ISSN(Online): 2320-9801
ISSN (Print): 2320-9798

International Journal of Innovative Research in Computer and Communication Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 8, August 2016

A λμ-B-Spline Curve with Shape Parameters

Urvashi Mishra[1]

Assistant Professor, Department of Mathematics, Mata Gujri Mahila Mahavidyalaya, Jabalpur, Madhya Pradesh, India[1]

ABSTRACT: Spline curve and surface play an important role in CAD and computer graphics. In this paper, we propose several extensions of cubic uniform B-spline. Then, we present the extensions of interpolating $\lambda\mu$-B-spline based on the new B-splines and the singular blending technique. The advantage of the extensions is that they have global and local shape parameters. In order to provide more flexible approaches for designers, the λμ-B-spline curves are constructed as a generalization of the traditional cubic uniform B-spline curves. Possessing multiple local shape control parameters, λμ-B-spline curves not only inherit the properties of cubic uniform B-spline curves, but also exhibit better performance when adjusting its local shapes through two local shape control parameters., which suggest the much wider applications to the pattern design system of apparel CAD/CAM. Besides inheriting the properties of classical B´ezier curves of degree *n*, the corresponding λμ-B-spline curves have a good performance on adjusting their shapes by changing shape control parameter. Specially, in the case where the shape control parameter equals zero, the λμ-B-spline curves degenerate to the classical B´ezier curves. In this paper, the shape modification of λμ-B-spline curves by constrained optimization is investigated. The definition and properties of *λ*-B´ezier curves are given in detail, and the shape modification is implemented by optimizing perturbations of control points. Which show that the proposed method is effective and easy to implement.

KEYWORDS: B-spline curve, $\lambda\mu$-B-spline, $\lambda\mu$-B-spline curves**,** shape parameter, blending function, B-spline surface, Shape Modification.

I. INTRODUCTION

Curve and surface modeling is an important subject in CAD and computer graphics. Spline curve/surface modeling is the most traditional modeling method based on the theory of computer aided geometric design (CAGD)Several kinds of splines have been proposed in the field of CAGD, such as B-splines and T-spline. In practice, we often use cubic uniform B-spline for curve/surface modeling. However, once the control points of the cubic uniform B-spline curve are determined, the shape of the curve is determined. In order to overcome this disadvantage, several extensions of cubic uniform B-spline have been proposed. On the other hand, cubic B-spline interpolation is the traditional global interpolation method [9¡10]. Unfortunately, it also has some disadvantages, making them less desirable for certain applications First, it cannot provide parameter for curve local modification; second, it may exhibit undesirable oscillations; and solving the linear system is very expensive computationally.

To overcome the drawbacks of B-splines, many attempts aiming at new modeling methods have been explored in recent years. In order to enhance the flexibility of B-spline models, some researchers have suggested many types of curves with shape parameters incorporated into the basis functions. For instance, Han presented quadratic trigonometric polynomial curves with one shape parameter. Zhang constructed C-B-spline curves in the space {1, *t*, cos *t*, sin *t*}. Ksasov and Sattayatham gave explicit formulation and recurrence relations for the calculation of generalized B-splines (GB-spline). Juh*α*´ sz and Bancsik considered the sequences of closed B-spline curves, where the elements differ only in their degrees, *i.e.*, they share the same control polygon. Yan and Liang constructed a class of algebraic–trigonometric blended splines with two parameters *x* and *y*. Ma es et al. showed that the normalized Powell–Sabin B-splines form a stable basis for the max norm. Based on quartic blending functions, piecewise polynomial curves with a shape parameter were constructed by Han and Liu. Wang et al. presented piecewise algebraic and trigonometric uniform B-spline curves with a shape parameter of degree k $(k \leq 2)$. Shen and Wang extended B-spline basis functions to changeable degree spline (CD-spline) basis functions, each of which may consist of polynomials of different degrees

ISSN(Online): 2320-9801
ISSN (Print): 2320-9798

International Journal of Innovative Research in Computer and Communication Engineering

(An ISO 3297: 2007 Certified Organization)

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on its support interval. Yin and Tan introduced trigonometric polynomial uniform B-splines curves with multiple shape parameters.

Moreover, the interpolation method is global; thus, changes to any data point will require solving again all the linear systems. However the interpolating splines based on singular blending cannot conveniently modify the curves. If the users want to globally modify the curves, they must set all the local parameters to be equal. In this paper, we propose the extensions of interpolating B-spline by the singular blending technique, and name them $\lambda\mu$ B-spline. They not only have local shape parameters, but also have global parameters. We also present their applications in data interpolation and shape deformation. However, a common problem of these improved designs/models is that they are unable to achieve local changes on individual elements because the shape parameters take effect globally which implies that they can only adjust the global shape of the curves. In other words, they are lack of the local shape adjustability which is especially important. Researchers have introduced extended Bézier and B-spline models with some parameters in [1–15]. Xu and Wang in presented two classes of polynomial blending functions with local shape parameters λi and proposed a method of generating piecewise polynomial curves. By changing the values of the local shape parameters λ_i, the shape of the curves can be manipulated locally to a certain extent. However, since only one kind of local shape parameters λ_i is included, the local adjustability of curves is limited. In this paper, we introduce and construct a new kind of B-spline curves, which are extensions of the traditional cubic uniform B-spline curves and will be called the "$\lambda\mu$-B-spline curves". Here, λ_i and μ_i are two kinds of local shape parameters of the $\lambda\mu$- B-spline curve. While inheriting the properties of the traditional cubic uniform B-spline curves, $\lambda\mu$-B-spline curves have some other advantages including local shape adjustability and better approximation to the control polygon.
The remainder of the paper is organized as follows. The definition and properties of $\lambda\mu$ -B-Spline are given in Section 2. In Section 3, we present the shape modification of $\lambda\mu$-B-spline curves by constrained optimization of single point constraint. In Section 4 shape control of $\lambda\mu$ -B-Spline curve are discussed. In Section 5 Approximation of $\lambda\mu$ -B-Spline curve are discussed. In Section 6, we present some applications. At last, a short conclusion is given in section 7.

II. THE DEFINITION AND PROPERTIES OF $\lambda\mu$ -B-SPLINE CURVES

2.1. Extension Basis Function. The definition of extension basis functions is given as follows:

Definition. Let $\lambda \in [-1, 1]$; for $t \in [0, 1]$, the following polynomial functions

We define the quartic blending functions with two parameters λ_i and μ_i ($\lambda\mu$ - B functions for short) as follows:

$$\begin{aligned}
b_0^4 &= \frac{1}{6}(1-\lambda_i t)(1-t)^3 \\
b_1^4 &= \frac{1}{6}[4-6t^2+(3+\mu_i)t^3-\mu_i t^4] \\
b_2^4 &= \frac{1}{6}[1+(3+\lambda_i)t+3(1-\lambda_i)t^2-3(1-\lambda_i)t^3-\lambda_i t^4) \\
b_3^4 &= \frac{1}{6}[1-\mu_i(1-t)]t^3
\end{aligned} \tag{1}$$

Where - $2 \le \lambda_i,\ \mu_i \le 1$.

ISSN(Online): 2320-9801
ISSN (Print): 2320-9798

International Journal of Innovative Research in Computer and Communication Engineering

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2.2. Properties of λμ-B-spline curves

Theorem 1. The λμ-B-spline basis functions $b_j^4(t)$ (j = 0, 1, 2, 3) (1) have the following properties:

(a) **Degeneracy**. In the particular case where the shape control parameter λ and μ are equals zero, the λμ-B basis functions are just the classical ones of the same degree.

(b) **Nonnegativity**. When $-2 \le \lambda_i,\ \mu_i \le 1$, there are $b_j^4(t) \ge 0$ (j = 0, 1, 2, 3).

(b) **Partition of Unity**. One has $\sum_{i=0}^{n} b_j^4(t) = 1;$

(c) **Symmetry**. When $\lambda_i = \mu_i$, $b_j^4(t)$ (j = 0, 1, 2, 3) are symmetric,

that is $b_0^4(t) = b_3^4(1-t)$ and $b_1^4(t) = b_2^4(1-t)$.

2.3 λμ -B-Spline Curves:

Given control points $\mathbf{P}_i \in \mathbf{R}^n$ (n = 2, 3; i = 0, 1, . . . n) and knots $u_1 < u_2 < \cdots < u_{n+1}$, the curves

$$C_i^4(\lambda_i, \mu_i; t) = \sum_{i=0}^{n} P_{i+j-3} b_j^4(t) \qquad (2)$$

$$t \in [0,1]; i = 3, 4, \ldots\ldots, n;$$

are called quartic λμ -B-spline curve segments, where $b_j^4(t)$ (j = 0, 1, 2, 3) are λμ -B functions. All the curve segments make up the piecewise quartic blending spline curves with parameters λ_i and μ_i, defined as

$$C(\lambda_i, \mu_i; u) = C_i^4(\lambda_i, \mu_i; \frac{u - u_i}{\Delta u_i}) \;;\quad u \in [u_i,\ u_{i+1}] \subset [u_3,\ u_{n+1}] \qquad (3)$$

where, $\Delta u_i = u_{i+1} - u_i$ i = 3, 4, . . . n. We call the Formula (3) as a λμ -B-spline curve for short. In particular, when $\lambda_i = \mu_i = 0$, $C_4(\lambda_i, \mu_i; u)$ degenerates to the traditional cubic uniform B-spline curve.

2.4 Properties of λμ -B-Spline Curves:

According to Definition 1 and Theorem 1, it is easy to obtain the following properties for quartic λμ-B-spline curves:

(1) Geometric property at the endpoints: From the definition of a quartic λμ-B-spline curve, we can easily obtain

$$\begin{aligned} C_i^4(\lambda_i, \mu_i; 0) &= [\mathbf{P}_{i-3} + 4\mathbf{P}_{i-2} + \mathbf{P}_{i-1}] \\ C_i^4(\lambda_i, \mu_i; 1) &= [\mathbf{P}_{i-2} + 4\mathbf{P}_{i-1} + \mathbf{P}_i] \\ (C_i^4)'(\lambda_i, \mu_i; 0) &= (3 + \lambda_i)[\mathbf{P}_{i-1} - \mathbf{P}_{i-3}] \\ (C_i^4)'(\lambda_i, \mu_i; 1) &= (3 + \mu_i)[\mathbf{P}_i - \mathbf{P}_{i-2}] \end{aligned} \qquad (4)$$

ISSN(Online): 2320-9801
ISSN (Print): 2320-9798

International Journal of Innovative Research in Computer and Communication Engineering

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(2) Geometric invariance: The shape of a quartic λμ-B-spline curve is dependent on the choice of coordinate system because $C_4(\lambda_i, \mu_i; u)$ is an affine transformation of the control points. An affine transformation for the curve can be achieved by carrying out the same affine transformation on the control polygon.

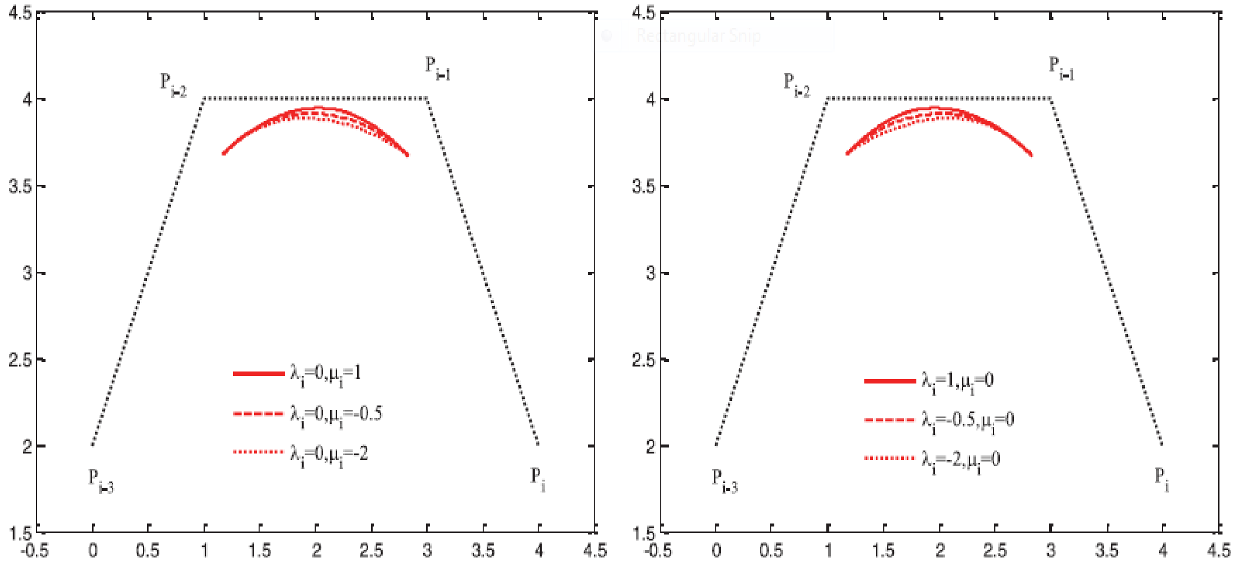
(3) Symmetry: When $\lambda_i = \mu_i$, the control points of a quartic λμ-B-spline curve can be labeled as or $\mathbf{P}_0, \mathbf{P}_1, \ldots, \mathbf{P}_n$ or $\mathbf{P}_n, \mathbf{P}_{n-1}, \ldots, \mathbf{P}_0$ without changing the shape of the curve.

(4) Convex hull property: When $-2 \le \lambda_i,\ \mu_i \le 1$, the basic functions are nonnegative by applying Theorem1.

A curve segment $C_i^4(\lambda_i, \mu_i; t)$ defined on $t \in [0, 1]$ lies inside the convex hull $\mathbf{H_i}$ of the control points $\mathbf{P}_{i-3}$, $\mathbf{P}_{i-2}$, $\mathbf{P}_{i-1}$, $\mathbf{P}_i$ and the entire curveg $C(\lambda_i, \mu_i; u)$ given in (3) lies inside the union of $\mathbf{H_i}$.

(5) Shape adjustable property: Given a control polygon, the shape of the traditional cubic uniform B-spline curve will be completely determined. But for a quartic λμ-B-spline curve $C_4(\lambda_i, \mu_i; u)$, the shape has not been fixed yet. Fixing a control polygon, the shape of the quartic λμ-B-spline $C_4(\lambda_i, \mu_i; u)$ curve remains adjustable by alerting the local shape parameters. Although the rational B-spline curve also has weights and shape modification in a control polygon, it suffers from several other issues due to the relative complexity of rational basis functions. For example, the rational form may be unstable with derivatives and integrals being difficult to compute. Compared to rational B-spline curve, the structure of quartic λμ-B-spline curve is simple.

Now we turn to the analysis of local shape adjustability and a better approximation to the control polygon of quartic λμ-B-spline curve segments, with a brief comparison to the traditional B-spline curve (see Fig. 1).



(a) λ_i fixed and μ_i changed (b) μ_i fixed and λ_i changed

ISSN(Online): 2320-9801
ISSN (Print): 2320-9798

International Journal of Innovative Research in Computer and Communication Engineering

(An ISO 3297: 2007 Certified Organization)

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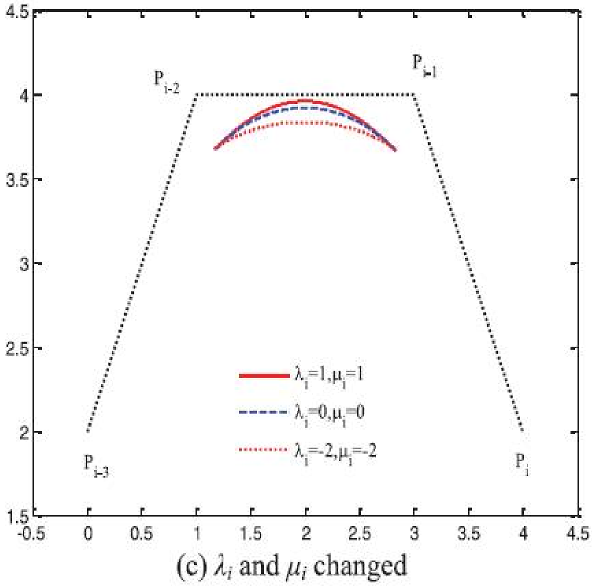

Fig. 1. The effect of changing the shape parameters on quartic λμ-B-spline curve segments. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article).

Fig. 1 illustrates the quartic λμ-B-spline curve segment $C_i^4(\lambda_i, \mu_i; t)$ with the same control polygon but different shape parameters. When λi= μi= 0, $C_i^4(\lambda_i, \mu_i; u)$ degenerates to the traditional cubic uniform B-spline curve (blue dashed line). From Fig. 1 we can get the following results:

(1) When we select the same λ_i but different μ_i, the quartic λμ-B-spline curve segment $C_i^4(\lambda_i, \mu_i; t)$ approaches to (or departs from) control points $\mathbf{P}_{i-1}$ and line $\mathbf{P}_{i-2}\,\mathbf{P}_{i-1}$ with the increasing (or decreasing) of shape parameters μ_i .

(2) When we select the same μ_i but different λ_i, the quartic λμ-B-spline curve segment $C_i^4(\lambda_i, \mu_i; t)$ approaches to (or departs from) control points $\mathbf{P}_{i-2}$ and line $\mathbf{P}_{i-2}\,\mathbf{P}_{i-1}$ with the increasing (or decreasing) of shape parameters λi.

(3) When we select different λ_i and μ_i, the quartic λμ-B-spline curve segment $C_i^4(\lambda_i, \mu_i; t)$ approaches to (or departs to)overall the control polygon with the concurrent increasing (or decreasing) of shape parameters λ_i and μ_i.

Therefore, quartic λμ-B-spline curve segments have local shape adjustability in comparison to the traditional cubic uniformB-spline curves.

2.5. Performance Comparison of λμ -Bezier Curves, Bezier Curves, And NURBS:

A B´ezier curve is defined as a parametric one which forms the basis of the Bernstein function. However, once the control points and their corresponding Bernstein polynomials are given, the shape of a B´ezier curve is formed uniquely and there is no possibility to adjust it anymore. Modifying the shape of B´ezier curves essentially requires the adjustments of vertexes of the control polygon, which is very inconvenient. For these reasons, the problem of shape modification of curves is proposed. Although the Curves, and NURBS. A B´ezier curve is defined as a parametric one which forms the basis of the Bernstein function.
However, once the control points and their corresponding Bernstein polynomials are given, the shape of a B´ezier curve is formed uniquely and there is no possibility to adjust it anymore. Modifying the shape of B´ezier curves essentially requires the adjustments of vertexes of the control polygon, which is very inconvenient. For these reasons, the problem

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ISSN (Print): 2320-9798

International Journal of Innovative Research in Computer and Communication Engineering

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of shape modification of curves is proposed. Although the weights in NURBS method can adjust the shapes of NURBS curve and the NURBS curve has good properties and can express the conic section, the NURBS curve also has disadvantages, such as difficulty in choosing the value of the weight, the increased order of rational fraction caused by the derivation, and the need for a numerical method of integration.

The shape parameters are applied to generate some curves whose shape is adjustable as an extension of the existing method. The $\lambda\mu$ -B´ezier curves (4) have most properties of the corresponding classical B´ezier curves. Moreover, the shape parameter can adjust the shape of the $\lambda\mu$ -B´ezier curves without changing the control points. With the increasing of the shape parameter, the $\lambda\mu$ -B´ezier curves approach to the control polygon or control net, and the $\lambda\mu$ - B´ezier model can approximate the control polygon or control net better than the classical B´ezier model.

In addition, the expressions of $\lambda\mu$ - B´ezier curves defined in this paper are more concise compared with the B´ezier curves and NURBS curves. Particularly, when the shape parameter $\lambda=\mu=0$ equals zero, the $\lambda\mu$ -B´ezier curves (4) degenerate to the classical B´ezier curves. To sum up, with the extra degree of freedom provided by the shape parameter λ and μ in $b_i^n(\lambda_i,\mu_i)\geq 0$ (i = 0, 1, . . . , n),the curves can be freely adjusted and controlled by changing the value of λ_i and μ_i instead of changing the control points $\mathbf{P}_0, \mathbf{P}_1, \ldots, \mathbf{P}_n$. Performances of $\lambda\mu$ -B´ezier curves, B´ezier curves, and NURBS curves are compared in detail in Table 1.

Table 1 :Performance comparisons of $\lambda\mu$-Bézier curves, Bézier curves, and NURBS.

Property	curves	curves	curves
Property of basis functions			
Nonnegativity	✓	✓	✓
Partition of unity	✓	✓	✓
Symmetry	✓	✓	✓
Shape parameters	✓	M	-
Linear independence	✓	✓	✓
Degeneracy	✓	M	✓
Property of the Curves			
Variation diminishing property	✓	✓	✓
Affine invariability	✓	✓	✓
Convex hull property	✓	✓	✓
Symmetry	✓	✓	✓
End-point properties	✓	✓	✓
Extra degree of freedom	✓	M	✓

III. SHAPE MODIFICATION FOR $\lambda\mu$-BÉZIER CURVES BY CONSTRAINED OPTIMIZATION

$\lambda\mu$ - B´ezier curve of degree n with control points $\mathbf{P}_0, \mathbf{P}_1, \ldots, \mathbf{P}_n$ ($n \geq 2$) is

$$C_1(\lambda_1,\mu_1;t)=\sum_{i=0}^{n} P_i b_i^4((\lambda_1,\mu_1;t) \qquad t \in [0, 1] \tag{5}$$

ISSN(Online): 2320-9801
ISSN (Print): 2320-9798

International Journal of Innovative Research in Computer and Communication Engineering

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Modified curves are

$$C_2(\lambda_1,\mu_1;u)=\sum_{i=0}^{l-1}P_ib_i^4(\lambda_1,\mu_1;u)+\sum_{i=l}^{m}(P_i+\delta_i)b_i^4(\lambda_1,\mu_1;u)+\sum_{i=m+1}^{n}P_ib_i^4(\lambda_1,\mu_1;u);$$
$$C_3(\lambda_2,\mu_2;u)=\sum_{i=0}^{l-1}P_ib_i^4(\lambda_2,\mu_2;u)+\sum_{i=l}^{m}(P_i+\delta_i)b_i^4(\lambda_2,\mu_2;u)+\sum_{i=m+1}^{n}P_ib_i^4((\lambda_2,\mu_2;u); \qquad (6)$$

Here $\delta_i=(\delta_i^x\delta_i^y\delta_i^z)^T$; (i = l, l +1,......,m) reperturbations of control points **P***i* (*i* = *l*, *l* + 1, . . . , *m*).

IV. SHAPE CONTROL OF THE $\lambda\mu$- BÉZIER CURVE

For control points $\mathbf{P}_i \in \mathbf{R}^n$ (n = 2, 3; i = 0, 1, . . . n) and knots $u_1 < u_2 < \cdot\cdot\cdot < u_{n+1}$, the curves

$$C_i^4(\lambda_i,\mu_i;t)=\sum_{i=0}^{n}P_{i+j-3}b_j^4(t) \qquad t\in[0,1]; i=3,4,.....,n;$$

We rewrite this equation as follows;

$$C(\lambda_i,\mu_i;t)=\sum_{i=0}^{4}P_ib_i^4(t)$$
$$C(\lambda_i,\mu_i;t)=\frac{1}{6}\left\{\sum_{i=0}^{4}P_iD_i(t)+\lambda_it(1-t)^3(P_2-P_0)+\mu_it^3(1-t)(P_1-P_3)\right\}$$

where

$$D_0(t)=(1-t)^3$$
$$D_1(t)=4-6t^2+3t^3$$
$$D_2(t)=1+3t+3t^2-3t^3$$
$$D_3(t)=t^3$$

Obviously, shape parameter λ_i and μ_i affects the curve on the control edges (P_2-P_0) and (P_1-P_3).The shape parameter λ_i and μ_i serves to effect local control in the curve: as λ_i and μ_i increases, the curve moves in the direction of edges (P_2-P_0) and (P_1-P_3) and as λ_i and μ_i decreases, the curve moves in the opposite direction to the edges (P_2-P_0) and (P_1-P_3).Figure 1 shows a computed example of $\boldsymbol{\lambda\mu}$ Bézier Curves with different values of shape parameter. These curves are generated by setting $\lambda_i=-1$, $\mu_i=1$ (red solid), $\lambda_i=10$, $\mu_i=0$ (blue solid lines) and λ_i = -2, μ_i=-2 (red dotted lines).

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International Journal of Innovative Research in Computer and Communication Engineering

(An ISO 3297: 2007 Certified Organization)

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V. APPROXIMABILITY

Control polygon provides an important tool in geometric modeling. It is an advantage if the curve being modeled tends to preserve the shape of its control polygon. Now we show the relations of the cubic trigonometric Polynomial B-spline curve and cubic Bézier curves corresponding to their control polygon.

Suppose P_0,P_1,P_2,and P_3 are not collinear; the relationship between cubic trigonometric Polynomial B-spline curve $C(\lambda_i,\mu_i;t)$ and the quartic Bézier curve;

$$B_i(t) = \sum_{j=0}^{4} p_{i+j-1}(4,j)(1-t)^{4-j}t^j$$

$t \in [0, 1]$ with the same control points P_i (i=0,1,2,3) are given by

$$C(\lambda_i,\mu_i;\frac{1}{2}) - P_2 = \frac{1}{6}\left\{\frac{1}{8}\left(P_0 + 23P_1 + 15P_2 + P_3\right) + \frac{\lambda_i}{16}(\mathbf{P}_2 - \mathbf{P}_0) + \frac{\mu_i}{16}(\mathbf{P}_1 - \mathbf{P}_3)\right\}$$

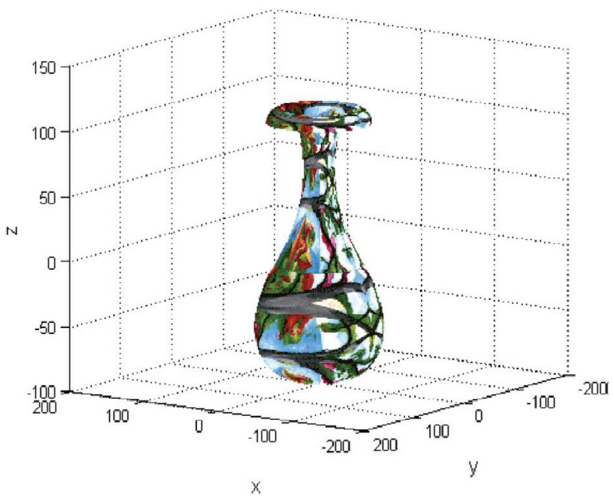
and

$$B_i(1/2) - P_2 = (1/16)\,(P_{i-1} + 4P_i - 10P_{i+1} + 4P_{i+2} + P_4)\ ;$$

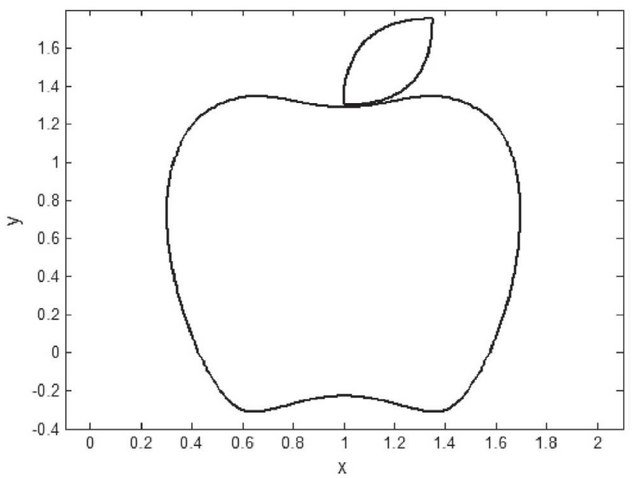
These equations shows that quartic $\boldsymbol{\lambda\mu}$ Bézier Curves can be made closer to the control polygon by altering the values of shape parameters.

VI. APPLICATIONS

The $\lambda\mu$-B-spline curves are an extension to the classical cubic uniform B-spline curves. Besides of inheriting the properties of them, $\lambda\mu$-B-spline curves have a good performance on adjusting their local shapes by changing multiple shape parameters. Since $\lambda\mu$-B-spline rotational surfaces are produced by utilizing a $\lambda\mu$-B-spline curve called generating line, they not only inherit the outstanding properties and advantages of $\lambda\mu$-B-spline curves, but also possess multiple local shape parameters. Moreover, these shape parameters can adjust the shape of the engineering complex rotational surfaces without changing the control points of generating line. To sum up, the methods described in this paper can provide a new class of mathematical theory for application software development of CAD/CAM, which includes manufacturing industry, computer graphics, computer vision, computer animation, multimedia technology, etc. By the use of $\lambda\mu$-B-spline curves and $\lambda\mu$-B-spline rotational surfaces, we can expediently construct various kinds of complex curves and rotational surfaces. In order to demonstrate the approaches which can be applied to curve and surface deformation.



. **Coloured drawing surface**



Complex curves of apple (λ_i =-2, μ_i =-2)

ISSN(Online): 2320-9801
ISSN (Print): 2320-9798

International Journal of Innovative Research in Computer and Communication Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 8, August 2016

VI. CONCLUSION

$\lambda\mu$-B-spline curves constructed in this paper have the similar structure as the cubic uniform B-spline ones. While inheriting the main advantages of the B-spline curves, they exhibit better performance when adjusting their local shapes by changing the two local shape control parameters, with many advantages including shape adjustability and a better approximation to the control polygon . $\lambda\mu$-B-spline curve segments have the shape adjustability in comparison to the traditional ones and possess a better approximation to control polygons. The examples provided show the efficacy of this algorithm and how it can be applied to CAD/CAM modeling systems. Finally, we know that many different shapes of $\lambda\mu$-B-spline rotational surfaces can be obtained through adjustment of its shape parameters. In this paper, we give the definition of $\lambda\mu$-B´ezier curves and discuss their properties in detail. It is shown that $\lambda\mu$--B´ezier curves of degree n with shape parameter keep many properties of the corresponding traditional B´ezier curves and are more convenient than traditional ones. We can alter the shape of $\lambda\mu$-B´ezier curve by modifying the values of the shape parameter without changing its control points. Further, we investigate the shape modification of $\lambda\mu$-B´ezier curves for constrained optimization of single point constraint (including modification of shape parameter and control points)..Future work will focus on studying the shape modification for λ-B´ezier surfaces.

VII. ACKNOWLEDGEMENTS

This work is supported by the University Grant Commission of India (Grant F.No. MS-108/106045/13-14/CRO). We are very grateful to the editors and reviewers for their helpful suggestions and comments.

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