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Numerical Solutions of Burger's Equation Arising in Longitudinal Dispersion Phenomenon in Fluid Flow Through Porous Media by Reduced Differential Transform Method

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ABSTRACT: The longitudinal dispersion phenomenon of miscible phenomenon of miscible fluids in porous media is discussed in which the flow is unsteady. After the mathematical formulation of the phenomenon get a non-linear partial differential equation explicitly Burger's equation. We get the series solution of the problem by using Reduced differential transform method (RDTM) & also derived numerical solution by MATLAB. This type of phenomenon has been of great importance to hydrologist & in oil industries.

KEYWORDS: Miscible fluids, longitudinal dispersion phenomenon, Burger's equation, RDTM.

I. INTRODUCTION

The present paper discuses the solution of longitudinal dispersion phenomenon, which arising in the miscible fluid flow through homogeneous porous media. In a miscible displacement process a fluid is displaced in a porous medium by another fluid that is miscible with the first fluid. Miscible displacement in porous media plays an important role in many engineering and science fields. Among many flow problems in porous media, one involves fluid mixtures called miscible fluids. A miscible fluid is a single phase fluid consisting of several completely dissolved homogeneous fluid species, a distinct fluid-fluid interface doesn't exist in a miscible fluid. So that the interfacial tension between them is zero. The problems of dispersion have been receiving considerable attention from chemical, environmental and petroleum engineers, hydrologist, mathematicians and soil scientists. Over the past ten decades longitudinal dispersion in porous media has been studied and correlated extensively for gaseous and aqueous systems.

II. RELATED WORK

Several researchers have discussed this problem with common assumption of homogeneous porous media with constant porosity, steady seepage flow velocity and constant dispersion coefficient but their techniques are different such as P.H.Bhathawala [4] derived analytical solution of miscible fluid flow through homogeneous porous media by applying a two parameter singular perturbation method. K.J.Chauhan, Falguni Dabhade & D.M. Patel [5] discussed analytical solution of the longitudinal dispersion problem by using infinitesimal transformations group technique of similarity analysis. R.Meher, S.K.Meher & M.N.Mehta [6, 7] studied numerical & graphical solution of dispersion phenomenon by Adomain decomposition method and they also employed a New approach to Backlund transformation for longitudinal dispersion of miscible fluid flow through porous media in oil reservoir during secondary recovery process. Ravi N. Borana, V.H.Pradhan & M.N.Mehta [8, 9] obtained Numerical solution of Burger's equation by using finite difference method and they also discussed numerical solution of Burger's equation in longitudinal dispersion phenomenon in fluid flow through porous media by Cranck-nicolson scheme. Kunjan shah & Twinkle Singh [10, 11]

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solved Burger's equation in one dimensional ground water recharge by spreading numerically by q-Homotopy Analysis method & they also derived Mixture of New Integral transform & Homotopy perturbation method to solve Burger's equation arising in the longitudinal dispersion phenomenon. R. Meher & M.N.Mehta [12] discussed a new approach to backlund transformations to solve Burger's equation. Kajal patel, M.N.Mehta & Twinkle R. singh [13] studied a solution of one-dimensional dispersion phenomenon by Homotopy Analysis Method. Rudraiah et al. [14] have provided analytical study of the dispersion in saturated deformable or non-deformable porous media with or without chemical reaction, considering a series of particular cases selected through different practical problems using different dispersion models. They obtained basic equation using mixture and homogenation. Different analytical and numerical models are valid for long time (asymptotic) and for all time (transient) are explained.

III. STATEMENT OF THE PROBLEM

Miscible displacement in porous media is a type of double-phase flow in which the two phases are completely soluble in each other. Therefore capillary forces between the fluids do not come into effect. The longitudinal dispersion of the impure or saline water with the concentration c(x, t) flowing in the x-direction has been considered, that the ground water recharge takes place over a large basin contain homogeneous porous medium is saturated with fresh water. The miscible flow under conditions of complete miscibility could be thought to behave locally at least, as a single-phase fluid, which would obey Darcy's law. The change of concentration, in turn, would be caused by the bulk coefficients of diffusion of the one fluid in the other. There is no mass transfer between solid and liquid phases, is assumed [2]. The miscible flow takes place both longitudinally and transversely, but the spreading caused by dispersion is greater in the direction of flow than the transverse direction. One dimensional treatment of these systems avoids treatment of a radial or transverse component of dispersion. We only consider the dispersion phenomenon in the direction of flow i.e. longitudinal dispersion, which takes places when miscible fluids flow in homogeneous porous media. The problem is to describe the growth of mixed region. i.e. to find concentration c(x, t) of the impure water as a function of time t and position x, as two miscible fluids flow through homogeneous porous media. Outside of the mixed zone (on either side) the single fluid equation describes the motion of fluid.

IV. FORMULATION OF THE PROBLEM

According to Darcy's law, the equation of continuity for the mixture, in the case of incompressible fluids is given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\bar{v}) = 0 \qquad (1)$$

Where $'\rho'$ is the density for mixture and $'\bar{v}'$ is the pore seepage velocity vector.
The equation of diffusion for a fluid flow through a homogeneous porous medium, without increasing or decreasing the dispersing material is given by

$$\frac{\partial C}{\partial t} + \nabla.(C\bar{v}) = \nabla.\left[\rho\bar{D}\nabla\left(\frac{C}{\rho}\right)\right] \qquad (2)$$

Where 'C' is the concentration of a fluid in a porous media. D is the coefficient of dispersion with nine components D_{ij}. In a laminar flow for an Incompressible fluid through homogeneous porous medium, density $'\rho'$ is constant. Then equation (2) becomes,

$$\frac{\partial C}{\partial t} + \bar{v}.(\nabla C) = \nabla.[\bar{D}\nabla C] \qquad (3)$$

Let us assume that the seepage velocity $\bar{v}$ is along the x-axis, then $\bar{v} = u(x,t)$ and the non zero components will be $D_{11} \approx D_L = \gamma$ (coefficient of longitudinal dispersion) and other Components will be zero [6].
Equation (3) becomes,

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = \gamma\frac{\partial^2 C}{\partial x^2} \qquad (4)$$

Where u is the component velocity along x-axis which is time dependent as well as concentration along x-axis in $x \geq 0$ direction and $D_L > 0$ and it is the cross sectional flow velocity in porous media.
$u = \frac{C(x,t)}{C_0}$, Where $x > 0$ *and for* $C_0 \cong 1$ by [6]. Equation (4) becomes

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$$\frac{\partial C}{\partial t} + C\frac{\partial C}{\partial x} - \gamma\frac{\partial^2 C}{\partial x^2} = 0 \qquad (5)$$

Where γ is the coefficient of the longitudinal dispersion.

This is the non linear Burger's equation for longitudinal dispersion of miscible fluid flow through porous media. The theory that follows is confined to dispersion in unidirectional seepage flow through semi finite homogeneous porous media. The seepage flow velocity of impure water is assumed unsteady. Here the initial concentration of dispersion is considered as an input highest constant concentration of impurity at $x = 0$ is C_0. The porous medium is considered as nonadsorbing.

The governing partial differential equation (5) for longitudinal hydrodynamic dispersion with in a semi finite nonadsorbing porous medium in a unidirectional flow field in which γ is the longitudinal dispersion coefficient, C is the average cross-sectional concentration, u is the unsteady seepage velocity, x is a coordinate parallel to flow and t is time.

We can write equation (7) as $C(x,t) = \gamma C_{xx} - C\,C_x$

The problem is solved by using Reduced Differential Transform method. The numerical values are shown by table. Curves indicate the moisture content corresponding to various time periods.

V. PROPOSED ALGORITHM

$$\frac{\partial C}{\partial t} + C\frac{\partial C}{\partial x} - \gamma\frac{\partial^2 C}{\partial x^2} = 0 \qquad (5)$$

Taking $\gamma = 1$ & the initial condition $C(x,0) = C_0 = f(x)$

$$f(x) = e^{-x} \qquad (6)$$

The problem is solved by reduced differential transform method because our equation is partial differential equation.

Reduced differential Transform Method

The Basic definition of RDTM is given below

If the function u(x, t) is analytic and differential continuously with respect to time t and space x in the domain of interest then let

$$U_k = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]$$

Where the t-dimensional spectrum function $U_k(x)$ is the transformed function. u(x,t) represent transformed function. The differential inverse transform of $U_k(x)$ is defined as follow

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k$$

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]t^k$$

Apply RDTM on (5)

$$(k+1)C_{(k+1)}(x,t) = ((C_k)_{xx} - \left[\sum_{r=0}^{k}(C_r)\left(C_{(k-r)}\right)_x\right] \qquad (7)$$

Now let k=0 then put initial condition (6) into eq. (7), So we have the values of $C_k(x)$ as following

$$(1)C_1(x,t) = ((C_0)_{xx} - \left[\sum_{r=0}^{0}(C_r)\left(C_{(k-r)}\right)_x\right]$$

$$C_1(x,t) = ((C_0)_{xx} - \left[(C_0)\left(C_{(0-0)}\right)_x\right]$$

$$C_1(x,t) = e^{-x} - [(e^{-x})(-e^{-x})]$$

$$C_1(x,t) = e^{-x} + e^{-2x}$$

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Now let k=1

$$(2)C_2(x,t) = ((C_1)_{xx} - \left[\sum_{r=0}^{1}(C_r)\left(C_{(k-r)}\right)_x\right]$$

$$(2)C_2(x,t) = ((C_1)_{xx} - \left[(C_0)\left(C_{(1-0)}\right)_x + (C_1)\left(C_{(1-1)}\right)_x\right]$$

$$= e^{-x} + 4\,e^{-2x} - [(e^{-x})(-e^{-x} - 2\,e^{-2x}) + (\,e^{-x} + \,e^{-2x})(-\,e^{-x})]$$

$$= e^{-x} + 4\,e^{-2x} - [-\,e^{-2x} - 2\,e^{-3x} - \,e^{-2x} - \,e^{-3x}]$$

$$C_2(x,t) = \frac{e^{-x} + 6\,e^{-2x} + 3\,e^{-3x}}{2}$$

Now let k=2,

$$(3)C_3(x,t) = ((C_2)_{xx} - \left[\sum_{r=0}^{2}(C_r)\left(C_{(k-r)}\right)_x\right]$$

$$(3)C_3(x,t) = ((C_2)_{xx} - \left[(C_0)\left(C_{(2-0)}\right)_x + (C_1)\left(C_{(2-1)}\right)_x + (C_2)\left(C_{(2-2)}\right)_x\right]$$

$$= e^{-x} + 24\,e^{-2x} + 27\,e^{-3x} - [(e^{-x})(-e^{-x} - 12\,e^{-2x} - 9\,e^{-3x}) + 2.(\,e^{-x} + \,e^{-2x})(-\,e^{-x} - 2\,e^{-2x}) + (\,e^{-x} + 6\,e^{-2x} + 3\,e^{-3x})(-\,e^{-x})]$$

$$= \frac{1}{2}[\,e^{-x} + 24\,e^{-2x} + 27\,e^{-3x} - [-e^{-2x} - 12\,e^{-3x} - 9\,e^{-4x} + (-2\,e^{-2x} - 4\,e^{-3x} - 2\,e^{-3x} - \,4e^{-4x}) + (-e^{-2x} - 6\,e^{-3x} - 3\,e^{-4x})]]$$

$$C_3(x,t) = \frac{1}{6}[\,e^{-x} + 28\,e^{-2x} + 51\,e^{-3x} + 16\,e^{-4x}]$$

In this way we can generated other polynomials by putting different values in equation (7)
Now by inverse Transform

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k$$

$$C(x,t) = C_0(x,t)t^0 + C_1(x,t)t + C_2(x,t)t^2 + C_3(x,t)t^3 + \cdots$$

$$C(x,t) = e^{-x}t^0 + (e^{-x} + e^{-2x})t^1 + (\,e^{-x} + 4\,e^{-2x} + 2\,e^{-2x} + 3\,e^{-3x})\frac{t^2}{2!} + (\,e^{-x} + 28\,e^{-2x} + 51\,e^{-3x} + 16\,e^{-4x})\frac{t^3}{3!}\ldots$$

VI. RESULTS

The following table shows the approximate values of concentration of impure water at different distance x and time t.

x	t=0	t=0.1	t=0.2	t=0.3	t=0.4	t=0.5	t=0.6	t=0.7	t=0.8	t=0.9	t=1
0	1	1	1	1	1	1	1	1	1	1	1
0.1	0.8494	0.8393	0.8284	0.819	0.8116	0.8059	0.8014	0.7979	0.7951	0.7928	0.7909
0.2	0.7132	0.6971	0.6799	0.6652	0.6536	0.6446	0.6376	0.6321	0.6277	0.6242	0.6212
0.3	0.59	0.571	0.5509	0.5338	0.5231	0.5099	0.5018	0.4955	0.4904	0.4863	0.4828

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0.4	0.4784	0.459	0.4385	0.4212	0.4075	0.397	0.3888	0.3823	0.3772	0.373	0.3696
0.5	0.3775	0.3293	0.3403	0.3241	0.3115	0.3017	0.2941	0.2882	0.2834	0.2796	0.2764
0.6	0.2862	0.2705	0.2541	0.2402	0.2294	0.221	0.2145	0.2094	0.2054	0.2021	0.1994
0.7	0.2036	0.1912	0.1783	0.1674	0.1589	0.1524	0.1473	0.1433	0.1402	0.1376	0.1355
0.8	0.1288	0.1203	0.1114	0.104	0.0982	0.0937	0.0902	0.0875	0.0854	0.0836	0.0822
0.9	0.0612	0.0568	0.0523	0.0486	0.0456	0.0434	0.0416	0.0402	0.0392	0.0383	0.0375
1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

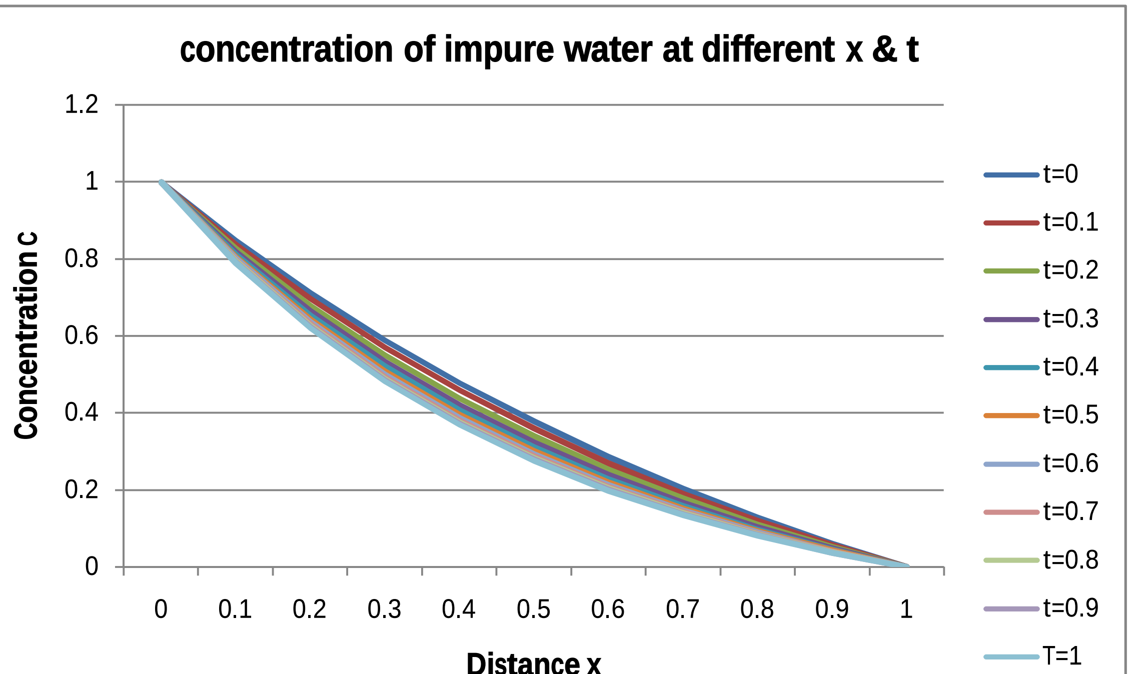

fig-1

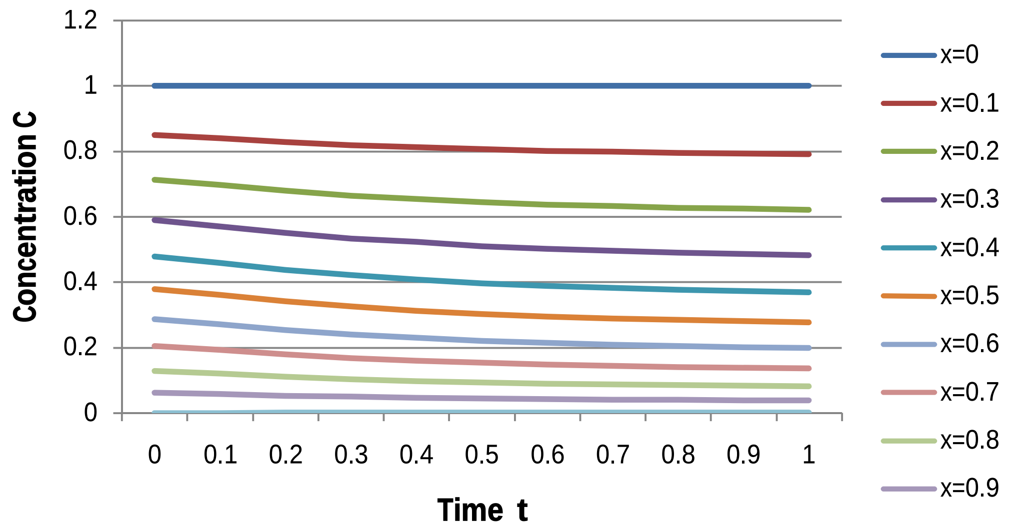

fig -2

VII. CONCLUSION

The graphical and numerical solutions have been obtained to predict the possible concentration of a impure water in unsteady unidirectional seepage flow through semi-finite homogeneous porous media. Focus to the source concentration that vary with the distance x and time $t \geq 0$. From the tabular values and graphs it is conclude that as distance x and time t increases the concentration of the impure water steadily decreases. The concentration C(x, t) of the impure water decreases as the distance x increases for the any fixed time t. Here the initial concentration of impure water at x=0 is highest and it is decreases as distance x increases for fixed time t. It is physically fact that at the source the concentration of impure water is always highest and it is decreasing & dispersing from the source. Here also conclude form the graph (fig-2) of concentration of impure water verses time t for given distance x, the concentration of impure water is decreasing for small time t and then it becomes steady and constant as time t increases for given different distance. Hence, it is clear that at the initial source the dispersion of impure water is not fast, therefore the concentration of impure water is slightly decreasing for small time t, for fixed distance x and then it becomes constant through the time for given distance x.

REFERENCES

[1]. Verma, A. P., : The Laplace transform solution of a one dimensional groundwater recharge by spreading Annali Di Geofision, 22, 1, p.25, 1969.
[2]. Bear, J., : Dynamics of fluids in porous media, American Elsevier publishing company Inc., New York, 1972.
[3]. Scheidegger, A. E. : Physics of flow through porous media, (Revised Ed.). Univ., Toronto Press.,1974.
[4]. Bhathawala, P.H., : Two parameter singular perturbation Method in the fluid flow through porous media, Ph.D Thesis, S.G.University, Surat (India), 1978.
[5]. Chauhan, K.J. et al. : The similarity solution of the longitudinal dispersion phenomenon of miscible fluids in porous media, Int. J. Adv. Appl. Math. and Mech. (IJAAMM), Vol. 1, issue 3, pp.77-82, ISSN: 2347-2529, 2014.
[6]. Meher, Ramakanta. et.al. : Adomain decomposition Method for dispersion phenomenon arising in longitudinal dispersion of miscible fluid flow through porous media, Adv. Theor. Appl. Mech., Vol. 3, Issue 5, pp.211-220,2010.
[7]. Meher, Ramakanta. et.al. : A new approach to Backlund transformations for longitudinal dispersion of miscible fluid flow through porous media in oil reservoir during secondary recovery process ,Theoret. Appl. Mech., Vol. 38, Issue 1, pp.1-16, Belgrade 2011.
[8]. Borana, Ravi.N. et al. : Numerical solution of Burger's equation in a one-dimensional groundwater recharge by spreading using finite difference Method, Int. J. of Advance Research in science and Engineering (IJARSE), ISSN :2319-8354(E), Vol.2, Issue 11, pp. 121-130, Nov- 2013.
[9]. Borana, Ravi N. et al. : Numerical solution of Burger's equation arising in longitudinal dispersion phenomenon in fluid flow through porous media by Crank-Nicolson Scheme , 5th International conference on porous media and their applications in science, engineering industry, Eds, ECI symposium series, Vol. (2014)

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ISSN (Print): 2320-9798

International Journal of Innovative Research in Computer and Communication Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 9, September 2016

[10]. Shah, Kunjan et al. : Solution of Burger's equation in a one-dimensional Groundwater recharge by spreading using q-Homotopy Analysis method, European Journal of pure and applied mathematics (EJPAM), ISSN : 1307-5543, Vol. 9, Issue 1,pp. 114-124, 2016.

[11]. Shah, Kunjan et al. : Solution of Burger's equation arising in the longitudinal dispersion phenomenon in fluid flow through porous media by mixture of new Integral transform method and Homotopy perturbation method, Journal of Geosciences and Environment Protection, Vol. 3, pp. 24-30, 2015.

[12]. Patel, kajal et al. : A solution of one-dimensional dispersion phenomenon by Homotopy Analysis Method, International Journal of modern Engineering Research (IJMER), Vol. 3, Issue. 6, pp. 3626-3631, ISSN: 2249-6645, Nov-Dec 2013.

[13]. Rudraiah, N. & Chiu-on Ng. : Dispersion in porous media with and without Reaction, A Review J. of Porous media, Vol. 3, 2007.

[14]. Abdul Monim Batiha & Belal Batiha: Differential transformation method for a reliable Treatment of the non-linear Biomedical Reaction model; advanced studies in Biology, Vol. 3, No. 8: pp. 355-360, 2011.

[15]. Patil, Narhari & Khambayat, Avinash : Reduced differential transform method for Gas Dynamics equation; Int. J. of Sci., spirituality ,Bussiness and Tech. (IJSSBT), Vol. 2, No. 2: ISSN. 2277-7261, 2014.

[16]. Patel, K.A. Bhathawala, P.H. Patel, Priyanka S. : Numerical Solutions of instabilities in displacement through porous media using differential transform method; Global journal for Research analysis (GJRA),Vol. 4, Issue 7,July-2015, ISSN No. 2277-8160.

[17]. Patel, K.A. Bhathawala, P.H. Patel, Priyanka S. : Numerical Solutions of Imbibition in double phase flow through porous medium with capillary pressure using differential transform method; Int. Journal of Sci. & Innovative Math. Research (IJSIMR), Vol. 3, Issue 8, August-2015,pp. 43-48.

[18]. Patel, Priyanka S. Bhathawala, P.H. Patel, K.A. : Numerical Solutions of Fingero-Imbibition in a Slightly dipping porous media Involving Magnetic Fluid; Int. Organization of Scientific Research Journal of Mathematics (IOSR-JM) e-ISSN: 2278-5728, p-ISSN: 2319-765X, Vol. 12, Issue 3,May-June-2016,pp. 49-53.