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# Image Segmentation using Morphological Component Analysis and Clustering Techniques

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**ABSTRACT:** Digital Image Segmentation is one of the major tasks in digital image processing. It is the process of separating the image into its constituent objects for analysis. This work proposes a new approach for segmentation, based on texture of the image. At first, the image is decomposed into morphological components, according to different texture characteristics such as coarseness, contrast, directionality and line-likeness using Morphological Component Analysis (MCA). This is followed by the modification and recombination procedures. Each morphological component is modified to enhance the texture characteristics and recombined to produce a texture enhanced image. Finally, the enhanced image is segmented using clustering techniques. For selecting an efficient clustering technique, comparison is made for three methods namely, k-means clustering, Fuzzy c-means clustering and GMM (Gaussian Mixture Model). Experimental results shows the effectiveness of the proposed enhancement technique when compared to the existing methods like unsharp masking (UM), VISU Shrink (VISU), Coherence-enhancing diffusion (CDF), shock filter (SHK). Brodatz and SIPI dataset is used for evaluating the performance of proposed technique.

**KEYWORDS**: MCA; Texture; Enhancement; Segmentation.

### I. INTRODUCTION

The methods we propose segment the enhanced image using clustering techniques. Image segmentation refers to the process of partitioning the image into multiple segmentation [8]. The goal of segmentation is to simplify and change the representation of image into something that is more meaningful and easier to analyze. Image segmentation is based on two basic properties of image 1) intensity values involving discontinuity that refers to sudden or abrupt changes in intensity as edges and 2) similarity that refers to partitioning a digital image into regions according to some pre-defined likeness criterion. There are many segmentation methods such as Edge Detection: Edge detection is a process of locating an edge of an image are assumed to represent object boundaries, and used to identify these objects. Detection of edges in an image is a very important step towards understanding image features. Edges consist of meaningful features and contained significant information. It reduces significantly the amount of the image size and filters out information that may be regarded as less relevant, preserving the important structural properties of an image. Since edges often occur at image locations representing object boundaries, edge detection is extensively used in image segmentation when images are divided into areas corresponding to different objects. Thresholding: Thresholding is the easiest way of segmentation. It is done through the threshold values which are obtained from the histogram of those edges of the original image. The threshold values are obtained from the edge detected image.

Region growing: In this technique pixels that are related to an object are grouped for segmentation. The thresholding technique is bound with region based segmentation. The area that is detected for segmentation should be closed. Region based segmentation is also termed as "Similarity Based Segmentation". There won't be any gap due to missing edge pixels in this region based segmentation. Clustering: Clustering is the process of grouping a set of objects in such a way that objects in the same group are more related to each other than to those in other groups. Cluster analysis involves applying one or more cluster algorithms with the goal of finding hidden patterns or grouping in a dataset. In this algorithm form groupings or clusters in such a way that data in a cluster have a higher measure of similarity than data in any other cluster [1].



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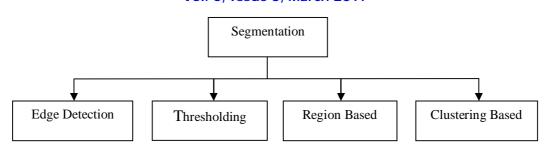


Fig.1 Types of Segmentation methods

Other methods enhance the textures in the image directly. Unsharp masking (UM) which enhances texture by emphasizing its high frequency contents [4], [5]. VISUShrink (VISU), which enhances texture by removing noise via shrinking wavelet coefficients in high-frequency, sub-bands not exceeding certain thresholds. Shock filtering (SHK), which enhance the texture edges well but it breaks some smooth regions. Coherences- Enhancing Diffusion (CDF) which prevents strong discontinuities at edges while removing artifacts from smooth regions, so that image textures are enhanced.

### II. RELATED WORK

Sotirios P. Chatzis et al, The HMRF model treat problem as an FCM-type clustering problem, effected by introduced the assumptions of the HMRF model into the fuzzy clustering procedure. In this approach utilizes a fuzzy objective function regularized by Kullback–Leibler divergence information used and applied mean-field-like approximation [1]. Yannis A. Tolias et al, This paper proposed a fuzzy segmentations at different resolutions are combined using a data fusion process in order to compute the final fuzzy partition matrix. The results provided segmentation, having lower fuzzy entropy when compared C-Means algorithm [3]. Dimitrios Charalampidis et al, A CK-means technique is proposed to reduce the possibility of converging at local minima and to estimated the correct number of clusters [6]. Christophe Biernacki et al, discussed about observed data are assigned to unknown clusters using a maximum a posteriori operator. Then, the Integrated Completed Likelihood (ICL) is approximated using a Bayesian information criterion (BIC. The resulting ICL criterion showed that it performs well both for choosing a mixture model and a relevant number of clusters [9].

### III. PROPOSED ALGORITHM

In the method presented herein, it is assumed that texture consist of several different components representing different visual characteristics. By modifying these components in different ways, distinct textures become more different in terms of the descriptors used to differentiate them.

The Morphological Components of different textures are then modified in different ways so that textures become more different with respect to these texture characteristics. Morphological Component Analysis (MCA) has proven successful in decomposing images into morphological distinct components.

A texture characteristic is broadly defined any property of a texture that can be quantified. Fig.1 shows a schematic of the proposed method to enhance textural differences by manipulating certain texture characteristics in certain ways. Finally the enhanced image is segmented.

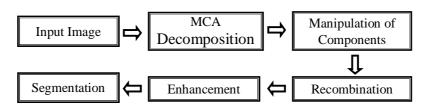


Fig.2 Architecture Diagram of proposed System



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#### 1. Decomposition using MCA

#### 1.1. Standard Cartoon + Texture Decomposition

Morphological component analysis (MCA) was proposed to seek components of the image. It can decompose the image into cartoon and texture part [2].

#### 1.2. Decomposition by multiple texture characteristics

The traditional MCA method by loosening the restrictions of dictionaries and seeking optimal parameters for those selected dictionaries, so that the image can be decomposed into different components corresponding to k textural characteristics described for solving the following k optimization problems using:

$$\left\{s_{s,n}^{opt}, s_{w,n}^{opt}\right\} = \arg\min_{\left\{s_{s,n}, s_{w,n}\right\}} \|T_{s,n}s_{s,n}\|_{1} + \|T_{w,n}s_{w,n}\|_{1} + \|I - s_{s,n} - s_{w,n}\|_{2}^{2}$$
(1)

where  $s_{s,n}$  and  $s_{w,n}$  are the components having strong and weak aspects of the n<sup>th</sup> texture characteristic, n = 1, 2, ..., *i*, e.g. a "coarse" component and a "non-coarse" component.  $T_{s,n}$  and  $T_{w,n}$  are dictionaries for  $S_{s,n}$  and  $S_{w,n}$  respectively, and I is the original image. The dictionaries used in this method have adjustable parameters so that the performance of decomposition can be more consistent for different images than the traditional MCA. The dictionaries proposed should satisfy two conditions 1) they can highlight the component corresponding to a certain characteristic. 2) They are insensitive to the other texture characteristics.

#### 1.3. Manipulation of the image components

After decomposing the image into pairs of strong and weak texture characteristic components, these components are manipulated to enhance the texture characteristics they are meant to capture. In general using the following equations:

$$s_{s,n}' = f_{s,n}\left(s_{s,n}\right) \tag{2}$$

$$s'_{w,n} = f_{w,n}(s_{w,n})$$
 (3)

where  $s_{s,n}$  and  $s_{w,n}$  are the components respectively exhibiting strong and weak aspects of the n<sup>th</sup> texture characteristic, n = 1, 2, ..., i, s'<sub>s,n</sub> and s'<sub>w,n</sub> are the manipulated strong and weak characteristic components, and  $f_{s,n}$  and  $f_{w,n}$  are the manipulation functions used to enhance the texture components  $s_{s,n}$  and  $s_{w,n}$  respectively.

#### 1.4. Re-combination of the manipulated components

After manipulating every component to enhance its own properties, the components are re-combined into a final texture-enhanced image I'.

### 2. Texture manipulation

The input image is decomposed into four specific texture characteristics like Coarseness: coarseness quantifies the number of edges in the local texture. Therefore, decomposing a texture into the coarse component and the fine component is to look for dictionaries corresponding to regions with few strong texture edges and region with many weak texture edges. Contrast: contrast measures the variance of the grey scale intensities in a local area. Dictionaries were chosen to represent either texture with high or low intensity variance. Directionality: Directionality of texture measures the orientation of the local texture at the range from 0 to  $\pi$ , we decompose the texture into horizontal and vertical direction. Line-Likeness: The Line-Likeness requires dictionary component corresponding to texture with very similar and very different direction in every local region. We need to separate the texture into components having similar but not identical directions respectively. By modifying each of the individual texture components and recombining them, the textures can be manipulated to be more different with respect to the specific characteristics represented by the modified components. Various manipulations are applied to transform the components.



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### 3. Texture Recombination

Recombination is the reverse process of decomposition. After re-combination of these manipulated components  $s'_{s,n}$  and  $s'_{w,n}$  as in following Equation:

$$I' = \frac{1}{i} \sum_{n=1}^{i} \left( S'_{s,n} + S'_{w,n} \right) \tag{4}$$

where  $s'_{s,n}$  and  $s'_{w,n}$  are calculated as the manipulated strong and weak characteristic components respectively, and *i* is the total number of characteristics used for image decomposition. Decomposition of image into different texture characteristics was done in order to manipulate the image. By doing such manipulations, weak edges are removed and pixel intensities are improved with high clarity.

After completing these steps, final enhanced image is obtained from these decomposed blocks by recombination. Recombination is the final stage that regains the input image as an enhanced image.





(a) Input Image (b) Enhanced Image Fig.3 Final Enhances Image

### 4. Segmentation

In this Phase, the enhanced image is segmented using the clustering techniques. In method involving visual inspection, it is often required to separate objects from background, in conditions of poor and non uniform illumination. In such cases one has to rely on adaptive methods that learn the illumination from the given images and base the Object background decision on this information. We here present a new method for image segmentation based on clustering. One can apply different algorithms to create different clustering of the data.

### 5. Clustering Techniques

Clustering refers to the process of grouping samples so that the samples are similar within each group. The groups are called clusters. Clustering is a data mining technique used in statistical data analysis, data mining, pattern recognition, image analysis etc. Different clustering methods include hierarchical clustering which builds a hierarchy of clusters from individual elements. Because of its simplicity and efficiency, clustering approaches were one of the first techniques used for the segmentation of (textured) natural images. In partitioned clustering, the goal is to create one set of clusters that partitions the data in to similar groups. Other methods of clustering are distance based according to which if two or more objects belonging to the same cluster are close according to a given distance, then it is called distance based clustering. In our work we have used:

- K-means clustering
- Fuzzy c-means clustering
- GMM (Gaussian Mixture Model)

#### K-means clustering

K-mean is one of the simplest unsupervised learning algorithms that solve the well known clustering problem. The procedure follows a simple and easy way to classify a given data set through a certain number of clusters (assume k clusters) fixed a priori.

In k-means clustering, it partition a collection of data into ak number group of data. It classifies a given set of data k number of disjoint cluster. K-means algorithms consist of two separate phases. In the first phase it calculate the k centroid and in the second phase it takes each point to the cluster which has nearest centroid from the respective data



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points. There are different methods to define the distance of the nearest centroid and one of the most used methods is Euclidean distance. Once the grouping is done ic recalculate the new centroid of each cluster and based on that centroid, a new Euclidean distance is calculated between each center and each data points and assign the points in the cluster which have minimum Euclidean distance. Each cluster in the partition is defined by its member objects and by its centroid. The centroid for each cluster is the point to which the sum of distances from all the objects in that cluster is minimized. So k-means is an iterative algorithm in minimization of the sum of distance from each object to its cluster centroid, over all clusters.

### ALGORITH: 1 K-MEANS CLUSTERING

- 1. Initialized number of cluster k and center.
- 2. For each pixel of an image, calculate the Euclidean distance d, between the center and each pixel of an image using the relation given below.

$$d = \parallel p(x, y) - c_k \parallel$$

(5)

- Assign all the pixels to the nearest center based on distance d.
- 4. After all pixels have been assigned, recalculate new position of the center using the relation given below.

$$c_k = \frac{1}{k} \sum_{y \in c_k} \sum_{x \in c_k} p(x, y) \tag{6}$$

- 5. Repeat the process until it satisfies the error values.
- 6. Reshape the cluster pixels into image.

Let us consider an image with resolution of  $x \times y$  and the image has to be cluster into k number of cluster. Let p(x,y) be an input pixels to be cluster and  $c_k$  be the cluster centers. The quality of the final clustering results is depends on the arbitrary selection of initial centroid. So if the initial centroid is randomly chosen, it will get different results for different initial centers. So the initial centers will be carefully chosen so that we get our desire segmentation. And also computational complexity is another term which we need to consider while designing the k-mean clustering. It relies on the number of data elements, number of cluster and number of iteration.

### Fuzzy c-means clustering

3.

Fuzzy c-means (FCM) is based on the same idea of finding clustering centers by iteratively adjusting their positions and evaluation of an objective function. It allows more flexibility by introducing the possibility of partial membership to clusters. The error function in fuzzy c-means is given below:

$$E = \sum_{j=1}^{C} \sum_{i=1}^{N} \mu_{ij}^{k} \parallel x_{i}^{(j)} - c_{j} \parallel^{2}$$
(7)

Where  $\mu_{ij}$  is the fuzzy membership of sample (or pixel)  $x_i$  and the cluster identified by its center  $c_j$ , and k is a constant that defines the fuzziness of the resulting partitions.

### ALGOTIRHM: 2 FUZZY C-MEANS CLUSTERING

The steps involved in fuzzy c-means image segmentation are:

- 1. Initialize the cluster centers  $c_i$  and Let t = 0.
- 2. Initialize the fuzzy partition membership functions  $\mu_{ij}$  according to Eq (8).
- 3. Let t = t + 1 and compute new cluster centers  $c_i$  using Eq (9).
- 4. Repeat step 2 and 3 until convergence.

E can reach the global minimum when pixels nearby the centriod of corresponding clusters are assigned higher membership values, while lower membership values are assigned to pixels distance from the centroid. Then the membership is proportional to the probability that a pixel belongs to a specific cluster where the probability is only dependent on the distance between the image pixel and each independent cluster center. The membership function and the cluster centers are updated by,



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$$\mu_{ij} = \frac{1}{\sum_{m=1}^{C} \left( \frac{\parallel x_j - c_i \parallel}{\parallel (x_j - c_m) \parallel^{2/(k-1)}} \right)}$$
(8)  
$$c_j = \frac{\sum_{j=1}^{N} \mu_{ij}^k x_j}{\sum_{i=1}^{N} \mu_{ij}^k}$$
(9)

Again, an initial setting for each cluster center is required and FCM also converges to a local minimum.

### **GMM (Gaussian Mixture Model)**

each of the rows sums to 1.

GMMs will find the clusters using a technique called "Expectation Maximization". In the "Expectation" step, we will calculate the probability that each data point belongs to each cluster (estimated mean vectors an covariance matrices). In the "Maximization" step, we'll re-calculate the cluster means and covariances based on the probabilities calculated in the expectation step.

### ALGORITHM: 3 GAUSSIAN MIXTURE MODEL

- 1. Given a set of data points, and assume that k cluster in the data, we need to:
  - a. Assign the data points to the clusters
  - b. Learn the Gaussian distribution parameters for each cluster.
- 2. Each cluster contains data generated from a Gaussian distribution.
- 3. Overall process of generating data:
  - a. First randomly select one of the clusters according to a prior distribution of the cluster.
  - b. Draw a random sample from the Gaussian distribution of that particular cluster.

Expectation Maximization define the EM (Expectation-Maximization) algorithm for Gaussian mixtures as follows. The algorithm is an iterative algorithm that starts from some initial estimate of  $\theta$  (e.g., random), and then proceeds to iteratively update  $\theta$  until convergence is detected. Each iteration consists of an E-step and an M-step.

**E-Step**: Denote the current parameter values as  $\theta$ . Compute  $w_{ik}$  (using the equation above for membership weights) for all data points  $x_i$ ,  $1 \le i \le N$  and all mixture components  $1 \le k \le K$ . Note that for each data point  $x_i$  the membership weights are defined such that  $\sum_{k=1}^{k} w_{ik} = 1$ . This yields an N × K matrix of membership weights, where

$$\alpha_k^{new} = \frac{N_k}{N}, \qquad i \le k \le K \tag{10}$$

**M-Step**: Now use the membership weights and the data to calculate new parameter values. Let  $\sum_{i=1}^{k} w_{ik}$ , i.e., the sum of the membership weights for the k<sup>th</sup> component is the effective number of data points assigned to component k.

$$\mu_k^{new} = \left(\frac{1}{N_k}\right) \sum_{i=1}^N w_{ik} \cdot x_i \qquad 1 \le k \le K$$
(11)

#### IV. EXPERIMENTAL RESULTS

The experiments are conducted on the texture and real world image database, which contains 82 textures and 10 real world images. Description of the datasets used in the proposed system is given in the Table 1

Database	Brodatz, SIPI
Number of textures	82 textures and
and real world	10 real world
Format	.tiff and .png
Size	256×256 pixels

Table.1 Dataset

The segmentation accuracy of each segmentation technique is computed by comparing the segmented result with the ground truth [7] using the metric:



(An ISO 3297: 2007 Certified Organization) Website: <u>www.ijircce.com</u> Vol. 5, Issue 3, March 2017  $C = \frac{|\{S(x,y)|S(x,y)=G(x,y)\}|}{N}$ 

(10)

where  $C \rightarrow Segmentation$  Accuracy,  $S \rightarrow Segmented label image, G \rightarrow Ground truth image, N \rightarrow Total number$ of pixels in the image. The segmentation accuracy of each images before and after enhancement were compared toevaluate the effect of proposed approach. Table.2 shows the segmentation accuracy of k-means, Fuzzy c-means andGMM while comparing the result, we observed that GMM performed well for all images than the other two techniques.Table.3 shows the segmentation accuracy of proposed method and existing methods.

	Segmentation Accuracy					
Images	k-means (%)	Fuzzy c-means (%)	GMM (%)			
	85.43	87.32	91.73			
	88.49	87.92	94.49			
*	89.59	90.43	91.73			
- A	81.01	83.45	91.03			
- Suss	92.43	92.83	93.52			

#### Table.2 Segmentation accuracy of proposed approach

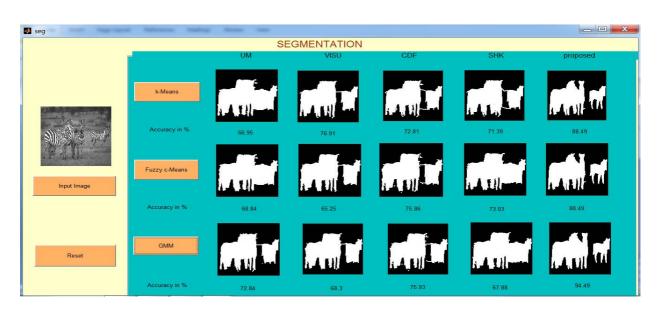


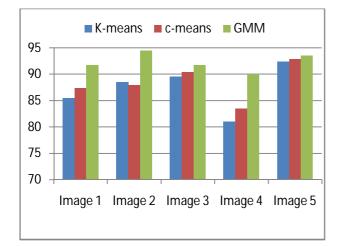
Fig.4 Segmentation using clustering result



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### Table.3 Segmentation accuracy of proposed approach and existing approaches

INPUT IMAGES	SEG TECH	UM	VISU	CDF	SHK	PROPOSED
	k-means	67.68%	72.45%	75.03%	78.43%	85.43%
	c-means	73.23%	73.84%	70.88%	78.34%	87.32%
	GMM	68.09%	75.67%	79.75%	68.22%	91.73%
Martin Rote	k-means	66.95%	76.91%	72.81%	71.39%	88.49%
	c-means	68.84%	65.25%	75.86%	73.03%	88.49%
	GMM	72.84%	68.03%	75.93%	67.88%	94.49%
	k-means	72.83%	81.67%	70.49%	73.68%	89.59%
	c-means	68.84%	81.45%	69.84%	75.23%	90.43%
	GMM	77.09%	78.88%	70.01%	72.09%	91.73%
ne sile	k-means	77.51%	67.64%	73.05%	69.45%	81.01%
Tr.X	c-means	65.72%	77.94%	74.01%	79.05%	83.45%
	GMM	65.63%	67.09%	74.23%	78.01%	91.03%
Son.	k-means	75.33%	76.05%	73.29%	81.63%	92.43%
	c-means	71.12%	74.52%	71.35%	81.39%	92.83%
	GMM	69.06%	74.49%	68.03%	81.46%	93.52%
TABA	k-means	72.23%	66.32%	65.81%	70.11%	92.83%
	c-means	69.83%	66.89%	67.32%	69.92%	93.72%
	GMM	71.85%	67.53%	66.67%	73.52%	91.45%
A Contraction	k-means	73.46%	68.73%	65.32%	66.14%	88.72%
	c-means	71.32%	65.51%	66.15%	73.21%	91.80%
	GMM	70.28%	69.32%	64.89%	66.45%	91.78%
	k-means	70.39%	68.26%	76.65%	70.01%	90.82%
	c-means	68.57%	69.56%	75.42%	67.82%	91.32%
	GMM	67.33%	68.81%	78.32%	70.89%	91.56%



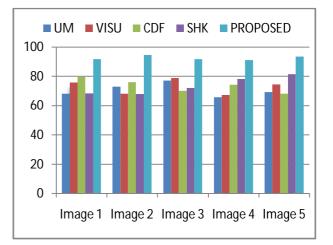


Fig.5 Comparisons chart of k-means, c-means and GMM

Fig.6 Comparisons chart of proposed approach and Existing approach



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V. CONCLUSION

We have proposed a novel system for segmentation using Morphological Component Analysis (MCA) and Clustering techniques. MCA is used to decompose textures into multiple morphological components according to texture characteristics: coarseness, contrast, directionality and line-likeliness. We have evaluated the proposed approach using Brodatz, SIPI and Berkeley dataset, which contains 92 images. Experimental results show an enhanced image quality. For performance evaluation we have calculated MSE and PSNR of original input image and enhanced output image. We found that PSNR of output image is greater than 30. Then the enhanced image is segmented using clustering techniques like k-means clustering, Fuzzy c-means clustering and GMM (Gaussian Mixture Model) and the performance is compared with the existing techniques. We found that our proposed approach outperforms the other existing techniques such as Unsharp masking (UM), VISUShrink, Coherence-Enhancing Diffusion (CDF) and Shock Filter (SHF).

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