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Numeric Technique in Computation of Concentration in Miscible Fluid Flow through Porous Media

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ABSTRACT: The main aim of the paper is to solve non-linear partial differential equation using numeric technique. It contains the method to convert a partial differential equation into difference equation or polynomial solutions. One of the famous Finite difference method namely Successive over Relaxation method is used to obtain the solution.

KEYWORDS: Concentration, Porous Media, Numeric Technique

I. INTRODUCTION

The present paper contains the phenomenon of longitudinal dispersion in the flow of two miscible fluids through porous media. The process by which miscible fluids in the laminar flow mix in the direction of the flow is known as longitudinal dispersion [9]. The hydrodynamic dispersion occurs due to the actual movements of individual tracer particles throughout the pores and various physical and chemical phenomenon that take place within the pores. This phenomenon simultaneously occurs due to molecular diffusion and convection [7,9].

This type of phenomenon takes place at river mouths where the sea water intrusion into reservoir. It also takes place in the underground recharge of waste water. When oil is displaced by one of the LPG (liquid petroleum gas) products like Ethane, Propane or Butane, it is known as immiscible flooding. If LPG is in the liquid phase then it is miscible with the oil and theoretically all residual oil can be recovered. However, practically it is not possible to recover all the residual oil.

Several authors like Scheidegger [1], Greenkorn [2], Schwartz[3] discussed this problem from different viewpoints.

This phenomenon yields a non-linear partial differential equation. The Present paper contains method of how to convert partial differential equation into difference equation with appropriate conditions.

II. STATEMENT OF THE PROBLEM

Water resources engineering involves many important problems which is related with the mass-transport of a miscible fluid in a flow.

Generally, a fluid is considered to be a continuous material and therefore in addition to the velocity of a fluid element, the molecules in this element have random motion. Due to the random motion, molecules of a certain material in high concentration at one point will spread with time. So the velocity considered here is time dependent. The net molecular motion from a point of higher concentration to one of lower concentration is known as molecular diffusion [6,8].

Fluid flows in nature are usually turbulent, but the porous medium through which the fluid flows, considered as homogeneous and for this reason, in the direction of flow, laminar flow is assumed in which miscible fluids mix.

Two Fluids are moving through the random passages of the medium. If these two fluids take different routes, then fluid elements adjacent to each other at one time will separate. Velocity across the cross-section of the passages is not uniform. Hence the geometrical dispersion is considered to be coupled with molecular diffusion and dispersion. Many authors have considered the passage as randomly connected tubes. By considering this they have shown that the dispersion in an isotropic medium which can be described with a coefficient D for longitudinal dispersion in the direction of seepage velocity [7, 8].

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III. MATHEMATICAL FORMULATION OF THE PROBLEM

The equation of continuity for the mixture is given by [10]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \quad (1)$$

Where ρ is the density the of mixture and $\vec{v}$ is the pore (seepage) velocity (vector).

If there is no addition or subtraction of the dispersing material, then the equation of diffusion for a fluid flow through homogeneous porous medium is given by

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\vec{v}) = \nabla \cdot \left[\rho \bar{D} \nabla \left(\frac{c}{\rho}\right)\right] \quad (2)$$

Where c is the concentration of the fluid say X in the other host fluid say Y (i.e. c is the mass of X per unit volume of the mixture), $\bar{D}$is the tensor co-efficient of dispersion with nine componentsD_{ii} [9].

In a laminar flow through homogeneous porous medium if temperature is considered constant, ρ may be considered to be constant. From equation (1)

$$\nabla \cdot \vec{v} = 0$$

Therefore, Equation (2) becomes

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\vec{v}) = \nabla \cdot [\bar{D}\, \nabla c] \quad (3)$$

When the seepage velocity $\vec{v}$ is along the X-axis, the non-zero components are $D_{11} = D_L$and$D_{22} = D_T$(coefficient of transverse dispersion),and other $D_{ij}$are zero [11].

Thus equation (3) becomes,

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D_L \frac{\partial^2 c}{\partial x^2} \quad (4)$$

Where u is the velocity component along X-axis which is time dependent and D_L is the longitudinal dispersion co-efficient.

An appropriate initial and boundary conditions are :

$$c(x,0) = \beta_0 \,, 0 \le x \le L \,\,, c(0,t) = \beta_1 \text{ and } c(L,t) = \beta_2, t > 0 \quad (5)$$

Where β_0 is the concentration of the tracer at$t = 0$

$\beta_1$is the concentration at $x = 0$

$\beta_2$is the concentration at $x = L$

Here $= -\frac{1}{\sqrt{t}}$, since it is the cross-sectional time dependent flow velocity through porous medium. Negative sign shows the decreasing of velocity in the direction of flow for definiteness.

From (4), we have

$$\frac{\partial c}{\partial t} - \frac{1}{\sqrt{t}}\frac{\partial c}{\partial x} = D_L \frac{\partial^2 c}{\partial x^2} \quad (6)$$

For simplicity of the equation let us set t=1 in equation (5)

$$\frac{\partial c}{\partial t} = \frac{\partial c}{\partial x} + D_L \frac{\partial^2 c}{\partial x^2} \quad (7)$$

With the boundary conditions (5) and considering L=1,

$$c(x,0) = 0 \,, 0 \le x \le 1 \,, c(0,t) = 0.5 \text{ and } c(1,t) = 0, t > 0 \quad (8)$$

IV. MATHEMATICAL SOLUTION OF THE PROBLEM

Since it is not easy to obtain analytical solution of equation (7), Finite difference method is used to obtain the solution. . A well-known Finite Difference Method, Successive over Relaxation method [4] is used.

Replacing all partial derivatives in equation (7) by finite differences [5] , we get

$$\frac{c_{i,j+1} - c_{i,j}}{k} = \frac{c_{i+1,j} - c_{i,j}}{h} + \frac{D_L[c_{i+1,j+1} - 2c_{i,j+1} + c_{i-1,j+1} + c_{i+1,j} - 2c_{i,j} + c_{i-1,j}]}{2h^2}$$

$$c_{i,j+1} = c_{i,j} + \frac{k(c_{i+1,j} - c_{i,j})}{h} + \frac{+D_L k[c_{i+1,j+1} - 2c_{i,j+1} + c_{i-1,j+1} + c_{i+1,j} - 2c_{i,j} + c_{i-1,j}]}{2h^2}$$

$$c_{i,j+1}\left(1 + \frac{D_L k}{h^2}\right) = c_{i,j}\left(1 - \frac{k}{h} - \frac{D_L k}{h^2}\right) + c_{i+1,j}\left(\frac{k}{h} + \frac{D_L k}{2h^2}\right) + \frac{D_L k[c_{i+1,j+1} + c_{i-1,j+1} + c_{i-1,j}]}{2h^2} \quad (9)$$

In (9), put $r = \frac{k}{h^2}$, $x_i = x_0 + ih$, $i = 0,1,2, \dots \dots \dots$

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Where h is the mesh size in the x-direction.
And $t_j = jk, j = 0,1,2, \ldots \ldots \ldots \ldots \ldots \ldots$
Where k is the step length in the t-direction.

$$c_{i,j+1}(1+D_L r) = c_{i,j}(1-hr-D_L r) + c_{i+1,j}\left(hr+\frac{D_L r}{2}\right) + \frac{D_L r[c_{i+1,j+1}+c_{i-1,j+1}+c_{i-1,j}]}{2}$$

$$c_{i,j+1} = c_{i,j}\frac{(1-hr-D_L r)}{(1+D_L r)} + c_{i+1,j}\frac{(hr+\frac{D_L r}{2})}{(1+D_L r)} + \frac{D_L r[c_{i+1,j+1}+c_{i-1,j+1}+c_{i-1,j}]}{2(1+D_L r)} \quad (10)$$

Let $\lambda_i = c_{i,j}\frac{(1-hr-D_L r)}{(1+D_L r)} + c_{i+1,j}\frac{(hr+\frac{D_L r}{2})}{(1+D_L r)} + \frac{D_L r c_{i-1,j}}{2(1+D_L r)}$ (11)

From (10) and (11)

$$c_i = \lambda_i + \frac{D_L r[c_{i+1}+c_{i-1}]}{2(1+D_L r)}$$

$$c_{i,j+1} = \lambda_i + \frac{D_L r[c_{i+1,j}+c_{i-1,j}]}{2(1+D_L r)} \quad (12)$$

Using Gauss-sheidal method

$$c_{i,j+1} = \lambda_i + \frac{D_L r[c_{i+1,j}+c_{i-1,j+1}]}{2(1+D_L r)} \quad (13)$$

Using Succesive over relaxation method

$$c_{i,j+1} = c_{i,j} + \omega\left[\lambda_i + \frac{D_L r[c_{i+1,j}+c_{i-1,j+1}]}{2(1+D_L r)} - c_{i,j}\right] \quad (14)$$

Substituting $D_L = 0.6$,h=0.1, k=0.1, $\omega = 1.9$ in(11) and (14), we have

$$\lambda_i = 0.428571428c_{i-1,j} - 0.8571428c_{i,j} + 0.57142857c_{i+1,j} \quad (15)$$

$$c_{i,j+1} = c_{i,j} + 1.9[\lambda_i + 0.428571428(c_{i+1,j} + c_{i-1,j+1}) - c_{i,j}] \quad (16)$$

From (8), (15) and (16) ,different value of concentration at different value of x and t are shown in the table.

t→	0.1	0.2	0.3
X↓		C	
0	0.5	0.5	0.5
0.1	0.407143	0.414704	0.418981641
0.2	0.331531	0.343845	0.350925264
0.3	0.269961	0.285001	0.293789488
0.4	0.219825	0.236155	0.245848049
0.5	0.179	0.195622	0.205642428
0.6	0.145757	0.161999	0.171941346
0.7	0.118688	0.134118	0.14370618
0.8	0.096646	0.111005	0.120061472
0.9	0.078698	0.091851	0.088104694
1	0.064082	0.02316	0.061615395

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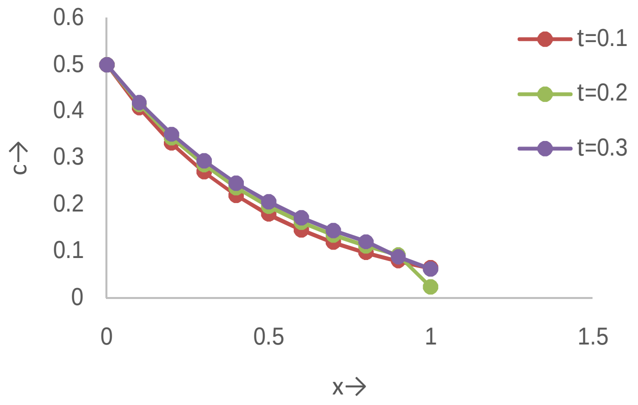

Figure-1

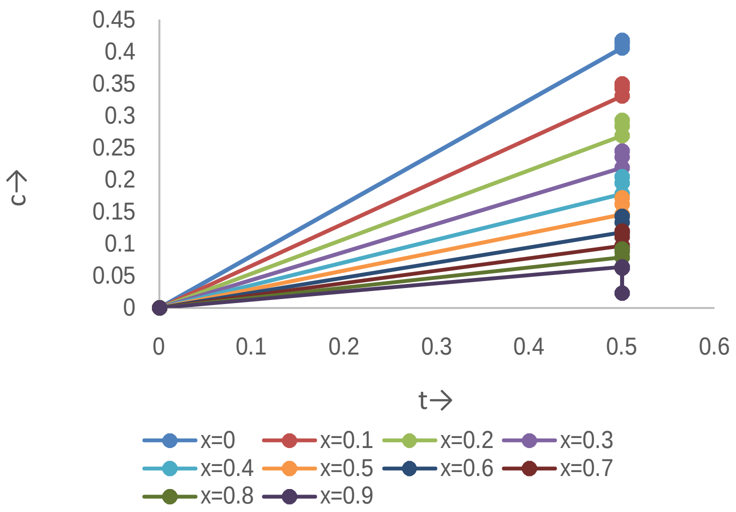

Figure- 2

V. CONCLUSION

Figure -1 shows that as x i.e. length increasing ,then concentration decreases and figure-2 shows that as time t increasing ,then concentration also increasing which is our suitable solution.

VI. SCOPE OF THE PROBLEM

This type of problem is applicable in spreading of contaminant in a canal .It is also very useful in evaporation of water vapour from water surface.

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When sewage is discharged into a body of water, it is very important to determine how the sewage is dispersed in the receiving water. To determine the rate of evaporation from the surface of a reservoir, it is necessary to know the rate at which water vapour near this surface is carried into the air above.

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