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Optimal Two Stage Flow Shop Scheduling Problem with Branch and Bound Method Including Transportation Time

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ABSTRACT: This paper studies two stage flow shop scheduling problem in which processing times are associated with their respective probabilities and transportation times from one machine to another are given. The objective of the study is to get optimal sequence of jobs in order to minimize the total elapsed time. The given problem is solved with branch and bound method. The method is illustrated by numerical example.

Keywords: Transportation Time, processing time, elapsed time, Branch and Bound.

I. INTRODUCTION

Flow shop scheduling models are effective tools for management that can be utilized for modelling many service and production processes such as continuous production systems. In some flow shop scheduling problems, each manufacturing centres has more than one machine, each job is started through the first machine and then goes to next machine in order, ending up with the last machine. In flow shop scheduling problems, all machines are arranged in constant series. The main objective is to find the best possible schedule and sequence for jobs in order to minimize the makespan. A flowshop scheduling problem has been one of classical problems in production scheduling since Johnson [6] proposed the well known Johnson's rule in the two-stage flowshop makespan scheduling problem. Yoshida and Hitomi [11] further considered the problem with setup times. Yang and Chern [10] extended the problem to a two-machine flowshop group scheduling problem. Maggu and Dass[9] introduced the concept of equivalent job for a job-block when the situations of giving priority of one job over another arise. Kim, et al.[7] considered a batch scheduling problem for a two-stage flowshop with identical parallel machines at each stage. Brah and Loo [1] studied a flow shop scheduling problem with multiple processors. Futatsuishi, et al. [4] further studied a multi-stage flowshop scheduling problem with alternative operation assignments.

Lomnicki [8] introduced the concept of flow shop scheduling with the help of branch and bound method. Further the work was developed by Ignall and Scharge [5], Chandrasekharan [3] , Brown and Lomnicki [2], with the branch and bound technique to the machine scheduling problem by introducing different parameters. Practically scheduling problem depends upon the significant factors namely, Transportation time. The transportation times (loading time, moving time and unloading etc.) from one machine to another are also not negligible and therefore must be included in the job processing. However, in some application, transportation time have major impact on the performance measures considered for the scheduling problem so they need to considered separately. In this paper we consider a two-stage flowshop scheduling problem including Transportation time with the help of branch and bound method. Branch and bound is an exact method usually used in scheduling problems to find optimal solutions. This method requires three components: a lower bound (LB), an upper bound and a branching strategy. Branch and bound provides a systematic enumeration procedure that considers bounds on the objective function for different subsets of solutions. Subsets are eliminated when their lower bound is dominated by the bound of other subset. The procedure is repeated until the search tree is exhausted and the optimal solution is found. The given method is very simple and easy to understand. Thus, the problem discussed here has significant use of theoretical results in process industries.

II. ASSUMPTIONS

i. No passing is allowed.

ii. Each operation once started must performed till completion.

iii. Jobs are independent to each other.

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iv. A job is entity, i.e. no job may be processed by more than one machine at a time.

v. $\sum p_i=1$, $\sum q_i=1$, $0\le p_i\le 1$ and $0\le q_i\le 1$.

III. NOTATIONS

We are given n jobs to be processed on two stage flowshop scheduling problem and we have used the following notations:

a_i : Processing time for i^{th} job on machine A
b_i : Processing time for i^{th} job on machine B
p_i : Probability associated with processing time a_i.
q_i : Probability associated with processing time b_i.
S_0 : Optimal sequence
t_i : Transportation time of i^{th} job from machine A to machine B
A_i : Expected processing time of i^{th} job on machine A
B_i : Expected processing time of i^{th} job on machine B
J_r : Partial schedule of r scheduled jobs
$J_{r'}$: The set of remaining (n-r) free jobs

IV. MATHEMATICAL DEVELOPMENT

Consider n jobs say i=1, 2, 3 … n are processed on two machines A & B in the order AB. A job i (i=1,2,3…n) has processing times a_i & b_i associated with their respective probabilities p_i and q_i on each machine A & B respectively. Let A_i be the expected processing time on machine A and B_i be the expected processing time on machine B. Let t_i be transportation time of i^{th} job from machine A to machine B.

The mathematical model of the problem in matrix form can be stated in tableau-1 :

Jobs	Machine A		t_i	Machine B	
i	a_i	p_i		b_i	q_i
1	a_1	p_1	t_1	b_1	q_1
2	a_2	p_2	t_2	b_2	q_2
3	a_3	p_3	t_3	b_3	q_3
-	---	-	-	--	-
n	a_n	p_n	t_n	b_n	q_n

Tableau – 1

Our objective is to obtain the optimal schedule of all jobs which minimize the total elapsed time, using branch and bound technique.

V.ALGORITHM

Step 1: Calculate Expected processing time $A_i = a_i \times p_i$ and $B_i = b_i \times q_i$

Step 2: (i) $G_i = A_i + t_i$ and (ii) $H_i = B_i + t_i$

Step 3: Calculate

(i) $l_1 = t(J_r,1) + \sum_{i\in J_r'} G_i + \min_{i\in J_r'}(H_i)$

(ii) $l_2 = t(J_r,2) + \sum_{i\in J_r'} H_i$

***Step* 4**: Calculate

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$$l = \max(l_1, l_2)$$

We evaluate l first for the n classes of permutations, i.e. for these starting with 1, 2, 3………n respectively, having labelled the appropriate vertices of the scheduling tree by these values.

Step 5**:** Now explore the vertex with lowest label. Evaluate l for the (n-1) subclasses starting with this vertex and again concentrate on the lowest label vertex. Continuing this way, until we reach at the end of the tree represented by two single permutations, for which we evaluate the total work duration. Thus we get the optimal schedule of the jobs.

Step 6**:** Prepare in-out table for the optimal sequence obtained in step 6 and get the minimum total elapsed time.

VI. NUMERICAL EXAMPLE

Consider 4 jobs 2 machine flow shop problem whose processing time of the jobs on each machine is given in tableau-2

Jobs	Machine A		t_i	Machine B	
i	a_i	p_i		b_i	q_i
1	80	0.1	5	40	0.4
2	65	0.2	3	85	0.2
3	20	0.4	6	30	0.1
4	70	0.3	4	30	0.3

Tableau – 2

Our objective is to obtain optimal schedule for above said problem.

Solution: **As per Step1:** The expected processing time as in tableau-3

job	Machine A	t_i	Machine B
i	A_i		B_i
1	8	5	16
2	13	3	17
3	8	6	3
4	21	4	9

Tableau – 3

Step2: Calculated values are as in tableau-4

job	Machine A	Machine B
i	G_i	H_i
1	13	21
2	16	20
3	14	9
4	25	13

Tableau – 4

Node	$LB(J_r)$
1	77
2	79
3	81
4	88
12	77
13	81
14	80
123	81
124	77

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Step3: Calculate

(i) $l_1 = t(J_r,1) + \sum_{i \in J'_r} G_i + \min_{i \in J'_r}(H_i)$

(ii) $l_2 = t(J_r,2) + \sum_{i \in J'_r} H_i$

For J_1 = (1).Then J′(1) = {2,3,4}, we get l_1 = 77 , l_2 = 76

LB(1) $l = \max(l_1, l_2)$ =77

Similarly, we have LB(2)= 79, LB(3)= 81, LB(4)= 88

Step 3 & 4:

Now branch from J_1 = (1). Take J_2 =(12).Then J'_2={3,4} and LB(12) = 77

Proceeding in this way, we obtain lower bound values as shown in the tableau- 5

Step 5 :

Therefore the sequence S_1 is 1-2-4-3 and the corresponding in-out table for sequence S_1 is as in tableau-5:

And the tree is as follows:

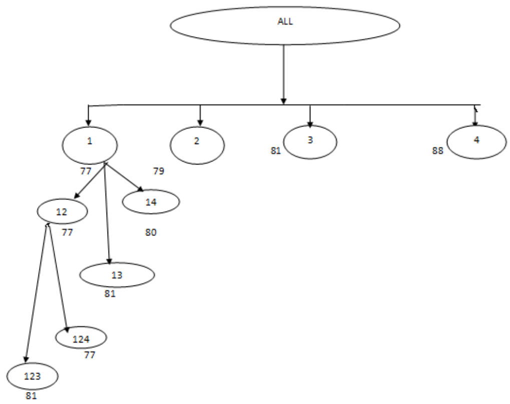

Figure 1: Lower bounds for the partial schedules

Hence the total elapsed time is 59 units.

VII. REMARKS

The study may further be extended by considering various parameters such as break down interval, due date, mean weightage time etc.

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