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Calculation of Moisture Content Arising in uni-Dimensional Flow through Homogenous Porous Media

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ABSTRACT: The present paper deals with the problem related to the hydrological situation of uni - dimensional vertical ground water recharge by dispersion. This type of problem plays an important role in water resource science, ground engineering and farming sciences. These type of flows occur in the water intrusion system and the underground disposal of leakage. This problem yields a nonlinear partial differential equation. The solution is obtained by Successive over Relaxation method.

KEYWORDS: Ground water, Homogeneous Porous Media, S.O.R.

I. INTRODUCTION

Due to content changes,which is a function of time,the unsteady flow of water through soils occurs.The entire pore spaces is considered not completely filled with flowing liquid. Because of this reason, unsaturated flow of water through soils occurs.The Knowledge concerning such flow is very useful to hydrologist, farmers and various fields of science.[6,12].

Drainage is a provision for the exclusion of additional water. The common objective of land projects to prevent or eliminate either water logging or accumulation. Practically every area where irrigation has been carried on for time has been affected by high water table. Therefore in planning, construction and operation of an irrigation project, provision for enough drainage is an essential part.

Many Authors [1,10] have discussed this problem in different ways and used different methods to obtain the solution.

II. STATEMENT OF THE PROBLEM

Consider a large basin of such geological configuration whose sides are limited by rigid boundaries and the bottom by a thick layer of water table. Consider the groundwater recharge which takes place over such a basin. The flow is assumed vertically downwards through unsaturated porous media [5].

Diffusivity co-efficient is considered as a constant and it isisassumed to be equivalent to its average value over the whole range of moisture content. Permeability of the medium is assumed as a continuous linear function of the moisture content [6]. The governing equation to the given model is a nonlinear partial differential equation. Solution is obtained by Successive over Relaxation method.

III. MATHEMATICAL FORMULATION

The continuity equation for an unsaturated porous medium [3,4] is given by

$$\frac{\partial}{\partial t}(\rho_m \gamma) = -\mathrm{div}M \qquad (1)$$

Where ρ_m is thebulk density of the medium, γ is its moisture content on a dry bulk basis, and M is the mass fluctuation of moisture.

According to Darcy's law [7,12] for the motion of water in a porous medium, we have,

$$\vec{V} = -k grad\varphi \qquad (2)$$

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Where $grad\varphi$ is the volume flux of moisture and k is the co-efficient of aqueous conductivity[13].
From (1) and (2) we obtain,

$$\frac{\partial}{\partial t}(\rho_m\gamma) = \mathrm{div}(\rho k \nabla\varphi) \qquad (3)$$

Where $M = \rho\vec{V}$, ρ is the flux density .
Our problem is concerned with the flow which takes place only in the vertical direction [8,9].
Let $\varphi = \psi - gz$ [11,12].
Where ψ is the capillary pressure, g is the gravitational force which is constant and the positive direction of the z-axis is the same as that of the gravity[6].
Therefore equation (3) becomes

$$\rho_m\frac{\partial\gamma}{\partial t} = \frac{\partial}{\partial z}\left(k\rho\frac{\partial\psi}{\partial z}\right) - \frac{\partial}{\partial z}(k\rho g) \qquad (4)$$

Let us consider γ and ψ to be connected by a single valued function, equation (4) can be written as,

$$\frac{\partial\gamma}{\partial t} = \frac{\partial}{\partial z}\left(\frac{\rho}{\rho_m}k\frac{\partial\psi}{\partial\gamma}\frac{\partial\gamma}{\partial z}\right) - \frac{\rho g}{\rho_m}\frac{\partial k}{\partial z}$$

Where $D_a = \frac{\rho k_0}{\rho_m}\frac{\partial\psi}{\partial\gamma}$ and is called average value of diffusivity co-efficient, and
Assuming $k = k_0\gamma$, where k_0 is the slope of the permeability vs moisture content plot ,from the above equation,

$$\frac{\partial\gamma}{\partial t} = \frac{\partial}{\partial z}\left(D_a\gamma\frac{\partial\gamma}{\partial z}\right) - \frac{\rho g k_0}{\rho_m}\frac{\partial\gamma}{\partial z} \qquad (5)$$

Let us consider the water table is situated at a depth L and substituting

$$\frac{z}{L} = \xi, \quad \frac{D_a}{L^2}t = T, \quad \beta = \frac{\rho k_{0g}}{\rho_{mD_a}}$$

Equation (5) reduces into dimensionless form as follow:

$$\frac{\partial\gamma}{\partial T} = \frac{\partial}{\partial\xi}\left(\gamma\frac{\partial\gamma}{\partial\xi}\right) - \beta\frac{\partial\gamma}{\partial\xi} \qquad (6)$$

ξ = Penetration depth,
T = Time
β= Flow parameter(cm^2)
ρ = Mass density of water(gm)
Initially the moisture content is assumed to be zero throughout the region. At z =0 it is γ_0, and at z = L it is assumed to remain 100% throughout the process of investigation. As capillary action at the stationary groundwater level is very small, , it is neglected.
The appropriate boundary conditions are given by

$$\gamma(0,T) = \gamma_0\,, \quad \gamma(1,T) = 0 \qquad (7)$$

$$\gamma(\xi,0) = 0 \qquad (8)$$

IV. MATHEMATICAL SOLUTION OF THE PROBLEM

Using Finite differences for (6),we have,

$$\frac{\gamma_{m,,n+1} - \gamma_{m,n}}{k} = \frac{1}{2h^2}\left[\gamma^2{}_{m+1,n} - 2\gamma^2{}_{m,n} + \gamma^2{}_{m-1,n}\right] - \frac{\beta}{h}(\gamma_{m+1,n} - \gamma_{m,n})$$

$$\gamma_{m,,n+1} = \gamma_{m,n} + \frac{k}{2h^2}\left[\gamma^2{}_{m+1,n} - 2\gamma^2{}_{m,n} + \gamma^2{}_{m-1,n}\right] - \frac{\beta k}{h}(\gamma_{m+1,n} - \gamma_{m,n}) \qquad (9)$$

Put $\boldsymbol{r} = \frac{\boldsymbol{k}}{\boldsymbol{h}^2}$ in (10) and using Gauss-Sheidal method [14]

$$\gamma_{m,,n+1} = \gamma_{m,n} + \frac{r}{2}\left[\gamma^2{}_{m+1,n} - 2\gamma^2{}_{m,n} + \gamma^2{}_{m-1,n+1}\right] - \beta rh(\gamma_{m+1,n} - \gamma_{m,n}) \qquad (10)$$

Setting $\lambda_m = (1+\beta rh)\gamma_{m,n} + \frac{r}{2}\left[\gamma^2{}_{m+1,n} - 2\gamma^2{}_{m,n}\right] - \beta rh\gamma_{m+1,n}$ (11)

Setting $\lambda_m = (1+\beta rh)\gamma_{m,n} + \frac{r}{2}\left[\gamma^2{}_{m+1,n} - 2\gamma^2{}_{m,n}\right] - \beta rh\gamma_{m+1,n}$ (11)

$$\gamma_{m,,n+1} = (1-\omega)\gamma_{m,n} + \omega\left[\lambda_m + \frac{r}{2}\gamma^2{}_{m-1,n+1}\right] \qquad (12)$$

Choose k = 0.01, h = 0.1, $\beta = 2.5$, $\omega = 1.67$

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From (12),

$$\lambda_m = 1.25\gamma_{m,n} + 0.5[\gamma^2_{m+1,n} - 2\gamma^2_{m,n}] - 0.25\gamma_{m+1,n} \quad (13)$$

$$\gamma_{m,,n+1} = -0.67\gamma_{m,n} + 1.67[\lambda_m + 0.5\gamma^2_{m-1,n+1}] \quad (14)$$

Numerical calculation at different values of T and ξ are shown in the following table.

t→	T=0.01	T=0.02	T=0.03	T=0.04
ξ ↓	γ			
0	0.8	0.8	0.8	0.8
0.1	0.5344	0.762911	0.769186	0.719063
0.2	0.238462	0.711114	0.651465	0.695958
0.3	0.047482	0.485003	0.599018	0.655455
0.4	0.001883	0.199076	0.50272	0.597124
0.5	2.96E-06	0.033096	0.255731	0.530279
0.6	7.31E-12	0.000915	0.055903	0.307738
0.7	4.46E-23	6.99E-07	0.00261	0.082763
0.8	1.66E-45	4.07E-13	5.69E-06	0.005728
0.9	2.31E-90	1.39E-25	2.7E-11	2.74E-05
1	4.5E-180	1.6E-50	6.1E-22	6.27E-10

V. GRAPHICAL REPRESENTATION

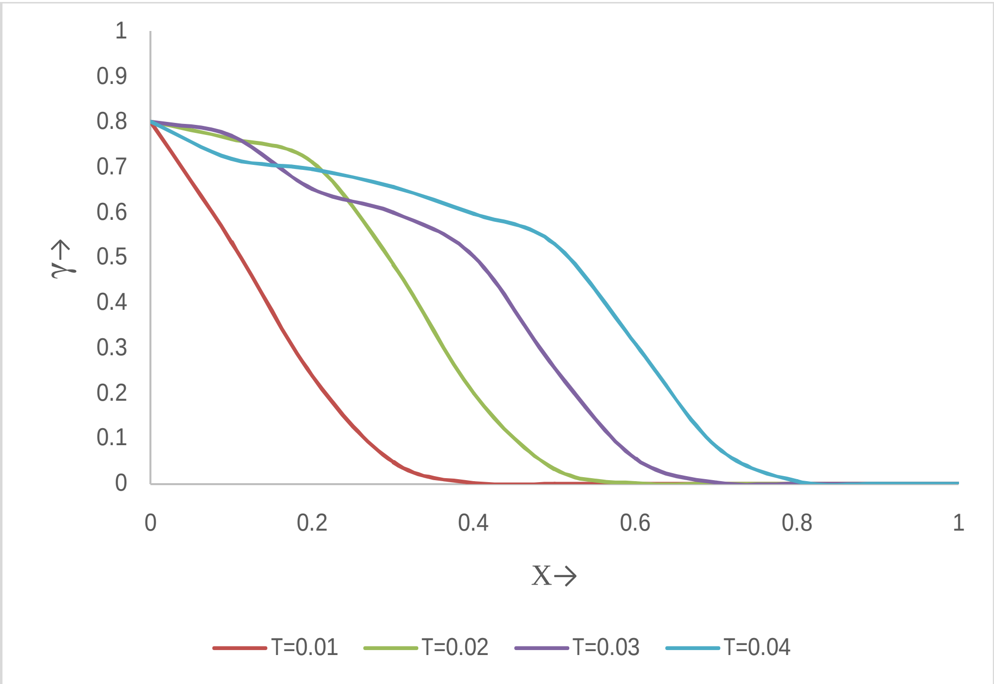

Figure -1

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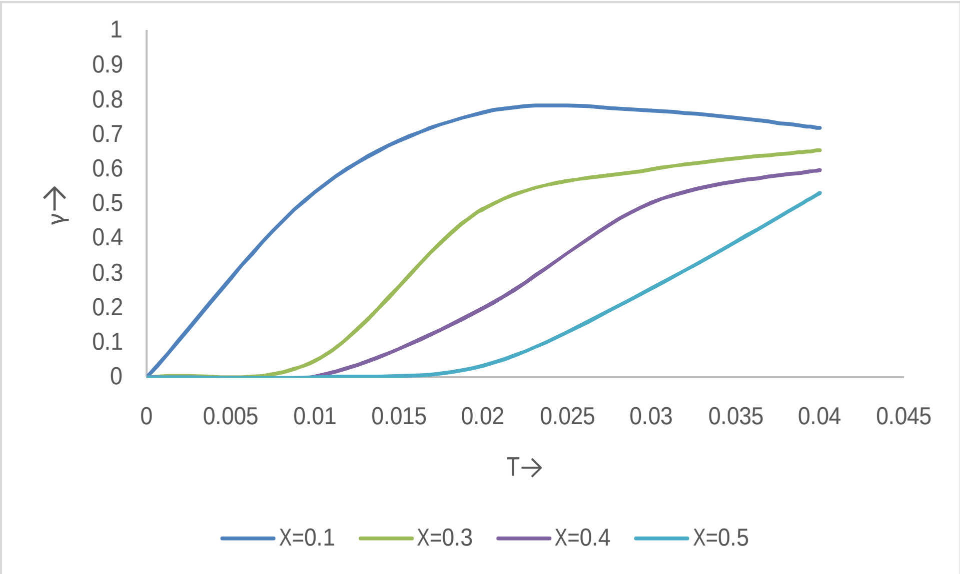

Figure -2

VI. CONCLUSION

From figure -1, it is clear that moisture content is $\gamma = \gamma_0 = 0.8$ at layer $\xi = 0$ and then it start decreasing. It decreases to to 0. i.e. as length is increasing the moisture content is decreasing.

From figure-2, it is clear that initially $\gamma = 0$ throughout the region. As time increases, the moisture content also increases and after some time and then it becomes constant.

VII. SCOPE OF THE PROBLEM

It is applicable to problems of water supply, land reclamation .It is also useful to the fields of petroleum production and agriculture.

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