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# Study of Counter-Current Imbibition Arising In Doublephaseflow in the Context of Homogeneous Porous Media

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**ABSTRACT**:The present paper consists phenomenon of counter-current imbibition arising in double phase flow through homogeneous porous media. Different fluid has different wet abilities. Counter current phenomenon occurs due to the difference in the wetting abilities of the fluid. The solution is obtained by using well-known Finite Difference Method, Successive over-Relaxation method.

**KEYWORDS:** Homogeneous Porous media, Counter-current Imbibition, Double Phase, S.O.R.

## I. INTRODUCTION

The present paper deals with the phenomenon of counter-current imbibitionwhich occurs in two immiscible phase flow through homogeneous porous media [1]. When a porous medium is filled with some fluid which preferentially wets the medium then there is animpulsive flow of the native fluid from the medium.The solution of governing equation is obtained by a Successive over Relaxation method [23].

Verma[4] considered the presence of heterogeneity in the medium marginally. He obtained an approximate solution for Imbibitions phenomenon.He [5,6] discussed this phenomena in Cracked porous media. He alsodiscussed the existence and uniqueness of similarity of imbibition phenomenon [7]. Graham and Richardson [9], formally discussed this phenomenon.

## II. STATEMENT OF THE PROBLEM

Consider a finite cylindrical piece of homogenous porous matrix of length L. It is considered fully saturated with a native fluid. Also it is completely surrounded by an impermeable surface except one end [10]. This end is uncovered to an adjacent formation of injected fluid. An injected fluid is assumed more wetting than that of native liquid. Due to this reason there isa unstructured linear flow of injected fluid into the medium and a counter flow of the resident fluid from the medium [12]. This occurrence is known as linear counter-current Imbibition.

The governing equation to this phenomenon is a partial differential equation, whose solution is obtained by a well known Finite difference method, Successive over Relaxation method [13].

## III. MATHEMATICAL FORMULATION OF THE PROBLEM

Assuming Darcy's law, governing law of the seepage velocity of flowing fluids are written as:

$$V_i = -\frac{k_i}{\mu_i} k \frac{\partial P_i}{\partial x} \qquad (1)$$

$$V_n = -\frac{k_n}{\mu_n} k \frac{\partial P_n}{\partial x} \qquad (2)$$

Where $V_i$ and $V_n$ are seepage velocity of injected and native fluid respectively, k is the permeability of the homogeneous medium, $k_i$ and $k_n$ are relative permabilities of injected and native fluid respectively, $P_i$and $P_n$ are the pressures and $\mu_i$ and $\mu_n$ are viscosities of injected and native fluid respectively.

Continuity equations for the flowing phase are:

$$P \frac{\partial S_i}{\partial t} + \frac{\partial V_i}{\partial x} = 0 \qquad (3)$$

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$$P\frac{\partial S_n}{\partial t}+\frac{\partial V_n}{\partial x}=0 \qquad (4)$$

Where $P$ is the porosity of the medium and $S_i$ and $S_n$ are saturation of injected and native fluid respectively.
For counter-current imbibition phenomenon an analytic condition is given by

$$V_i = -V_n \qquad (5)$$

$$P_c = P_n - P_i \qquad (6)$$

From equations (1),(2) and (5), we have

$$\frac{k_n}{\mu_n}\frac{\partial P_n}{\partial x}+\frac{k_i}{\mu_i}\frac{\partial P_i}{\partial x}=0 \qquad (7)$$

Substituting (6) in(7),we have

$$\frac{k_n}{\mu_n}\left\{\frac{\partial P_c}{\partial x}+\frac{\partial P_i}{\partial x}\right\}+\frac{k_i}{\mu_i}\frac{\partial P_i}{\partial x}=0$$

$$\therefore \frac{\partial P_i}{\partial x}=\frac{-\frac{k_n}{\mu_n}}{\left\{\frac{k_n}{\mu_n}+\frac{k_i}{\mu_i}\right\}}\frac{\partial P_c}{\partial x} \qquad (8)$$

From (1) and (3), we have

$$P\frac{\partial S_i}{\partial t}-\frac{\partial}{\partial x}\left\{\frac{k_i}{\mu_i}\,k\,\frac{\partial P_i}{\partial x}\right\}=0 \qquad (9)$$

From equations (8) and (9), we get

$$P\frac{\partial S_i}{\partial t}+\frac{\partial}{\partial x}\left\{k\frac{k_n k_i}{k_n\mu_i+k_i\mu_n}\frac{dP_c}{dS_i}\frac{\partial S_i}{\partial x}\right\}=0 \qquad (10)$$

Setting $\varepsilon=\frac{k_n k_i}{k_n\mu_i+k_i\mu_n}$

It is called co-efficient of saturation which is assumed to be a constant.
Now using the condition for capillary pressure depending upon phase saturation as

$$P_c=-\beta S_i \qquad (11)$$

Equation (10) can be written as

$$P\frac{\partial S_i}{\partial t}-\frac{\partial}{\partial x}\left[k\varepsilon\beta\frac{\partial S_i}{\partial x}\right]=0 \qquad (12)$$

With $S_i(0,t)=s_0$, $S_i(L,t)=s_1$, $S_i(L,t)=0, 0\le x\le L$

$$\frac{\partial S_i}{\partial t}-k\varepsilon\beta\frac{\partial^2 S_i}{\partial x^2}=0 \qquad (13)$$

Let $\xi=\frac{x}{L}$, $T=\frac{k\varepsilon\beta}{L^2}t$

$$\frac{\partial S_i}{\partial T}-\frac{\partial^2 S_i}{\partial \xi^2}=0 \qquad (14)$$

With $S_i(0,T)=s_0$, $S_i(1,T)=s_1$, $S_i(1,T)=0, 0\le\xi\le 1$

## IV. SOLUTION TECHNIQUE

Using S.O.R. method [12] for solving (14), we have

$$s_{i_{m,n+1}}=s_{i_{m,n}}+\frac{k}{2h^2}\left(s_{i_{m+1,n}}-2s_{i_{m,n}}+s_{i_{m-1,n}}+s_{i_{m+1,n+1}}-2s_{i_{m,n+1}}+s_{i_{m-1,n+1}}\right)$$

Let $\boldsymbol{r}=\frac{\boldsymbol{k}}{\boldsymbol{h^2}}$

$$(1+r)s_{i_{m,n+1}}=s_{i_{m,n}}+\frac{r}{2}\left(s_{i_{m+1,n}}-2s_{i_{m,n}}+s_{i_{m-1,n}}+s_{i_{m+1,n+1}}+s_{i_{m-1,n+1}}\right)$$

$$\boldsymbol{\lambda_i}=s_{i_{m,n}}+\frac{r}{2}\left(s_{i_{m+1,n}}-2s_{i_{m,n}}+s_{i_{m-1,n}}\right)$$

$$s_{i_{m,n+1}}=(1-\omega)s_{i_{m,n}}+\omega\left[\frac{r}{2(1+r)}\left(s_{i_{m+1,n}}+s_{i_{m-1,n+1}}\right)+\frac{\boldsymbol{\lambda_i}}{(1+r)}\right]$$

Choose $k=0.01$, $h=0.1$, $\omega=1.67$, $s_0=0.7$, $s_1=0$

$$s_{i_{m,n+1}}=-0.67s_{i_{m,n}}+1.67\left[0.25\left(s_{i_{m+1,n}}+s_{i_{m-1,n+1}}\right)+\frac{\lambda_i}{2}\right]$$

Numerical calculations for different values of saturation at different time and different length are shown in the following table.

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| T→ | T=0.01 | T=0.02 | T=0.03 | T=0.04 | T=0.05 |
|---|---|---|---|---|---|
| ξ ↓ | Si | | | | |
| 0 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| 0.1 | 0.29225 | 0.490575 | 0.495974 | 0.543039 | 0.528736 |
| 0.2 | 0.122014 | 0.287615 | 0.348313 | 0.371022 | 0.416974 |
| 0.3 | 0.050941 | 0.154649 | 0.227849 | 0.254527 | 0.301308 |
| 0.4 | 0.021268 | 0.078998 | 0.139335 | 0.164299 | 0.210856 |
| 0.5 | 0.008879 | 0.039007 | 0.080718 | 0.099688 | 0.143073 |
| 0.6 | 0.003707 | 0.018801 | 0.04482 | 0.057467 | 0.093806 |
| 0.7 | 0.001548 | 0.0089 | 0.024068 | 0.031805 | 0.059392 |
| 0.8 | 0.000646 | 0.004154 | 0.012582 | 0.017013 | 0.036223 |
| 0.9 | 0.00027 | 0.001917 | 0.006402 | 0.008648 | 0.020729 |
| 1 | 0.000113 | 0.000838 | 0.002912 | 0.003395 | 0.010899 |

## V. GRAPHICAL REPRESENTATION

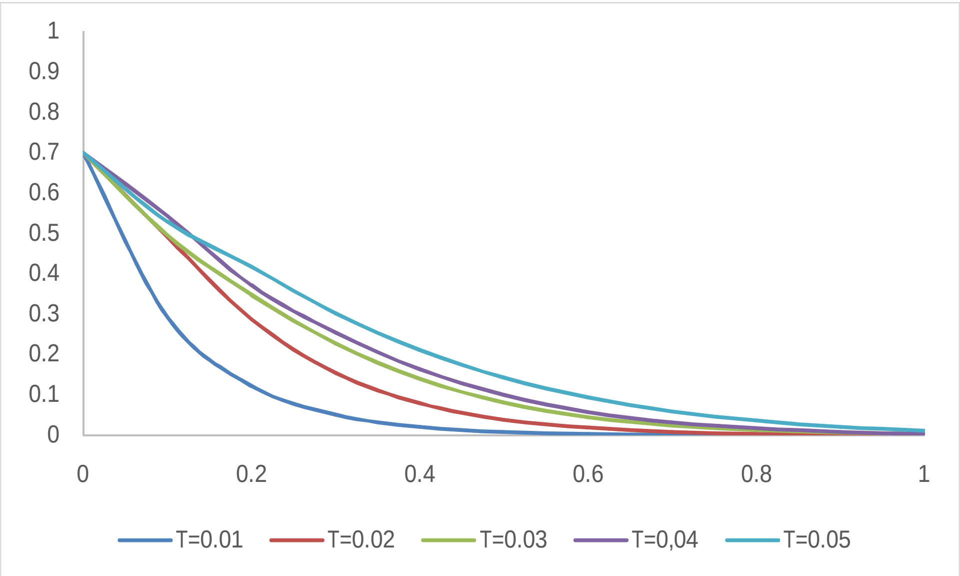


Figure –A: Length → Saturation

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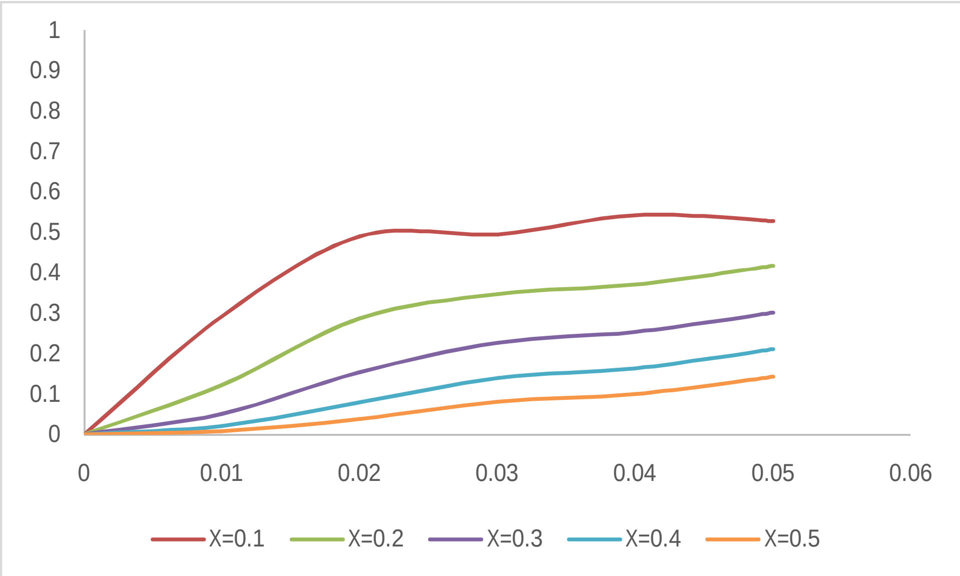


Figure-B: Time → Saturation

## VI. CONCLUSION

From figure-A, it is clear that keeping time constant as length increases the saturation decreases. From figure -B, it is clear that keeping length constant as time increases saturation increases.

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