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Study of Fingero-Imbibition Phenomena in the Context with Double Phase Flow through Porous Media Using Numeric Technique

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ABSTRACT: Fingero-imbibition phenomenon arises due to the simultaneous occurrence of two important phenomena viz. fingering and imbibition. The solution is obtained by assuming the saturation co-efficient as constant. The solution is obtained by Successive over Relaxation method, a well-known finite difference method.

KEYWORDS: Fingero-imbibition, Porous media, Double phase, Successive over Relaxation

I. INTRODUCTION

Porous medium is considered to be filled with some resident fluid known as native fluid. Medium is saturated with resident fluid and assume that less viscous fluid is injected into porous. Medium contains fluid which preferentially wets the medium. When this medium is carried into contact with this fluid, there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the native fluid from the medium. This phenomena is called imbibition[1] .This arises due to different wetting abilities. On the other hand, when a fluid inside of a porous medium is displaced by another fluid having lesser viscosity, instead of displacing the whole front, protuberances occur which shoot through the porous medium at comparatively great speeds. This phenomenon is called fingering. Occurrence of phenomena of fingering and imbibition simultaneously is known as finger-imbibition phenomena.Scheidegger[2] discussed the statistically behavior of instabilities. Graham and Richardson[3] discussed role of imbibition phenomena in recovery of oil.Olaniyi and Samson [4] discussed an approximate analytical study of Fingero-Imbibition Phenomena of Time-Fractional Type in Double Phase Flow through Porous Media.Verma [5,6] discussed this phenomena for a cracked porous medium and slightly dipping porous medium.Mehta and Verma [7] obtained Composite expansion of finger-
imbibition in double phase flow. R. Meher and S. Meher [8]obtained series solution for porous medium equation arising in fingero-imbibition phenomenon.

II. STATEMENT OF THE PROBLEM

A finite cylindrical mass of porous medium is of length L. Medium is assumed to be saturated with native fluid. It is completely surrounded by an impermeable surface whose one is opened. This face is labelled as the imbibition face (x=0) and this end is exposed to an adjacent formation of 'injected' liquid. It is assumed that the later fluid is preferentially wetting and less viscous. Due to this arrangement displacement process takes place in which the injection of the fluid is initiated by imbibition and the consequent displacement of native liquid produces protuberances. This arrangement is known as Fingero-Imbibition. Native fluid is considered as oil and injected fluid is considered as water.

III. MATHEMATICAL FORMULATION OF THE PROBLEM

By Darcy's law, the seepage velocity of injected fluid and native fluid are,

$$V_w = -\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \tag{1}$$

$$V_o = -\frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x} \tag{2}$$

Where k = Permeability of the homogeneous medium

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k_w= Relative permeability of injected fluid is assumed to be a functions of S_w
k_o = Relative permeabilitie of native fluid, is assumed to be functions of S_o.
P_w , P_o= Pressures of injected fluid and native fluid respective
μ_o , μ_w= Constant viscosities
S_w, S_o= Saturation of of injected fluid and native fluid respectively

Neglecting the variation in phase densities, the equation of continuity for both fluids are

$$P\frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \quad (3)$$

$$P\frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \quad (4)$$

Where P is the porosity of the medium.

From the definition of phase saturation, it is obvious that

$$S_w + S_o = 1 \quad (5)$$

The analytic condition for imbibition phenomenon is given by

$$V_w + V_o = 0 \quad (6)$$

Due to the pressure discontinuity of the flowing phases across their common interface, the capillary pressure (P_c), arise which is given by

$$P_c = P_o - P_w \quad (7)$$

From equations (1), (2) and (5), we have

$$\frac{k_o}{\mu_o}\frac{\partial P_o}{\partial x} + \frac{k_w}{\mu_w}\frac{\partial P_w}{\partial x} = 0 \quad (8)$$

From (7) and (8), we have

$$\frac{k_o}{\mu_o}\left\{\frac{\partial P_c}{\partial x} + \frac{\partial P_w}{\partial x}\right\} + \frac{\partial P_w}{\partial x} = 0$$

$$\therefore \frac{\partial P_w}{\partial x} = \frac{-\frac{k_o}{\mu_o}\frac{k_w}{\mu_w}}{\left\{\frac{k_o}{\mu_o}+\frac{k_w}{\mu_w}\right\}}\frac{\partial P_c}{\partial x} \quad (9)$$

From (1) and (3), we have

$$P\frac{\partial S_w}{\partial t} - \frac{\partial}{\partial x}\left\{\frac{k_w}{\mu_w}\, k\, \frac{\partial P_w}{\partial x}\right\} = 0 \quad (10)$$

From equations (9) and (10), we get

$$P\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left\{k\frac{k_o k_w}{k_o\mu_w + k_w\mu_o}\frac{dP_c}{dS_w}\frac{\partial S_w}{\partial x}\right\} = 0 \quad (11)$$

Setting $D(S_w) = \frac{k_o k_w}{k_o\mu_w + k_w\mu_o} = D$

It is called co-efficient of saturation which is assumed to be a constant.

Capillary pressure in terms of phase saturation is defined as :

$$P_c = -\beta S_w \quad (12)$$

Equation (11) can be written as

$$P\frac{\partial S_w}{\partial t} - \frac{\partial}{\partial x}\left[kD\beta\frac{\partial S_w}{\partial x}\right] = 0 \quad (13)$$

With $S_w(x,0) = s_0$, $S_w(0,t) = s_1$, $S_w(L,t) = 0, 0 \le x \le L$

$$\frac{\partial S_w}{\partial t} - kD\beta\frac{\partial^2 S_w}{\partial x^2} = 0 \quad (14)$$

Let $X = \frac{x}{L}$, $T = \frac{kD(S_w)\beta}{L^2}t$

$$\frac{\partial S_w}{\partial T} - \frac{\partial^2 S_w}{\partial X^2} = 0 \quad (15)$$

With $S_w(X,0) = s_0$, $S_w(0,T) = s_1$, $S_w(1,T) = 0, 0 \le X \le 1$

The governing equation of the problem is solved by Successive Over Relaxation (S.O.R.) method [10].

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IV. MATHEMATICAL SOLUTION

Using Crank –Nicolson method [9] for (15), we have

$$s_{w_{m,n+1}} = s_{w_{m,n}} + \frac{k}{2h^2}\left(s_{w_{m+1,n}} - 2s_{w_{m,n}} + s_{w_{m-1,n}} + s_{w_{m+1,n+1}} - 2s_{w_{m,n+1}} + s_{w_{m-1,n+1}}\right)$$

Let $\boldsymbol{r} = \frac{\boldsymbol{k}}{\boldsymbol{h^2}}$

$$(1+r)s_{w_{m,n+1}} = s_{w_{m,n}} + \frac{r}{2}\left(s_{w_{m+1,n}} - 2s_{w_{m,n}} + s_{w_{m-1,n}} + s_{w_{m+1,n+1}} + s_{w_{m-1,n+1}}\right)$$

$$\boldsymbol{\lambda_i} = s_{w_{m,n}} + \frac{r}{2}\left(s_{w_{m+1,n}} - 2s_{w_{m,n}} + s_{w_{m-1,n}}\right)$$

$$s_{w_{m,n+1}} = (1-\omega)s_{w_{m,n}} + \omega\left[\frac{r}{2(1+r)}\left(s_{w_{m+1,n}} + s_{w_{m-1,n+1}}\right) + \frac{\boldsymbol{\lambda_i}}{(1+r)}\right]$$

Choose k = 0.01, h=0.1,$\omega = 1.68$, $s_0 = 0, s_1 = 0.4$

$$s_{w_{m,n+1}} = -0.68s_{w_{m,n}} + 1.68\left[0.25\left(s_{w_{m+1,n}} + s_{w_{m-1,n+1}}\right) + \frac{\lambda_i}{2}\right] \quad (16)$$

Equation (16) is obtained using S.O.R. method [10]
Numerical calculations for different values of saturation at different time and different length are shown in the following table.

T→	T=0.01	T=0.02	T=0.03	T=0.04	T=0.05
X↓	S_w				
0	0.4	0.4	0.4	0.4	0.4
0.1	0.168	0.2810304	0.283923978	0.310827384	0.303398251
0.2	0.07056	0.165505536	0.19987582	0.212810562	0.236215202
0.3	0.0296352	0.089450888	0.131226164	0.146370973	0.16872135
0.4	0.012446784	0.045943569	0.080606042	0.094836656	0.116403006
0.5	0.005227649	0.022813461	0.046927127	0.057801376	0.077544389
0.6	0.002195613	0.011058862	0.026193747	0.033486509	0.049724243
0.7	0.000922157	0.00526515	0.014142226	0.018630771	0.030695628
0.8	0.000387306	0.002471942	0.00743421	0.010019091	0.018202664
0.9	0.000162669	0.001147659	0.003803435	0.005117331	0.010064983
1	6.83208E-05	0.00050388	0.001736821	0.002013847	0.005007156

IV. CONCLUSION

At zero distance, the saturation assumed to be is 0.4. As length is increasing, the saturation is decreasing and it tending to zero. Also from the table it is clear that initially saturation is zero and as time increasing, saturation also increasing.

REFERENCES

(1) Joshi, M. S.. "An Analytic Solution Of Fingero-Imbibition In Double Phase Flow Through Underground Porous Medimum", International Journal of Applied Mathematics & Mechanics/09730184, 20091101
(2) Scheidegger, A. E.; Johnson, E. F. The statistically behavior of instabilities in displacement process in porous media, Canadian Journal of Physics, 39(2) (1961), 326-334
(3) Graham, J.W.andRichardson,J.G. (1959): Theory of application of imbibition phenomena in recovery of oil. Trans. A.I.M.E.,216,377.

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(4) Olaniyi S. Iyiola, Samson BabatundeFolarin: Approximate Analytical Study of Fingero-Imbibition Phenomena of Time-Fractional Type in Double Phase Flow through Porous Media, European Journal Of Pure And Applied Mathematics Vol. 7, No. 2, 2014, 210-229
(5) Verma A.P. (1970): Fingero-imbibition in Artificial Replenishment of Ground water cracked porous medium, Water Resources Research,6,3.
(6) Verma,A.P.(1970): Fingero-imbibition in slightly dipping porous medium, journal of Applied Physics,41(9).
(7) Mehta M.N.,Verma A.P.: Composite expansion of finger-imbibition in double phase flow through underground porous medium, proceeding of the Indian Academy of Sciences-section A 87(2):65-70.
(8) MeherR.andMeherS.: Series Solution for Porous Medium Equation Arising in Fingero-Imbibition Phenomenon during Oil Recovery Process, Appl. Math. Inf. Sci. Lett. 3, No. 1, 25-30 (2015)
(9) Jain M.K., Numerical solution of differential equations.2nd edition, Wiley Eastern, 1984.
(10) S.S.Shastry: Introductory Methods of Numerical Analysis (4th Edition).