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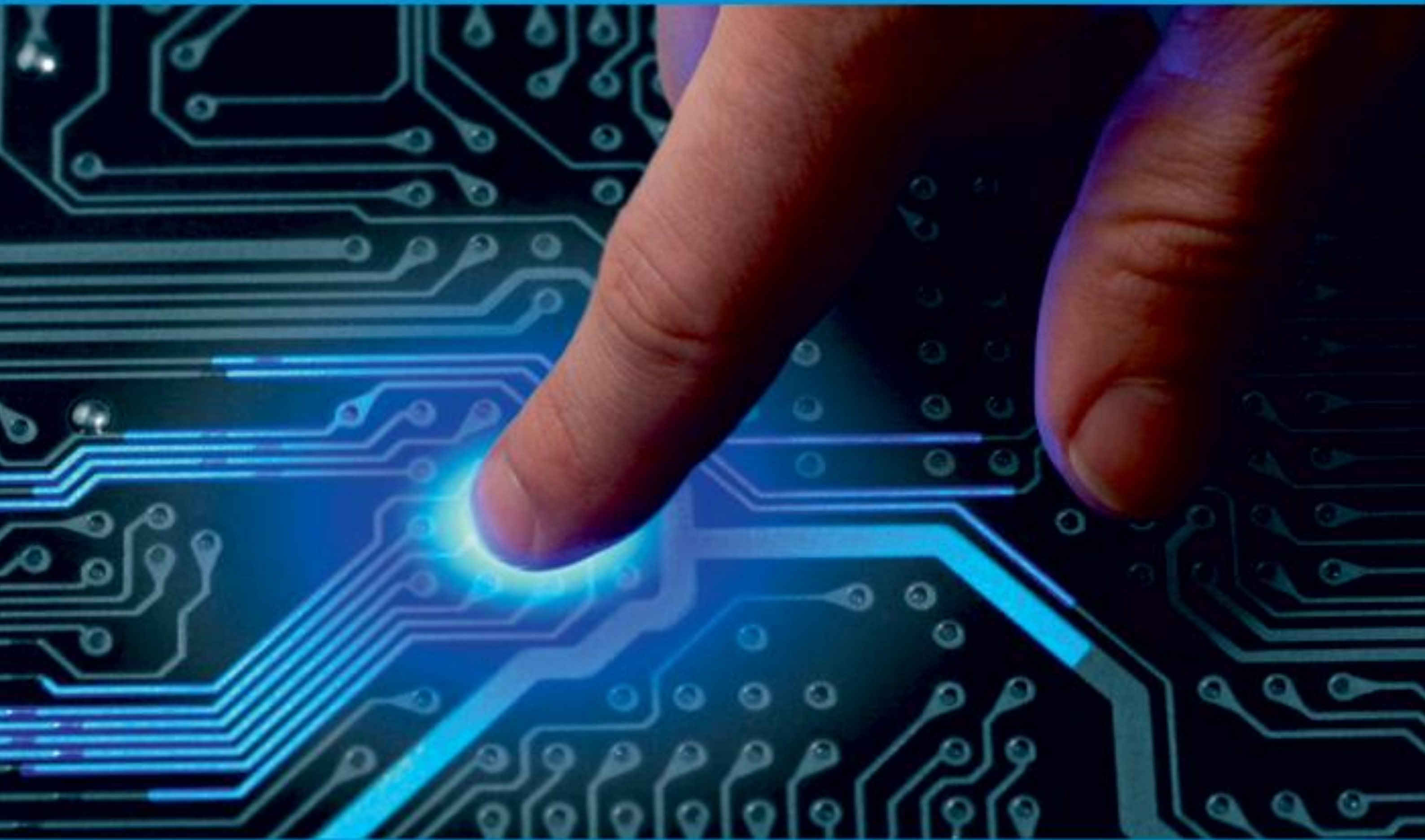

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Root-free Prony's method for Sparse signal Reconstruction

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ABSTRACT: In order to reconstruct a class of discrete-time signals with few nonzero coefficients (sparse signals) from a small number of observations, finite rate of innovation (FRI) and compressed sensing algorithms has been presented. Traditionally, Prony's and orthogonal matching pursuit algorithms are proposed to reconstruct sparse signals from the measurement samples when the dictionary is the union of Fourier and identity matrices. However, Prony's technique requires computing the polynomial roots of the annihilating filter, and this fact yields an unstable recovery of sparse of signal in the high noise environment. In this paper, we propose root-free Prony's method, which avoids polynomial root-finding. In the noiseless scenario, the proposed method is able to reconstruct perfectly the original sparse signal. Simulation results for the noisy environment demonstrate significant improvement in the performance in terms of MSE over the traditional methods, especially in the smaller SNR.

KEYWORDS: Discrete-time signals, Finite rate of innovation, Orthogonal matching pursuit, Root-free Prony's method, Sparse signal.

I. INTRODUCTION

Consider the issue of determining the signal's sparse representation in union of Fourier and identity bases when additive noise is contaminated by observations. Assume that the clean observations are given by

$$f[n] = \frac{1}{\sqrt{N}} \sum_{l=1}^{L_p} c_l \, e^{j\frac{2\pi}{N}nk_l} + \sum_{l=1}^{L_q} d_l \, \delta[n-n_l], \qquad 0\leq n<N-1 \tag{1}$$

where $0 \leq k_1 < \ldots < k_{L_p} <$ N-1 and $0 \leq n_1 < \ldots < n_{L_q} <$ N-1 are integers which corresponding to the indices of non-zero parameters and $c_l, d_l \in \mathbb{C}\backslash\{0\}$ their amplitudes. Due to presence of noise, the observed signal is given by

$$\tilde{f}\,[n] = f[n] + \gamma[n], \qquad 0\leq n<N-1, \tag{2}$$

where $\gamma[n]$ is i.i.d Gaussian noise. The observed signal is rewritten in matrix notation as follows

$$\tilde{f} = D\,g + \gamma = [F, I]\,[g_p^T, g_q^T]^T + \gamma \tag{3}$$

where $\tilde{f}, \gamma \in \mathbb{C}^N$ g_p, $g_q \in \mathbb{C}^{2P}$ and $g \in \mathbb{C}^{2P}$.

Various techniques are available in compressed sensing framework to address the noisy sparsity reconstruction issue. The two conventional methods for resolving this problem are orthogonal matching pursuit (OMP) algorithm and basis pursuit denoising (BPDN) [1,4]. Least absolute shrinkage and selection operator (LASSO) approach [2] was introduced to address the sparse reconstruction problem. Recently, a new class of greedy algorithms called subspace pursuit and compressive sampling matching pursuit (CoSaMP) [3] have been presented. These algorithms are faster than OMP while maintaining similar performance. More recently, Pro-Sparse [5] is based on a variation of Prony's methodology that makes full use of the special structure of observed matrix D = [F, I] has been proposed. Prony's method [6,7] involves solving polynomial roots of the annihilating filter which leads to unstable reconstruction when the observed signal corrupted with noise. In this work, to avoid solving polynomial roots of the annihilating filter, we proposed a root-free Prony's method (a. k. a finite dimensional FRI) [8] which is based on null space basis of linear system that yields better reconstruction performance than traditional Prony's method. The crucial insight is that, presuming the signal g_q are also a part of the noise, we may try to reproduce the original Fourier observations of observed signal based

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on Cadzow's method. Subsequently, we use root-free Prony's method to recover the signal g_q from Fourier observations.

II. PROPOSED ALGORITHM

In this section we propose a novel method based on root-free Prony's approach for sparse signal reconstruction.
First, we find g_q sparse vector by using Cadzow algorithm and then find g_p sparse vector by using root-free Prony's method which avoids solving polynomial roots of the annihilating filter. The following steps are involved in the proposed method

1. First, we assume g_q as '0' vector of size P X 1 and the corresponding index vector is Ω then we denoise the observed signal $\tilde{f}$ using Cadzow algorithm which gives denoised signal $\tilde{f}'$
2. Next, we use for loop to estimating non-zero values in g_q. Since the vector g_q having L_q non-zeros, we iterate the loop for L_q times. In each iteration we find one non-zero value and removed from the observed signal. In each iteration the following steps are involved
3. Start with computing residual vector r[n] by subtracting denoised vector $\tilde{f}'$ from original observed vector $\tilde{f}$.
4. Second step involves finding non-zero n_l location by taking maximum value of |r[n]|
5. Then store the value of the spike in g_q
6. Next remove spike obtained in previous step from observed signal '$\tilde{f}$'
7. This process repeats upto L_q iterations, after completion of L_q iterations all the spikes are removed from observed signal $\tilde{f}$.

Once all the non-zero values of g_q are removed then we find g_p sparse vector by using root-free Prony's approach which avoids solving polynomial roots of the annihilating filter. The main idea behind the root-free Prony's method is as follows:

1. Estimate annihilating filter q.

Consider filter q with L_p+ 1 components $\{q[n]\}_{n=0}^{Lp}$ whose transfer function has L_p zeros at $\eta_l = e^{j\frac{2\pi}{N}k_l}$, that is, $Q(z) = \prod_{l=1}^{Lp}(1-\eta_l z^{-1})$. It explicitly follows that

$$f[n]*q[n] = \sum_{i=0}^{Lp} q[i]\, f[n-i] = 0 \quad (4)$$

The filter $q[n]$ is called annihilating filter because it annihilates the sequence $f[n]$.
The corresponding matrix notation for above equation is given by

$$\begin{bmatrix} f[L_p] & f[L_p-1] & \cdots & f[0] \\ f[L_p+1] & f[L_p] & \ldots & f[1] \\ \vdots & \vdots & \ldots & \vdots \\ f[N-1] & f[N-2] & \ldots & f[N-L_p-1] \end{bmatrix} \begin{bmatrix} q[0] \\ q[1] \\ \vdots \\ q[L_p] \end{bmatrix} = 0 = F_q \quad (5)$$

2.Compute solutions of underdetermined system (3).

$$g_p = F^H f + \sum_{t=1}^{T} \alpha_t s_t \quad (6)$$

where F^H is the Hermitian matrix of F, $T = P - N$ is the dimension of null space of F, $\{s_t\}_{t=1}^{T}$ are null space basis of F and $\{\alpha_t\}_{t=1}^{T}$ are unknown values.

3.Find discrete Fourier transform of g_p.

Premultiplying (6) by unitary DFT matrix $U \in \mathbb{C}^{P\times P}$, we obtain DFT of g_p as

$$\hat{f} = Ug_p = U F^H f + \sum_{t=1}^{T} \propto_t U s_t \quad (7)$$

By defining $x = U F^H f$ and $U s_t = h_t$, Eq.(7) reduces to

$$\hat{f} = x + \sum_{t=1}^{T} \propto_t h_t \quad (8)$$

4. Determine unknown values $\{\alpha_t\}_{t=1}^{T}$

Let Tr{.} be an operator which maps $P \times 1$ vector $\hat{f}$ into an $(P - L_p) \times (L_p + 1)$ Toeplitz matrix $\hat{F}$= Tr{ $\hat{f}$ }. Since P elements in vector $\hat{f}$ are of the form f, the

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homogeneous system from (5) is also satisfied:

$$\hat{F}q = 0 \quad (9)$$

Substitute (8) in (9) to generate new linear system with T unknowns

$$[H_1 q\ H_2 q\ \ldots\ H_T q]\begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_T\end{bmatrix} = -Xq,$$

where $X = Tr\{x\}$ and $H_t = Tr\{h_t\}$. This is an overdetermined system, and since q is known, which yields a unique solution.

5. Build g_p - sparse signal.
From the estimated values $\{\alpha_t\}_{t=1}^{T}$ and available observations f, we build g_p -sparse vector using Eq.(6).

Algorithm: Reconstructing sparse signal g from observed signal $\tilde{f}$
1: Initialise vector $g_q = 0$.
2: Initialise indices $\Omega = \{0, 1, \ldots, P-1\}$.
3: Denoise $\tilde{f}' = \text{Cadzow}(\tilde{f}, L_p)$.
4: for i = 1 to Lq do
5: Compute residual $r = \tilde{f} - \tilde{f}'$.
6: Estimate non-zero location $n_l = \arg\max_{z\in\Omega}\{|r[n]|\}$.
7: Store non-zero value $g_q[n_l] = h[n_l]$.
8: Remove non-zero location from indices $\Omega \leftarrow \Omega \setminus \{n_l\}$.
9: Remove non-zero value from the observation $\tilde{f}' = \tilde{f} - g_q$.
10: Denoise $\tilde{f}' \leftarrow \text{Cadzow}(\tilde{f}', L_p)$.
11: end for
12: Estimate vector g_p = Finite dimensional FRI $(\tilde{f}', L_p)$.

III. SIMULATION RESULTS

We simulated two experiments and presented results to evaluate the performance of the root-free Prony's method in the presence of noise. Here the mean squared error (MSE) is used to evaluate the reconstruction performance.

EXPERIMENT-1

First experiment based on varying number of observations. For this we assume the signal with 5 non-zero coefficients having length 256 which is shown in fig.2. The signal is contaminated by Gaussian noise with signal to noise ratio (SNR) ranging from 5 dB to 50 dB and then measured 130 and 150 observations.

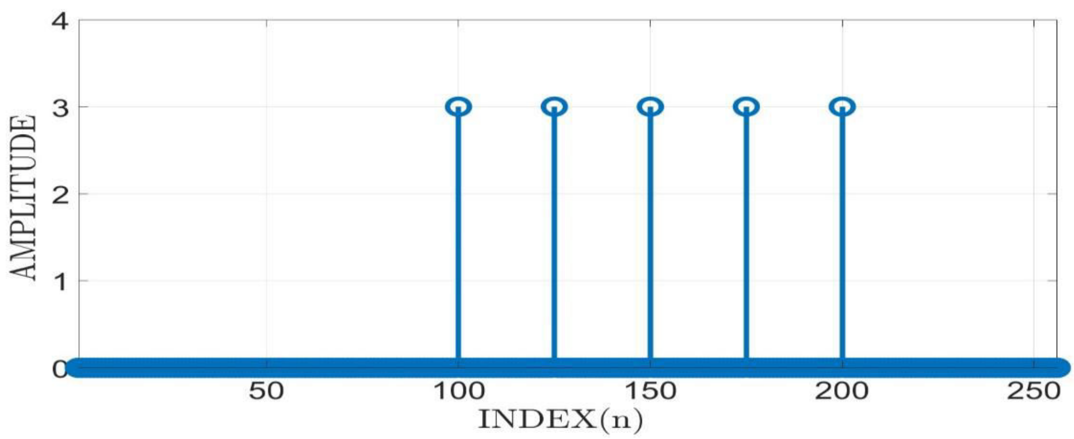

Fig .2: Original sparse signal

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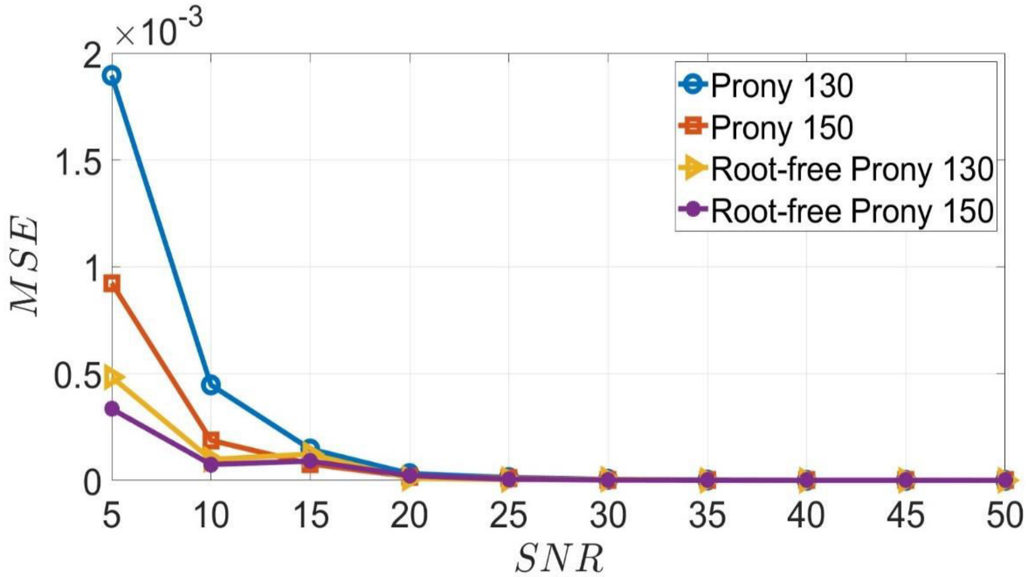

Fig. 3: Average MSE of root-free Prony's method and Prony's method relative to the number of observations at various SNR's

To reconstruct input sparse signal, we employed Prony's method and root-free Prony's method are these observations. Fig.3 shows average MSE w.r.t the number of observations at various SNRs. In both methods reconstruction performance increases when SNR increases. The plot also shown that when number of observations increases, reconstruction performance increases. Moreover, from fig 3, it is cleared that the proposed root-free Prony's method gives better reconstruction performance than Prony's method.

EXPERIMENT-2

Second experiment is based on varying signal sparsity. For this we assume two signals with 4 and 6 non-zero coefficients having length 256, which are shown in fig.4 and fig.5. The signals are contaminated by Gaussian noise with SNR ranging from 5 dB to 50 dB and then measured 130 observations.

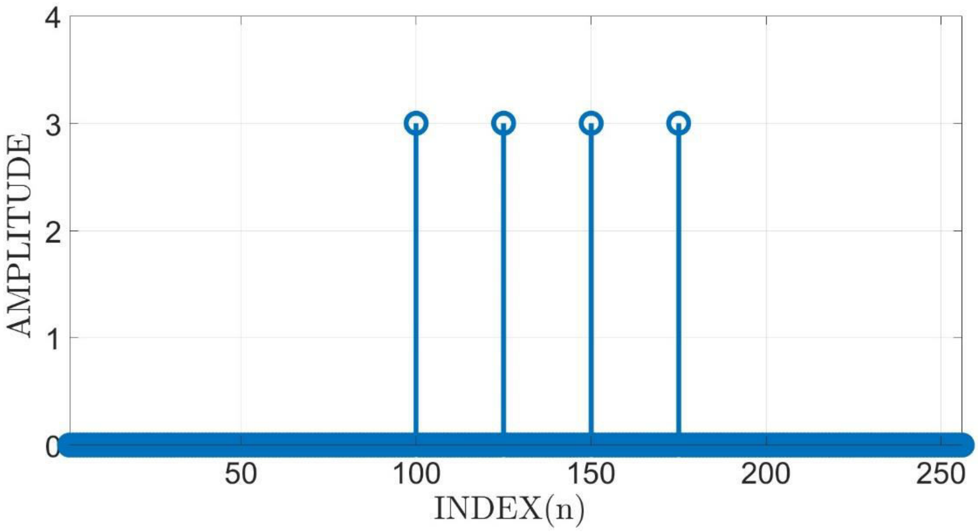

Fig. 4: Sparse signal with 4 non-zero coefficients

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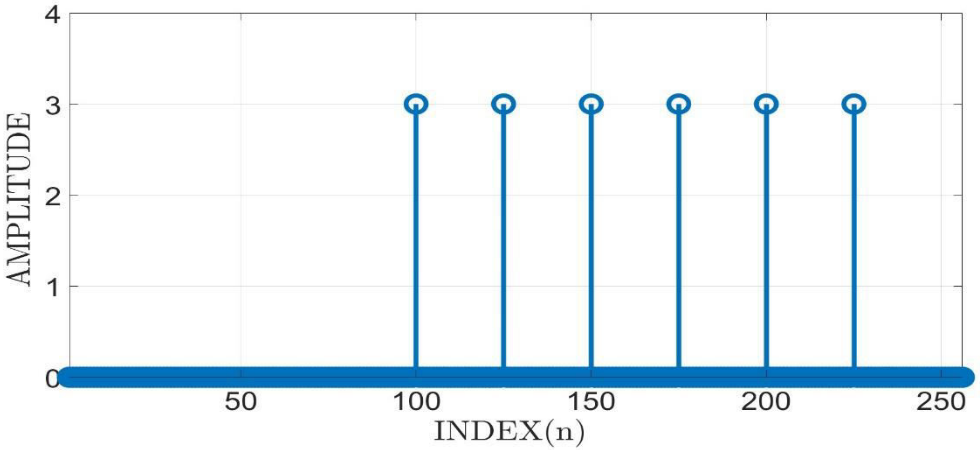

Fig. 5: Sparse signal with 6 non-zero coefficients

To reconstruct input signals, we employed Prony’s and root-free Prony’s methods to these observations.

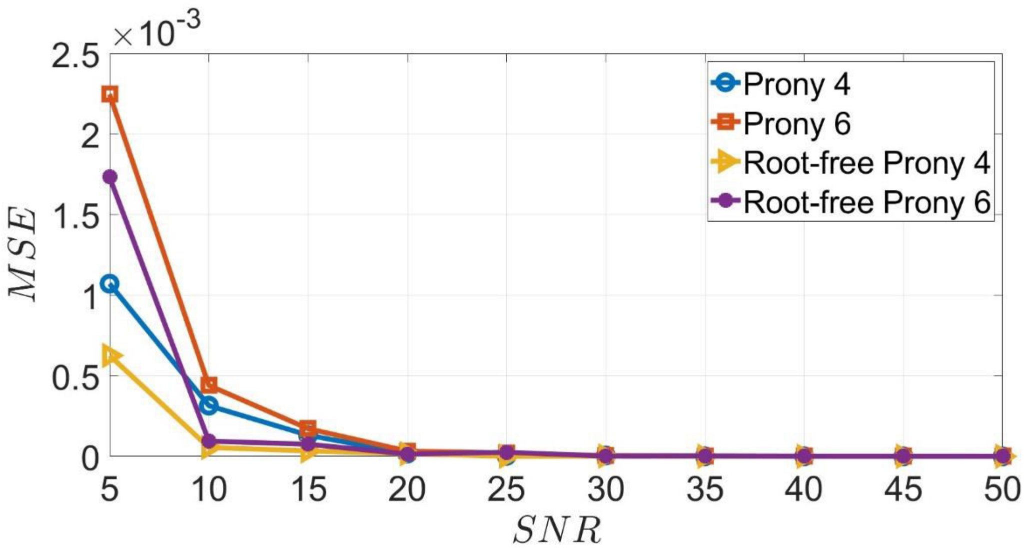

Fig. 6: Average MSE of root-free Prony’s method and Prony’s method relative to the signal sparsity at various SNR’s

Fig.6 shows average MSE w.r.t the signal sparsity at various SNRs. In both methods, reconstruction performance increases when SNR increases. The plot also shows that when signal sparsity increases, reconstruction performance decreases. Moreover from fig.6 it is cleared that the proposed gives better reconstruction performance than Prony’s method.

IV. CONCLUSION AND FUTURE WORK

In this work, using the complete solution of an underdetermined linear system, a novel root-free Prony's method is proposed for sparse signal reconstruction. In the noisy case, simulation results show that the proposed root-free Prony's method performs better than the traditional Prony's method especially in low signal-to-noise ratios.

Natural images are sparsely represented in the wavelet transform domain, therefore compressive imaging can be performed with fewer observations by utilizing root-free Prony's method. In sonar imaging, image transmission over underwater acoustic channel challenging because of channel characteristics, such as limited bandwidth. Since sonar images sparse in bandelet or wavelet transform domain, root-free Prony's method fair successfully compressing and transmitting images. Moreover, in radar imaging, the goal is to identify speed, altitude, and direction of steady and moving targets. By solving a reconstruction problem using the root-free Prony's method, received radar signal can be

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reconstructed from lesser observations. As a result, the price and complexity of receiver's hardware are drastically decreased.

REFERENCES

1. D. L. Donoho. "Compressed Sensing". IEEE Transactions on Information Theory 52.4 (Apr. 2006), pp. 1289–1306
2. R. Tibshirani. "Regression Shrinkage and Selection via the Lasso". Journal of the Royal Statistical Society. Series B (Methodological) 58.1 (1996), pp. 267– 288. URL: http //www.jstor.org/stable/2346178
3. D. Needell and J. A. Tropp. "CoSaMP: Iterative signal recovery from incomplete and inaccurate samples". Applied and Computational Harmonic Analysis 26.3 (May 2009), pp. 301–321.
4. W. Dai and O. Milenkovic. "Subspace Pursuit for Compressive Sensing Signal Recontruction". IEEE Transactions on Information Theory 55.5 (May 2009), pp. 2230–2249.
5. Jon Onativia, Yue M. Lu and Pier Luigi Dragotti ." Prosparse denoise: Prony's based sparse pattern recovery in the presence of noise" ,ICASSP 2016,pp.4044-4088
6. M. Vetterli, P. Marziliano, T. Blu, Sampling signals with finite rate of innovation. IEEE Trans. Signal Process. 50(6), 1417–1428 (2002)
7. P.L. Dragotti, M. Vetterli, T. Blu, Sampling moments and reconstructing signals of finite rate of innovation:Shannon meets Strang–Fix. IEEE Trans. Signal Process. 55(5), 1741–1757 (2007)
8. J. Oñativia, Y.M. Lu, P.L. Dragoni, Finite dimensional FRI, in 2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP) (IEEE, 2014), pp. 1808–1812

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