Assume

$$\gamma_1 = \gamma_1(a,b) = X^2 - 40Y^2 \; ; \qquad a,b > 0 \qquad (4)$$

(3) is solved through different approaches and different patterns of solutions thus obtained for (1) are illustrated below:

Pattern I

Equation (3) can be written as

$$X_1 + \sqrt{40}\; T_1 = \left[(X + \sqrt{40}\; Y)(X - \sqrt{40}\,Y)\right]^3 \qquad (5)$$

Which is equivalent to the system of equations

$$\left.\begin{aligned} X_1 + \sqrt{40}T_1 &= \left(X_1 + \sqrt{40}\,Y\right)^3 \\ X_1 - \sqrt{40}T_1 &= \left(X_1 - \sqrt{40}\,Y\right)^3 \end{aligned}\right\} \qquad (6)$$

Equating rational and irrational parts in (6) we get

$$\left.\begin{aligned} \alpha = \alpha(X,Y) &= X^3 + 200Y^3 + 15X^2Y + 120XY^2 \\ \beta = \beta(X,Y) &= X^3 + 320Y^3 + 24X^2Y + 120XY^2 \\ \gamma = \gamma(X,Y) &= X^2 - 40Y^2 \end{aligned}\right\}$$

Properties

1. $\beta(1,Y) - \alpha(1,Y) - 60\,SO_y \equiv 0(\,mod\; 23)$
2. $\beta(X,1) - \alpha(X,1) - t_{4,3x} - 120 \equiv 0$
3. $\alpha(X\,,1) + \beta(X,1) - SO_x - 39Pr_x \equiv 520(mod\; 101)$
4. $\alpha(X,1) - CP_x^6 - 15Pr_x \equiv 200(mod\; 21)$
5. $\gamma(X,Y) - t_{4,x} - 40 \equiv 0$
6. Each of the following expression represents a nasty number
   a. $\beta(0,1) - \alpha(0,1)$
   b. $\gamma(1,2) + \gamma(1,1)$
7. $\frac{1}{12}\,\alpha(1,1)$ represents a perfect number.
8. Each of the following expression can be expressed as a difference of two square numbers
   a. $\alpha(1,1)$
   b. $\beta(2,2)$
   c. $\gamma(2,2)$
9. Each of the following expression represents a perfect square
   a. $\alpha(2,2) + \alpha(1,1) + \alpha(1,0)$
   b. $\beta(2,2) + \beta(1,0)$
   c. $\beta(0,1) + \beta(1,1) - \beta(1.0)$
   d. $\gamma(2,2) - \gamma(0,1)$
10. Each of the following expression represents a cubical integer
    a. $\alpha(0,2) - 3\alpha(0,1)$
    b. $\alpha(1,2) - \alpha(0,2) + \alpha(1,0)$
    c. $\alpha(2,0) + \alpha(1,0)$

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Pattern II

One may write (3) as

$$X_1^2 - 40\,T_1^2 \;= \gamma^3 * 1 \tag{7}$$

Write 1 as

$$1 = \frac{(7+\sqrt{40})(7-\sqrt{40})}{9} \tag{8}$$

Using (4), (5) *and* (8) *in* (7) and applying
the method of factorization and equating positive factors, we get

$$X_1 + \sqrt{40}\,T_1 = \frac{1}{3}\;(7+\sqrt{40}\;)(X+\sqrt{40}\;Y)^3 \tag{9}$$

Equating rational and irrational parts of (9), we have

$$X_1 \;= \frac{1}{3}\,(7X^3 + 1600\,Y^3 + 120\,X^2Y + 840XY^2)$$

$$T_1 = \frac{1}{3}(X^3 + 280Y^3 + 21X^2Y + 120XY^2)$$

employing (2) , the values of X and Y satisfying (1) are given by

$$\left.\begin{array}{c} \alpha = \alpha(X,Y) = 4X^3 + 1000Y^3 + 75X^2Y + 480XY^2 \\ \beta = \beta(X,Y) = 5X^3 + 1280Y^3 + 96X^2Y + 600XY^2 \\ \gamma = \gamma(X,Y) = X^2 - 40Y^2 \end{array}\right\}$$

Properties

1. $4\beta(X,1) - 5\alpha(X,1) - t_{4,3x} - 120 \equiv 0$
2. $\alpha(X,1) + \beta(X,1) - 18\,P_x^5 - 162Pr_x \;\equiv 2280(mod\,918)$
3. $4\beta(1,y) - 5\alpha(1,y) - 120CP_y^6 \equiv 0(mod\,9)$
4. $\beta(x,1) - \alpha(x,1) - 2P_x^5 \;+ 20Pr_x \equiv 280(mod\,100)$
5. $4\beta(1,y) - 5\alpha(1,y) - 60SO_y \;\equiv 0(mod\,23)$
6. Each of the following expression represents a cubical integer
    a. $\gamma(2,2) - \alpha(2,0) + \gamma(1,0)$
    b. $\alpha(3,0) - \alpha(1,0) - \beta(1,0) - \gamma(1,0)$
    c. $\gamma(3,3) - \gamma(1,3)$
7. Each of the following expression represents a perfect number
    a. $\beta(3,0) - \alpha(3,0) + \gamma(1,0)$
    b. $\beta(1,0) + \gamma(1,0)$
8. Each of the following expression represents a Nasty number
    a. $\frac{1}{2}\alpha(3,0)$
    b. $\gamma(1,1) - \gamma(1,2)$
    c. $\alpha(3,0) - 3\alpha(1,0)$
    d. $\frac{1}{2}\gamma(0,3)$
    e. $\alpha(3,0) + 3\alpha(1,0)$
    f. $\beta(3,0) - 3\beta(1,0)$
9. $\gamma(3,3)$ can be expressed as a sum of cube numbers.

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10. Each of the following can be expressed as a perfect squares
    a. $\alpha(1,0)+\beta(1,0)-\gamma(0,2)$
    b. $\beta(1,1)+\gamma(0,1)-\beta(1,0)$
    c. $\beta(1,1)-\beta(2,0)-\beta(1,0)$
    d. $\alpha(1,1)+\beta(2,0)+\gamma(1,0)$
    e. $\alpha(1,1)-\gamma(0,1)+\gamma(1,0)$

Pattern III

One may write (3) as

$$X_1^2-40\,T_1^2 = \gamma^3 * 1 \tag{10}$$

Write 1 as

$$1 = \frac{(11+\sqrt{40})(11-\sqrt{40})}{9*9} \tag{11}$$

Using (4), (5) *and* (11) *in* (10) and applying the method of factorization and equating positive factors, we get

$$X_1+\sqrt{40}\,T_1 = \frac{1}{9}\,(11+\sqrt{40}\,)(X+\sqrt{40}\,Y)^3 \tag{12}$$

Equating rational and irrational parts of (12), we have

$$X_1 = \frac{1}{9}\,(11X^3+1600\,Y^3+120\,X^2Y+1320XY^2)$$

$$T_1 = \frac{1}{9}(X^3+440Y^3+33X^2Y+120XY^2)$$

As our aim is to find integer solutions choosing X=3x , Y =3y, we obtain as follows

$$\left.\begin{aligned}\alpha = \alpha(X,Y) &= \frac{1}{9}(16X^3+3800Y^3+385X^2Y+1920XY^2)\\ \beta = \beta(X,Y) &= \frac{1}{9}(19X^3+5120Y^3+384X^2Y+2280XY^2\\ \gamma = \gamma(X,Y) &= X^2-40Y^2\end{aligned}\right\}$$

employing (2) , the values of X and Y satisfying (1) are given by

$$\left.\begin{aligned}\alpha = \alpha(x,y) &= 48x^3+11400y^3+855x^2y+5760xy^2\\ \beta = \beta(x,y) &= 57x^3+15360y^3+1152x^2y+68440xy^2\\ \gamma = \gamma(x,y) &= 9x^2-360y^2\end{aligned}\right\}$$

Properties

1. $\beta(x,1)-\alpha(x,1)-9CP_x^6-297Pr_x-Gx_0 \equiv 3960(mod\ 11)$
2. $\beta(1,y)-\alpha(1,y)-3960CP_x^6-297Pr_x-783t_{4,y}-9 \equiv 0$
3. $\beta(x,1)-\alpha(x,1)-18P_n^5-288Pr_x \equiv 3960\ (mod\ 792)$
4. $\beta(x,1)-114\,P_A^5-1095Pr_x \equiv 15360\ (mod\ 5745)$
5. $\alpha(2,y)-22800\,P_y^5-120Pr_x \equiv 384\ (mod\ 3300)$
6. Each of the following expression represents a perfect square

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a. $(0,3)+\gamma(1,0)$
b. $2\gamma(1,0)-\gamma(1,3)$
c. $\alpha(1,1)+\gamma(1,2)+\gamma(1,0)$

7. $\beta(2,2)-\gamma(3,3)$ represents a cubic number
8. $\gamma(3,3)$ can be expressed as a difference of two square numbers.

IV. CONCLUSION

In this paper, we have presented three different patterns of non- zero distinct integer solutions of ternary cubic Diophantine equation $8\alpha^2-5\beta^2=3\gamma^3$ and relations between solutions and special numbers are also obtained. To conclude, one may search for other patterns of solutions and their corresponding properties.

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