



# **The Performance of LDPC Convolutional Codes – A Quantitative Analysis**

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**ABSTRACT:** Low Density Parity Check Convolutional codes (LDPC-CC) is a class of forward error correction codes which combines the strengths of LDPC codes and convolutional codes. When compared with other codes used in forward error correction, Quasi Cyclic LDPC-CCs (QC LDPC – CC) have significant performance improvement due to the effective iterative sliding window decoding and sum product decoding which can be turned in to the reduction of transmission power, resulting in a convenient and practical receiver. The analysis done in this paper is based on the Bit Error Rate of the decoded output for multiple code rate, different number of iterations and various decoding schemes. For lower code rate, soft decision decoding and increased number of iterations quasi cyclic LDPC-CCs yields a better performance. A new generation method for LDPC-CC which gives similar performance to the conventional method at a reduced implementation complexity is also introduced in this paper.

**KEYWORDS:** Low density parity check codes, Quasi cyclic low density parity check convolutional codes, Iterative sliding window decoding, Sum product decoding

## **I. INTRODUCTION**

In the process of communication, data transmitted through the channel is vulnerable to modifications due to channel characteristics and external noises that could distort the signal representing the data. This results in reduced reliability of the received data. Channel coding techniques are used to overcome these difficulties. Channel coding involves data transformations that are used for improving a system's error performance by enabling a transmitted message to better withstand the effects of channel impairments such as noise, interference, and fading [1]. The two channel coding policies are Forward Error Correction (FEC) and Automatic Repeat Request (ARQ). The process of adding redundant bits for error detection and correction is termed as Forward Error Correction (FEC). Several error correction codes have been developed to encode, decode and correct the received data. Error Correction Codes includes convolutional codes, BCH codes, RS codes, turbo codes, Low Density Parity Check Codes (LDPC) and LDPC Convolutional Codes (LDPC-CC). Low density parity check (LDPC) codes are forward error-correction codes, which was first proposed in the 1962 PhD thesis of Gallager at MIT which was virtually ignored by the coding community until mid 1990s. But today LDPC codes are replacing the currently existing convolutional codes. But the combination of these two codes gives even more powerful LDPC-CCs. The organization of the paper is as follows. After the introduction given in Chapter 1, Chapter II gives an overview of LDPC- Convolutional Codes. The procedural concepts are explained briefly in Chapter III. The following Chapters IV and Chapter V present the experimental results and conclusion respectively.

## **II. RELATED WORK**

LDPC codes are block codes with parity check matrices that contain only a very small number of non-zero entries. The sparseness of H (Parity check) matrix guarantees a limited decoding complexity which linearly increases with the code length. It ensures a minimum distance which also increases linearly with the code length [2].

In telecommunication, a convolutional code is a type of error correcting code in which each m-bit information symbol (each m-bit string) to be encoded is transformed into an n-bit symbol, where m/n is termed as the code rate ( $n \geq m$ ) and the transformation is a function of the last k information symbols, in which 'k' is the constraint length of the code.

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Even though convolutional encoding using shift register is a simple procedure, decoding of a convolutional code which uses a Viterbi decoder is much more complex.

## A. LDPC Convolutional Codes

LDPC convolutional codes can be considered as the convolutional counterparts of LDPC block codes. Analogous to LDPC block codes, LDPC convolutional codes are defined by sparse parity a check matrix, which gives them iterative message passing decoding capability. LDPC convolutional codes are suitable for practical implementation in a number of different communication scenarios, including continuous transmission and block transmission. A rate  $R = b/c$  LDPC-CC can be defined as the set of sequences [3][4]

$$V = (v_0, v_1 \dots v_t \dots) \in F \quad (1)$$

satisfying the equality  $vH^T = 0$ , where

$$H^T = \begin{bmatrix} H_0^T(0) & \dots & H_{Ss}^T(Ss) & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ & \dots & H_0^T(t) & \dots & H_{Ss}^T(t + Ss) \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

The semi-infinite diagonal type transposed parity check matrix  $H^T$  which contains mostly zeros and relatively few non-zero elements, is known as syndrome former [3][5]. The value  $Ss$  is the syndrome former memory. The quantity  $Ls=(Ss+1)c$  is called the constraint length of the code and is equal to total number of symbols of the code sequence involved in the parity check constraint at any time instant  $t$ . Similar to a block code, for a generator matrix  $G$  of the convolutional code we have  $GH^T = 0$ .

The two major construction methods for LDPC-CC were proposed by Tanner and Jimenez-Feltstr'om and Zigangirov (JFZ). Tanner exploits similarities between quasi cyclic block codes and time invariant convolutional codes whereas JFZ method uses a matrix based unwrapping procedure to obtain the  $H$  matrix of a periodically time varying convolutional code from the parity check matrix of a block code. As in the case of a LDPC code LDPC-CCs can also be represented using Tanner graph. The check nodes are connected to the bit nodes if the corresponding entry in the  $H$  matrix is 1. There is a one to one correspondence between a syndrome former and its Tanner graph representation. But the unique features convolutional codes makes the Tanner graph representation of LDPC-CC different from that of LDPC codes in the following way, 1) The convolutional code is infinite so that the corresponding Tanner graph is also infinite 2) The memory of a convolutional code restricts the local structure in the Tanner graph i.e.) the memory determines the maximum separation between two symbol nodes connected by a single constraint node [3].

## B. Decoding Process

The large values of  $Ss$  make the trellis representation of LDPC-CC impossible. But the sparse syndrome former of the code enable message passing iterative decoding. The special features of the Tanner graph structure enable a highly parallel continuous sliding window decoder for LDPC convolutional codes as shown in Fig. 1[2]. The decoding of LDPC-CC tries to reconstruct the transmitted codeword 'c', from the possibly corrupted received word, 'y' utilizing the parity-check matrix,  $H$ . The set of parity check constraints which must be satisfied for the received codeword to be correct or the same as the transmitted codeword is defined by the condition that  $cH^T = 0$ . Decoding is performed through iterative processing based on the Tanner graph, to satisfy the parity check conditions. Different message passing algorithms are named based on the type of message passed or for the type of operations performed at the nodes.

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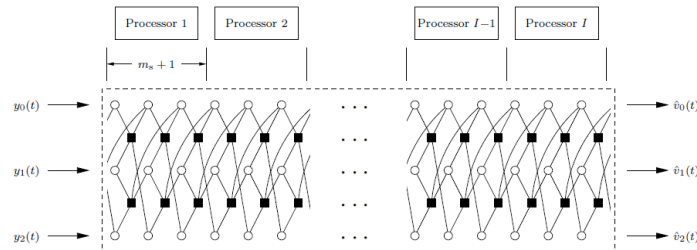


Fig. 1. Decoding of LDPC-CC Code  
Fig. 2.

## 1) Bit Flipping Algorithm

The bit-flipping algorithm is a hard decision message passing algorithm for LDPC-CCs which work as well for LDPC codes too. In this a binary (hard) decision about each received bit is made by the detector. On the contrary sum product algorithm iteratively computes an approximation of the MAP value for each code bit. A bit node sends a message announcing if it is a one or a zero, and each check node sends a message to each of the connected bit node, declaring what value the bit is based on the check node's information. The check node decides that its parity check equation is satisfied if the modulo-2 sum of the incoming bit values is zero. The current value of bit node is flipped when majority of message received are different from its received value. The basic principle of the bit-flipping algorithm is that a codeword bit involved in a large number of incorrect check equations is likely to be incorrect itself. This method has two benefits. Firstly additional iterations are avoided once a solution has been found and secondly the failure to converge to a codeword is always detected.

## 2) Sum-Product Decoding

The sum product algorithm is a soft decision message passing algorithm was the messages representing each decision are probabilities. The input bit probabilities are called the a priori probabilities for the received bits because they were known in advance before running the LDPC-CC decoder and the bit probabilities returned by the decoder are called the a posteriori probabilities. In sum product decoding these probabilities are expressed as log-likelihood ratios (LLR).

$$L(x) = \log \left( \frac{p(x=0)}{p(x=1)} \right) \quad (2)$$

The benefit of the logarithmic representation is that when probabilities need to be multiplied log-likelihood ratios need only be added, which reduces the implementation complexity. For an AWGN channel the priori LLR are given by (2).

$$r_i = 4 y_i \left( \frac{E_s}{N_o} \right) \quad (3)$$

The term  $y_i$  represents the received code word and for the simulation purpose assume that the quantity. The codeword is found by performing the extrinsic LLR and the total LLR calculation. The posteriori probabilities returned are exact MAP probabilities, only if Tanner graph is cycle free.

## III. PROPOSED SYSTEM

The system implemented in this paper is a Quasi Cyclic LDPC-CC (QC LDPC-CC) encoder – decoder scheme. The QC LDPC codes are codes in which rows and columns in the sub matrix have similar and cyclic connections. They can be formed by a concatenation of circularly shifted sub matrices. In the other construction method, the matrix is formed by isolated shifted identity sub matrices. Due to their quasi cyclic nature, QC-LDPC codes can be encoded efficiently with shift registers. The codes have general structure as:

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$$H_{QC\ LDPC} = [A_1, A_2, \dots, A_k] \quad (4)$$

Where  $A_i$  is a circulant matrix.

The QC LDPC-CC is generated by the matrix unwrapping of a QC LDPC block code. A system which is used for the performance analysis of QC LDPC-CC coding technique is represented in Fig 2.

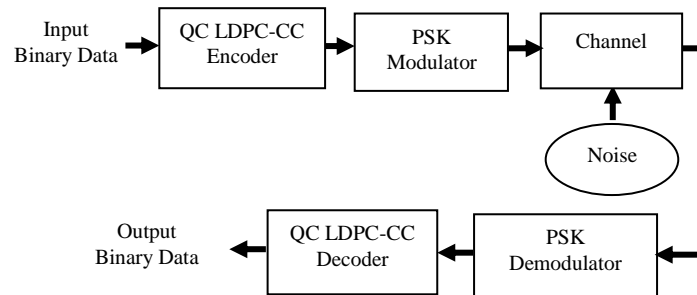


Fig. 3. Simplified block diagram for a system using QC LDPC –CC coding

Here input is a binary sequence, which is taken as message. It is given to the QC LDPC-CC encoder where it is converted to quasi cyclic LDPC-CCs. The coded data is then modulated using Phase Shift Keying Modulator (PSK Modulator) prior to transmission. Since the channel is not an ideal one, noise is added to the message when it passes through the channel. In the analysis of QC LDPC-CCs parameters such as code rate, number of iterations and decoding schemes are considered. The simulation is carried out over 50000 bits in each case. The code rate mainly defines the ratio of message bits to the total number of bits in a code word. The choice of appropriate code rate is usually a trade off between the desired error performance and required transmission power. The code rates used for the analysis in this paper are 1/2, 2/5 and 1/4. The effect of number of iterations is shown using 5, 15 and 30 iterations. The higher the number of iterations the better the chances for decoded data to be close to the original transmitted message. But the choice of number of iterations is often limited by the computational capacity of the system. Both hard decision decoding and soft decision are compared in this paper.

## IV. EXPERIMENTAL RESULTS

### A. Performance of uncoded and QC LDPC-CC coded data:

The performance variations of data without any coding and with QC LDPC-CC coding for 50000 bits is analysed in Fig 3. At first message is transmitted as such and BER of decoded data is analysed. Then QC LDPC-CC coded message is transmitted and the BER of decoded data is evaluated.

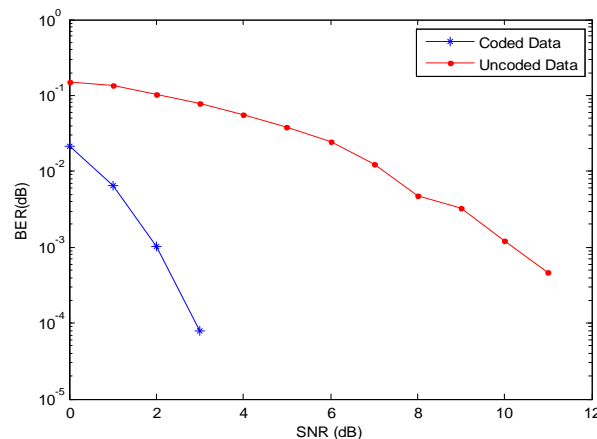


Fig. 4. Performance comparison of QC LDPC-CCs for coded and uncoded data

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It is evident that there is considerable variation in the performance of coded and uncoded data. For the QC LDPC–CC best BER at given conditions can be seen as  $10^{-4.1}$  dB at a SNR of 3 dB itself. A BER of  $10^{-6}$  dB can be obtained by increasing the number of bits.

## B. Performance of QC LDPC-CCs in comparison with QC-LDPC codes & Convolutional codes

When compared with the QC LDPC codes and convolutional codes, QC LDPC-CCs gives a superior performance which is clearly shown in Fig. 4.

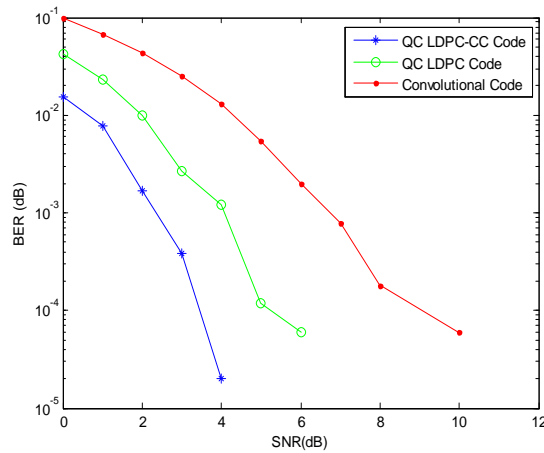


Fig. 5. Performance comparison of QC LDPC-CC, QC LDPC and Convolutional codes

QC LDPC-CCs provide a BER of  $10^{-4.8}$  dB at 4 dB SNR whereas a performance of  $\sim 10^{-4.2}$  dB is obtained for QC LDPC and convolutional codes at 6 dB and 10 dB SNR respectively. QC LDPC-CCs can deliver still higher performances with more number of bits and increased number of iterations.

## C. Performance of QC LDPC-CCs with different decoding method:

Performance of the two decoding algorithms i.e.) soft decision decoding algorithm (using sum product algorithm) and hard decision decoding algorithm (using bit flipping algorithm) are analysed and results are shown in Fig. 5.

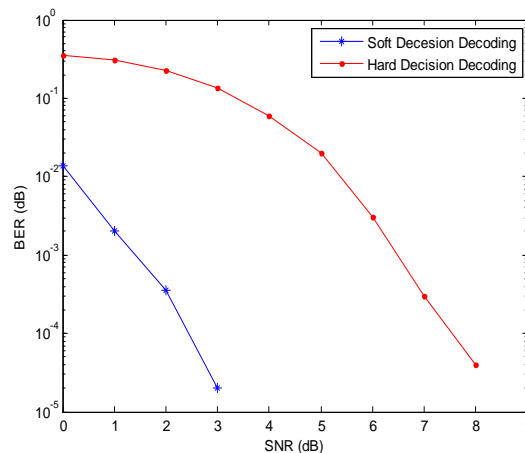


Fig. 6. Performance comparison of QC LDPC –CC code for soft decision and hard decision decoding

When the hard decision decoding and soft decision decoding methods are analysed it is evident that the performances of soft decision decoding methods are better. For soft decision decoding a BER of  $10^{-4.9}$  dB is obtained for a SNR of 3 dB whereas the same BER for hard decision decoding is obtained at almost 8 dB. Higher BER can be obtained at higher number of bits.

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## D. Performance of QC LDPC-CC with different code rate:

Next the analysis for different code rates for 50000 bits is given in Fig 6. Here the soft decision sum product algorithm is used. And it is seen that low code rate gives good BER.

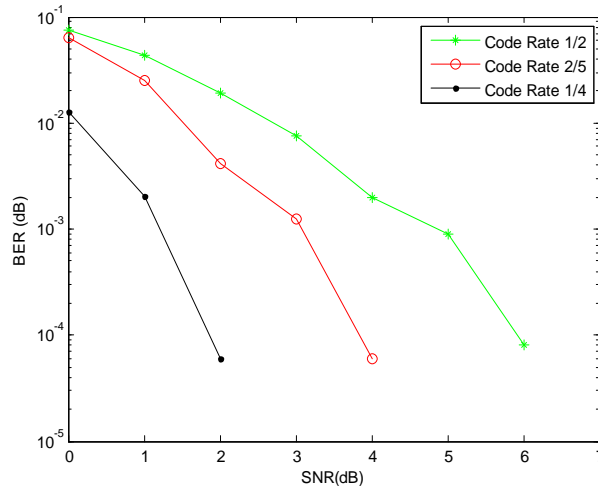


Fig. 7. Performance comparison of QC LDPC-CCs for different code rates

The performance is analysed using the code rates of 1/2, 2/5 and 1/4. Fig. 6 clearly shows that a BER of  $10^{-4.5}$  dB is obtained for code rate 1/4 whereas the same results for code rate 2/5 and 1/2 are reached at 4 dB and 6 dB respectively.

## E. Performance of QC LDPC-CC with different number of iterations:

The performance of QC LDPC-CCs also depends up on the number of iterations used in the message passing algorithm. The performance analysis based on the number of iterations, is given in Fig 7.

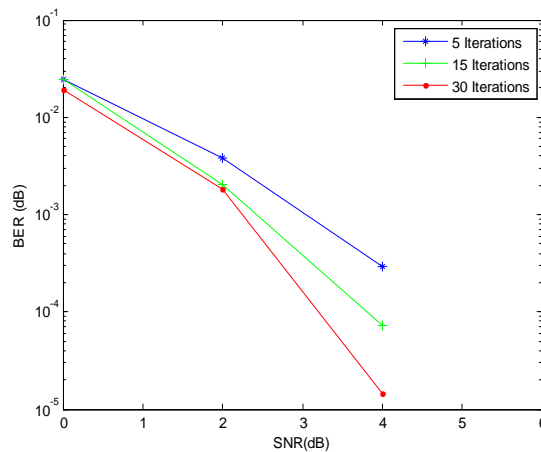


Fig. 8. Performance comparison of QC LDPC-CCs for different number of iterations

We can choose a specific number of iterations based on the hardware, power and computational resources available. The analysis shows that as the number of iterations is increased the BER performance also increases. For a SNR of 4 dB the BER for 30 iterations is  $10^{-5}$  dB, whereas for 15 iterations and 5 iterations it is  $10^{-4.1}$  dB and  $10^{-3.5}$  dB respectively. Better performance based on the BER can be obtained for higher number of bits.

## F. Performance of QC LDPC-CC in Rayleigh and Rician channel:

Rayleigh and Rician channels are considered to be fading channels were in addition to the noise introduced in the channel the signal is the faded version of transmitted signal. The presence of a Line of Sight (LOS) component or direct

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path differentiate Rician channel from Rayleigh channel. Fig. 8 shows the performance of QC LDPC-CC when compared to its QC LDPC counterpart under Rayleigh channel.

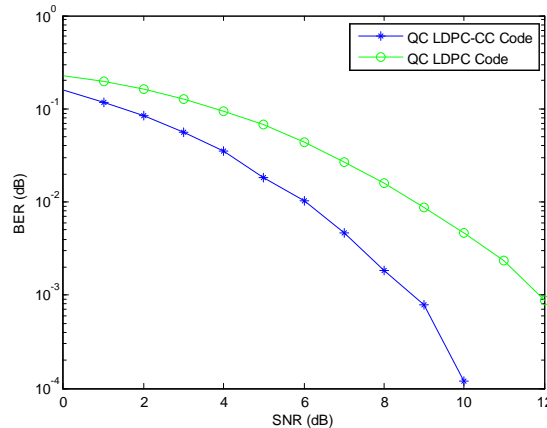


Fig. 9. Performance comparison of QC LDPC-CC and QC LDPC in Rayleigh channel

From Fig 8 it can be seen that quasi cyclic LDPC-CC achieves a bit error rate of  $10^{-4}$  dB at a SNR of 10 dB. But for a corresponding QC LDPC code BER is only  $10^{-3}$  dB even at a SNR of 12 dB. The performance of QC LDPC-CCs in Rician channel is analyzed in Fig 9.

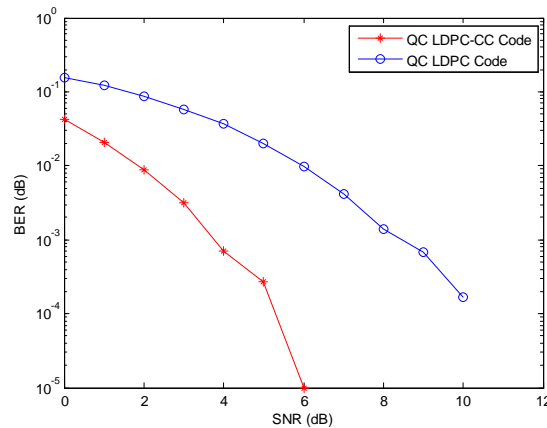


Fig. 10. Performance comparison of QC LDPC-CC and QC LDPC in Rician channel

As the result from Fig 9 indicates for the given conditions QC LDPC-CC works better than its QC LDPC code counterpart. QC LDPC code gives a BER of  $10^{-4}$  dB at a SNR of 10 dB whereas for quasi cyclic LDPC-CC  $10^{-5}$  dB BER is achieved at a SNR of 6 dB itself. The results in Fig 8 and Fig 9 confirms the fact that even when the channel is corrupted by fading and multipath propagation QC LDPC-CC gives a superior performance than its corresponding block code counterpart.

### G. Performance of LDPC-CC generated using new method in comparison with the conventional method:

Performance of a new type of LDPC-CC generated using the mirror image of the identity matrix in contrast to the concatenation of the shifted versions is introduced in Fig. 10.

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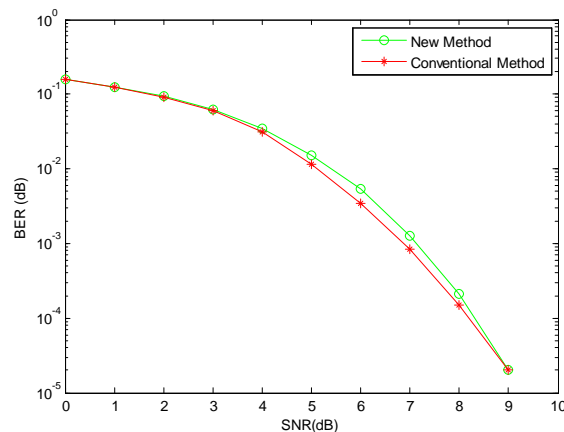


Fig. 11. Performance comparison of LDPC-CC using new method and conventional method

As evident from the plot, the new method gives approximately equal performance as that of the conventional method. The advantage of this method is that the comparable performance is obtained at a much lower generation complexity. The new method uses identity matrix and its mirrored version for the generation of base LDPC block code. But the conventional method requires identity matrix and multiple shifted versions to form the base matrix. From the result shown in Fig. 10 it can be concluded that generally the LDPC-CC using the new methods gives an equivalent if not better performance with lesser complexity at the transmission side. The performance can be improved by employing more suitable arrangements and more number of bits.

## V. CONCLUSION

In this paper, QC LDPC-CCs are analyzed on the basis of different parameters. The sparse syndrome former matrix and the iterative sliding window decoding allow the hardware implementation of the decoder to be less complex and enable parallel processing. Based on the above results considered under the conditions of different decoding algorithm, code rates, and different number of iterations we can clearly prove that, lower code rate, sum product algorithm (soft decision) and higher number of iterations yields best result. Also a new type of LDPC-CC which gives a comparable performance to the conventional method with reduced complexity is also introduced. The concept of QC LDPC-CCs can be effectively in the fields such as deep space communication, satellite and wireless communication.

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